Deriving the Equivalent Electrical Circuit from the Magnetic Device Physical Properties

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Purpose:

1. To define the electrical circuit equivalents of magnetic device structures to enable improved analysis of circuit performance.
2. To define the magnitude and location of relevant parasitic magnetic elements to enable prediction of performance effects
3. To manipulate parasitic elements to obtain improved or enhanced circuit performance
4. To encourage the circuit designer to be more involved with magnetic circuit design.

Magnetic Definitions:

Systeme International (SI) Units and Equations are used throughout this paper.

Simplifying the Magnetic Structure:

The first task is to take a magnetic device structure and reduce it to a few lumped elements – as few as possible for the sake of simplicity, because the equivalent electrical circuit will have just as many elements. This is not an easy task – just about everything is distributed, not lumped: the magnetic force from current in the windings, distributed flux, fringing flux adjacent to gaps, and stray fields. Boiling this down to a few elements with reasonable accuracy requires a little insight and intuition and experience – but it can be done.

Finite Element Analysis software on the other hand is extremely accurate because it does just the opposite – it chops the structure up unto a huge number of tiny elements.[1] It is a useful tool which can provide a great deal of knowledge about what goes on inside the magnetic device, but it does not provide a simple electrical equivalent circuit that lends itself to circuit analysis.

The magnetic structure shown in Figure 1 will be the first demonstration of this technique. This simple inductor is built upon a ferrite core, with air gaps to store the required inductive energy created by simply shimming the two core halves apart (not usually a good practice, but inexpensive).

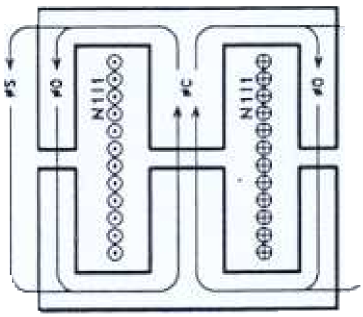

Fig 1. - Magnetic Structure - Inductor

The first thing to do is to simplify – judiciously. Looking at the inherent symmetry of the structure, the two outer legs can be combined into a single leg with twice the area. Fringing fields around the gaps will be ignored, except the effective gap area might be increased to take the fringing field into account.

The number of flux paths will be minimized, eliminating those that are trivial. For example, flux in the non-magnetic mateial adjacent to the core will be ignored, because it is trivial compared to the flux in the neighboring ferrite. On the other hand, if there were two windings, the small amount of flux between the windings must not be ignored – it constitutes the small but important leakage inductance between the two windings.

The core will be divided into regions of similar cross-section and flux density. The distributed magnetic force from the winding will be lumped and assigned a specific location in the physical structure.

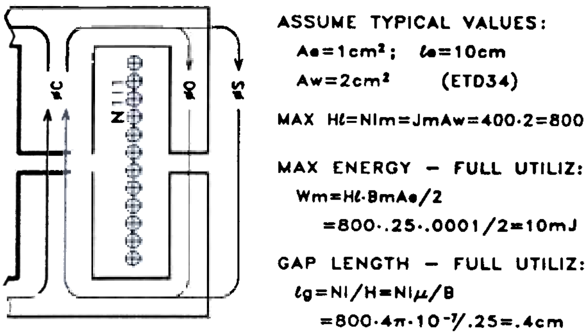

Fig 2. - Core Parameters and Utilization

For the purpose of illustration, the parameters of the core (ETD34) are given in Figure 2. The maximum ampere-turns obtainable is calculated from the window area times the max. current density in copper of $400 A/cm^2$. The maximum flux at saturation equals the saturation flux density (0.25T) times the core area. From this, the maximum possible energy storage (in an appropriate gap) equals ½BH, for a total of 10mJ. The gap length required to achieve this full utilization is calculated as shown in Figure 2. These calculations are not relevant to the modeling process, but they help indicate the suitability of this core for the intended application.

The Reluctance Diagram: Next, a reluctance diagram will be created, modeling the physical structure. Reluctance is essentially magnetic impedance. It is a measure of the opposition to flux within any region of the magnetic device.

$$R = \frac{F}{\phi} = \frac{H\ell}{BA} = \frac{\ell}{\mu A}$$

The reluctance of each significant region of the device is calculated from its area, length and permeability, and inserted with its specific value into the appropriate location in the reluctance model, as shown in Figure 3. Again, because of inherent symmetry, the model can be simplified by combining the two centerpost halves into a single element, likewise combining the outer leg portions on both sides of the gap. The magnetic field source, the ampere-turns of the winding, which is really a circulatory field is assigned to any discrete point where the flux is not divided. It would be incorrect to locate this source in series with the outer leg, because it would drive the flux in the stray field in the wrong direction.

