Integration once again of Eq. (10), Fig. 7(c), gives the potential distribution $V(x)$ and the built-in potential V_{bi}:

$$V(x) = \mathscr{E}_m\left(x - \frac{x^2}{2W}\right) \tag{13}$$

$$V_{bi} = \tfrac{1}{2}\mathscr{E}_m W \equiv \tfrac{1}{2}\mathscr{E}_m(x_n + x_p) \tag{14}$$

where W is the total depletion width. Elimination of $\mathscr{E}_m$ from Eqs. (12) and (14) yields

$$W = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)V_{bi}} \tag{15}$$

for a two-sided abrupt junction. For a one-sided abrupt junction, Eq. (15) reduces to

$$W = \sqrt{\frac{2\varepsilon_s V_{bi}}{qN_B}} \tag{15a}$$

where $N_B = N_D$ or N_A depending on whether $N_A \gg N_D$ or vice versa. The values of W as a function of the impurity concentration for one-sided abrupt junctions in silicon are shown in Fig. 9 (dashed line for zero bias).

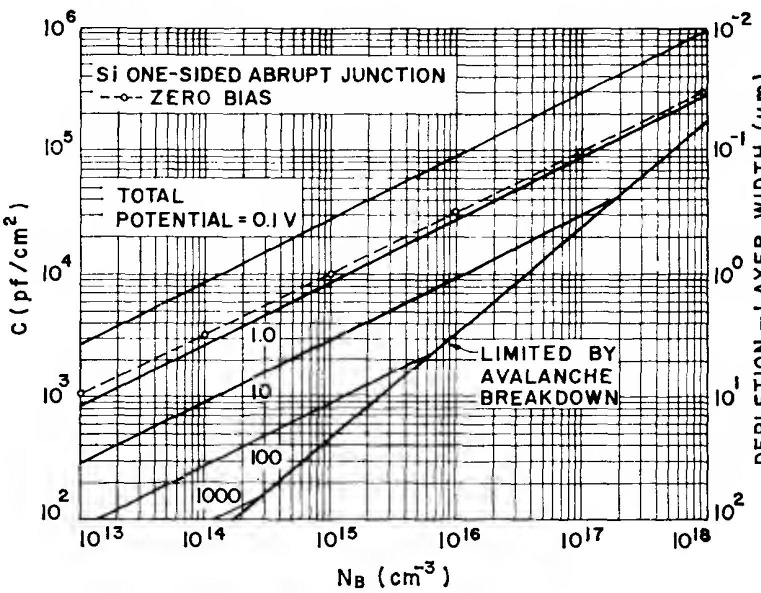

Fig. 9 Depletion-layer capacitance per unit area and depletion-layer width as a function of doping for one-sided abrupt junction in Si. The dashed line is for the case of zero bias voltage.

When a voltage V is applied to the junction, the total electrostatic potential variation across the junction is given by $(V_{bi} + V)$ for reverse bias (positive voltage on n region with respect to the p region) and by $(V_{bi} - V)$ for forward bias. Substitution of these values of voltage in Eqs. (15) or (15a) yields the depletion layer width as a function of the applied voltage. The results for one-sided abrupt junctions in silicon are shown in Fig. 9. The values above the zero-bias line (dashed line) are for the forward-biased condition; and below, for the reverse-biased condition.

These results can also be used for GaAs since both Si and GaAs have approximately the same static dielectric constants. To obtain the depletion-layer width for Ge, one must multiply the results of Si by the factor

$$\sqrt{\varepsilon_s(\mathrm{Ge})/\varepsilon_s(\mathrm{Si})} = 1.17.$$

B. Depletion-Layer Capacitance. The depletion-layer capacitance per unit area is defined as $C \equiv dQ_c/dV$ where dQ_c is the incremental increase in charge per unit area upon an incremental change of the applied voltage dV.

For one-sided abrupt junctions the capacitance per unit area is given by

$$C \equiv \frac{dQ_c}{dV} = \frac{d(qN_B W)}{d\left(\dfrac{qN_B}{2\varepsilon_s} W^2\right)} = \frac{\varepsilon_s}{W} = \sqrt{\frac{q\varepsilon_s N_B}{2(V_{bi} \pm V)}} \quad \mathrm{pf/cm^2}, \tag{16}$$

or

$$\frac{1}{C^2} = \frac{2}{q\varepsilon_s N_B}(V_{bi} \pm V), \tag{16a}$$

$$\frac{d(1/C^2)}{dV} = \frac{2}{q\varepsilon_s N_B} \tag{16b}$$

where the $\pm$ signs are for the reverse- and forward-bias conditions respectively. It is clear from Eq. (16a) that by plotting $1/C^2$ versus V, a straight line should result for a one-sided abrupt junction. The slope gives the impurity concentration of the substrate (N_B), and the intercept (at $1/C^2 = 0$) gives the built-in potential V_{bi} (more accurate consideration gives $V_{bi} - 2kT/q$). The results of the capacitance are also shown in Fig. 9. It should be pointed out that, for the forward bias, there is a diffusion capacitance in addition to the depletion capacitance mentioned above. The diffusion capacitance will be discussed later in Section 4(4).

For the cases of cylindrical p-n junctions, Fig. 6(c), the capacitance per unit length is equivalent to the capacitance of a coaxial transmission line and is given by[19]