Synchronization in Interleaved OFDMA Systems

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Abstract— **Recently, Orthogonal Frequency Division Multiple Access (OFDMA) as a new multiple access technology has received a lot of attention. Multi-carrier communications increase the efficiency and capacity of wireless networks, but are very sensitive to carrier frequency synchronization errors. The estimation of carrier frequency offset in the uplink of OFDMA system is extremely challenging due to the nature of multiple-parameter estimation, especially in an interleaved carrier assignment scheme. In this paper, the base station receiver signal structure are investigated and a structure based estimator is proposed for simultaneously estimating multiple users' carrier frequency offsets. The estimator presented here exploits the inner structure of the OFDMA signal block in the uplink and doesn't need the presence of training symbols or pilot tones.**

I. Introduction

Orthogonal Frequency Division Multiplexing has been proposed in the physical layer design for broadband wireless multiple access system, which is regarded as OFDMA system, such as in IEEE 802.16 [1]. OFDMA inherits from OFDM the weakness of being sensitive to inaccurate frequency references [2]. Frequency synchronization to eliminate carrier frequency offset (CFO) between the transmitter and the receiver is a major task in designing OFDM receiver.

While frequency synchronization in single user OFDM systems has been widely studied [3][4][5], it remains a challenging problem for the synchronization in the OFDMA system [6][7]. In the broadcast link (downlink) of OFDMA systems, the estimation problem is relatively simple in that all users' information are sent by a single transmitter – base station (BS), so each user just needs to estimate its own CFO and recover the orthogonality after adjustment. The really difficulty lies in the OFDMA uplink, where each user occupies a portion of the total carriers with mutually different CFO. Carrier frequency offset estimation in this case is a multiple-parameter estimation problem.

The CFO estimation in the OFDMA uplink are closely related to the carrier assignment scheme in OFDMA systems. There are two major carrier assignment schemes: one is to divide the whole bandwidth into small continuous sub bands and each user is assigned to one or more sub bands; in the other scheme, the carriers belonging to different users are regularly interleaved over the whole bandwidth [8]. Interleaved carrier assignment scheme provides the maximum separation among the carriers assigned to the same user, which maximizes the frequency diversity for each user. On the other hand, interleaved carrier assignment scheme complicates synchronization in the uplink, since it minimizes the separation of the carriers assigned to different users.

CFO estimation in the uplink of an OFDMA system have been discussed in a few recent papers, e.g. [6] and [7]. Sub-band based OFDMA uplink frequency and time synchronization using maximum likelihood is discussed in [6]. In [7], null subcarriers based CFO estimator is proposed for sub-band based uplink and interleaved broadcast link. Both papers assume that the sub-band carrier assignment scheme is used in the OFDMA uplink. The basic idea is to separate different users' data and perform synchronization one by one. To take advantage of frequency diversity of interleaved carrier assignment scheme, it is important to investigate CFO estimation and frequency synchronization issue in the uplink of interleaved OFDMA systems.

This paper develops a structure based algorithm for CFO estimation in the uplink of interleaved OFDMA. In such system, signal from each user has a special periodic structure within an OFDMA block, by which we formulate the received signals into a matrix form and reduce the number of unknown frequencies to the number of users. We then use a high resolution signal processing technique [9] to estimate the CFO's of all involving users deterministically using only one OFDMA block.

II. Signal Structure

Let N denote the FFT size of the OFDMA system in concern. It is the total number of all available subcarriers and virtual subcarriers in the guard band, with sequential index $\{n\}$, where $n \in \{0, 1, \ldots, N-1\}$. The N subcarriers are divided into Q sub-channels and each sub-channel has $P = N/Q$ subcarriers. Let K denote the number of users in one OFDMA block. Assume that the kth user is assigned with sub-channel $\{q\}$. The index of the subcarriers in sub-channel $\{q\}$ is $[q, Q+q, \ldots, (P-1)Q+q]$. $\{X_0^{(k)}, X_1^{(k)}, \ldots, X_{P-1}^{(k)}\}$ are its modulation symbols in one OFDMA block. Let $\mathbf{H}^{(k)} = [H_0^{(k)}, H_1^{(k)}, \ldots, H_{P-1}^{(k)}]$ denote the corresponding channel frequency responses of the kth user on its assigned P subcarriers and assume that the channel keeps unchanged during the period of an OFDMA block. Let $\Delta f^{(k)}$ denote the CFO of the kth user, where $|\Delta f^{(k)}| < 0.5\Delta f$ and Δf is the carrier spacing of the OFDMA system. The time domain samples of the kth user received at BS after the removal of cyclic prefix are

$$r^k(n) = e^{j2\pi n\theta^{(k)}/P} \sum_{p=0}^{P-1} H_p^{(k)} X_p^{(k)} e^{j2\pi pn/P} \tag{1}$$

where $n = 0, 1, \ldots, N-1$, $\xi^{(k)} = \Delta f^{(k)}/\Delta f$ is the normalized CFO, and $\theta^{(k)} = (q + \xi^{(k)})/Q$ is defined as *effective CFO.* It is important to note that

$$r^{(k)}(n + \nu P) = e^{j2\pi\nu\theta^{(k)}} r^{(k)}(n) \tag{2}$$

where ν is an integer. Eq.(2) indicates that the received signal samples from the kth user are periodic with every P samples. The signal samples in one period have a common phase shift compared with their peers in the previous period.