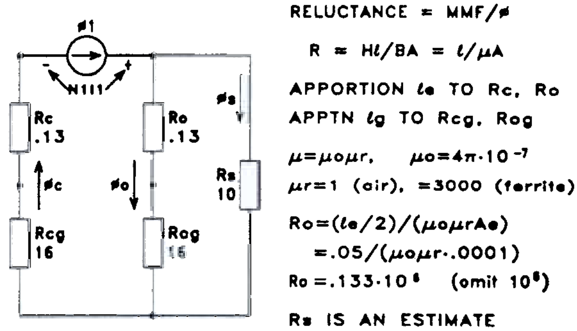

Fig 3. - The Simplified Reluctance Model

So the reluctance diagram includes the reluctance of the combined centerpost ferrite and also the centerleg gap, the combined outer leg ferrite and the outer leg gap, and the reluctance of the stray field outside the core (the calculation is an educated guess). This "magnetic circuit" can be analyzed just as though it were an electrical circuit. Remember that it is *not* an elecrical circuit – reluctance is definitely *not* the same as resistance – it stores energy rather than dissipate it. But the reluctance diagram does follow the same rules as an electrical circuit, and the amount of flux in each path can be calculated based on the magnetic force and the reluctance.

Much can be learned by examining the reluctance model and playing "what if" games. Note that in each leg, the calculated gap reluctances are more than 100 times greater than the adjacent ferrite core legs. This indicates that the core reluctances could be eliminated, impairing accuracy by less than 1%. Note also that the flux in the stray field is actully large than the flux in the outer leg of the core, because the stray field reluctance is less than the gap reluctance. This means that much noise is propageted outside the core, and the inductance value obtained depends heavily on the stray field, which is difficult to calculate. If the centerleg gap is eliminated and the outer leg gap correspondingly increase, the amount of stray field increases further.

On the other hand, if the outer leg gap is closed and the centerleg gap is widened, the stray field is almost eliminated, because the reluctance of the centerleg without gap is much less than the stray field reluctance. In this manner, the reluctance model is useful without even converting it to the equivalent electrical circuit.

Magnetic-Electrical Duality:

Fifty years ago, E. Colin Cherry published a paper showing the duality between magnetic circuits and electrical circuits.[3] It is well know that two electrical circuits can be duals of each other – the Cuk converter is the dual of the flyback (buck-boost), for example. Electrical circuits that are duals are *not* therefore equivalent. Magnetic circuits and electrical circuits are in a different realm, and yet in this case *the duals are truly equivalent.*

A dual is created by essentially turning the circuit inside out and upside down. Some of the magnetic-electrical duality relationships and rules are:

Nodes	**Meshes (loops)**
Open	**Short**
Series Elements	**Parallel Elements**
Magn. Force	**Ampere-Turns**
dϕ/dt	**Volts/turn**
Reluctance	**Permeance**

Polarity Orientation: Rotate in same direction

Circuits must be planar

Permeance is the reciprocal of reluctance:

$$P = \frac{1}{R} = \frac{\mu A}{\ell}$$

Permeance is actually the inductance for 1 turn. Multiply permeance by N^2 to obtain the inductance value referred to an N-turn winding.

Polarity Orientation means that for elements such as the windings that have polarity, to assign the proper polarity in the dual, rotate all polarity indications in the same direction from the original to the dual.

A **planar circuit** is defined as one that can be drawn on a plane surface with no crossovers. The duality process fails if there are crossovers, which can occur with complex core structures.[5] Actually, with only three windings on a simple core, if the reluctance model includes every theoretically possible flux linkage combination between windings, there will be crossovers. But most of these theoretical linkages are trivial, and should be ignored, for the sake of simplicity if nothing else. This is where common sense comes in.

Creating the Dual: The process for creating the electrical dual from the reluctance model is actually quite simple, as illustrated in Figure 4.