In OFDMA, an OFDMA block received by BS is the superposition of all users' signals. Let $r(n)$, $n = 0, 1, \ldots, N-1$, denote the signal samples of the superposed OFDMA block, we have $r(n) = \sum_{k=1}^{K} r^{(k)}(n)$. $\{r(n)\}_{n=0}^{N-1}$ can be arranged into a $Q \times P$ matrix, $\mathbf{Y}$, and the following relationship holds

$$\mathbf{Y} = \mathbf{V} \cdot \mathbf{R} \tag{3}$$

where $\mathbf{V}$ is a $Q \times K$ Vandermonde matrix and its element $v_{q,k} = e^{j2\pi(q-1)\theta^{(k)}}$ and

$$\mathbf{R} = \begin{bmatrix} r^{(1)}(0) & r^{(1)}(1) & \ldots & r^{(1)}(P-1) \\ r^{(2)}(0) & r^{(2)}(1) & \ldots & r^{(2)}(P-1) \\ \vdots & \vdots & \ddots & \vdots \\ r^{(K)}(0) & r^{(K)}(1) & \ldots & r^{(K)}(P-1) \end{bmatrix}. \tag{4}$$

III. Estimation Algorithm

To estimate the CFO's of all involved users, the first step is to estimate the effective CFO's, $\theta^{(k)}$, $k = 0, 1, \ldots, K-1$. It is noticeable that if sub-channel $\{q\}$ is occupied by the kth user, there will be one and only one effective CFO, $\theta^{(k)}$, to be in the range $(\frac{q-0.5}{Q}, \frac{q+0.5}{Q})$. The real CFO, $\Delta f^{(k)}$ can be derived by

$$\Delta f^{(k)} = \xi^{(k)} \Delta f = (2M\theta^{(k)} - q)\Delta f. \tag{5}$$

In an AWGN environment, sampled signals of an OFDMA block at BS is

$$\mathbf{Y} = \mathbf{VR} + \mathbf{N} \tag{6}$$

where $\mathbf{N}$ is white Gaussian matrix with zero mean and mean power of σ^2. Let $\mathbf{\Phi} = \mathbf{YY}^H/P$. $\mathbf{\Phi}$ is the estimate of the covariance matrix

$$\mathcal{Y} = \mathrm{E}[(\mathbf{VR} + \mathbf{N})(\mathbf{VR} + \mathbf{N})^H] = \mathbf{VRR}^H\mathbf{V}^H + \sigma^2\mathbf{I}. \tag{7}$$

Applying a subspace based analysis similar to that in MUSIC [9], we have $\theta^{(k)}$, $k = 0, 1, \ldots, K$, are the points corresponding to the largest K local-maximum of

$$\frac{1}{\|a^H(\theta) U_n {U_n}^H a(\theta)\|^2} \tag{8}$$

where $a(\theta) = [1, e^{j2\pi\theta}, \ldots, e^{j2\pi(2M-1)\theta}]^T$ and U_n is the noise sub space of $\mathcal{Y}$.

IV. Simulation and Performance

The OFDMA system parameters in simulation are selected from IEEE 802.16a (Draft 2) [1], as shown in the Table I. To quantify the performance, we define the normalized root of mean square error (NRMSE) of the estimation as

$$\text{NRMSE} = \sqrt{\frac{1}{\Gamma K} \sum_{v=1}^{\Gamma} \sum_{k=0}^{K-1} (\hat{\xi}^{(k)} - \xi^{(k)})^2} \tag{9}$$

where Γ is the simulation times, K is the number of users, $\hat{\xi}^{(k)}$ is the estimated normalized CFO of the kth user, $\xi^{(k)}$ is the real normalized CFO.

TABLE I
Transmission parameters of OFDM system

Carrier Central Frequency	5.15 GHz
Uplink Bandwidth	20 MHz
FFT Size (N)	2048
Inter Carrier Spacing	11.16 KHz

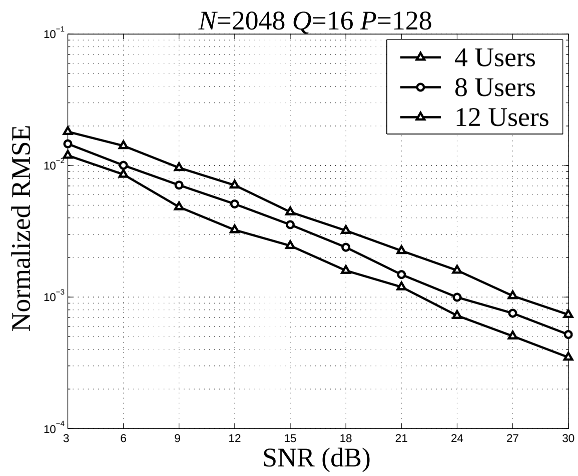

Fig. 1. The normalized RMSE vs. SNR when different number of users present in one OFDMA block.

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