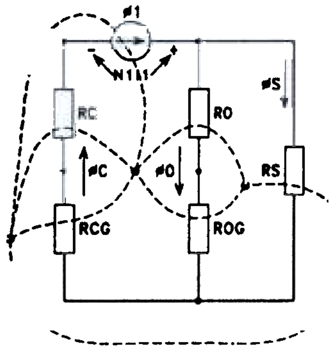

Fig 4. - The Magnetic / Electrical Dual

First, identify each mesh, or loop in the reluctance model. In this case there are 3 loops. (The outside is always considered a loop. Topologically a simple circle has two loops – the inside and the outside.) Put a dot in the center of each loop (any convenient location on the outside). These dots will be the nodes of the electrical circuit. Draw a dash line from electrical node to node though *every* intervening element. The dash lines are branches in the electrical circuit. The intervening elements become elements of the new circuit, but they are transformed: Reluctances become their reciprocal – permeances, the magnetic winding becomes the electrical terminals, with dϕ1/dt translating into V1/N1 (Faraday's Law), and magnetic force translating into N1I1 in series with the terminals. Note that the 5 nodes in the original reluctance model are automatically converted into 5 loops in the electrical dual. (Don't forget the outside is a loop!)

All of the values in the equivalent electrical circuit at this stage pertain to a one-turn winding. The electrical equivalent circuit is redrawn as shown in Figure 5:

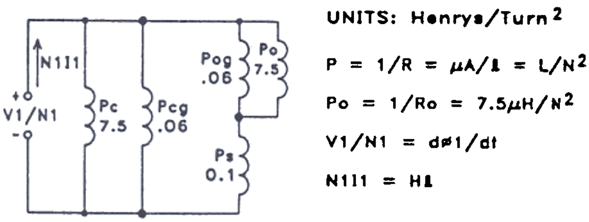

Fig 5. - The Equivalent Electrical Circuit

If the winding has fifteen turns, permeances are converted to inductance values by multiplying by 15^2. Likewise, terminal V/N is multiplied by 15 to become terminal voltage, and NI is divided by N to become terminal current. The final electrical equivalent circuit is shown below:

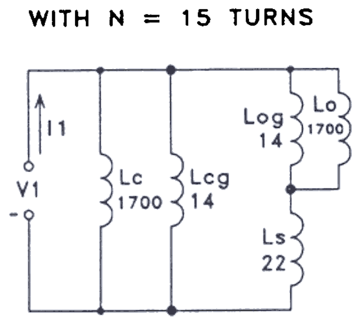

Fig 6. - Final Electrical Equivalent Circuit

This simple example obviously produces a trivial result. There are five inductive elements which represent the five reluctances in the reluctance model. The five inductances combine to form a single inductance value of 10μH. But each of the five elements is clearly identifiable back to the reluctance it represents. Note that the series reluctances of the centerleg ferrite and centerleg gap show up as parallel inductances in Fig. 6, and the stray field reluctance which was in parallel with the outer leg are now series inductances. The circuit shows that the high inductance values contributed by the centerleg ferrite and outer leg ferrite are irrelevant in parallel with the much lower gap inductances. It shows that the stray field inductance makes a very significant contribution to the overall inductance. But if the outer leg gap was closed up, its inductance would become infinite, making the stray field inductance irrelevant. The overall inductance would then equal the 14μH of the centerleg gap.

A simple transformer: A transformer with two windings is shown in Figure 7. The transformer has no gap — energy storage is undesirable. The flux between the two windings, although small, is very important because the energy contained between the windings constitutes leakage inductance.

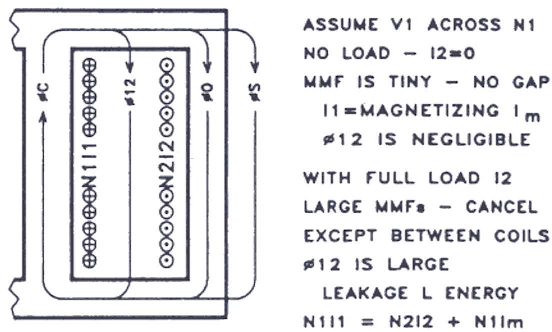

Fig 7. - Two-Winding Transformer

Figure 8 is the reluctance model with specific values calculated for the same ETD34 core used in the previous example, but ungapped. The ampere turns in the two windings cancel except through the region between the windings (R12). Thus, when the transformer is loaded, this is the only place the fields don't cancel, storing considerable leakage inductance energy as a function of load current.

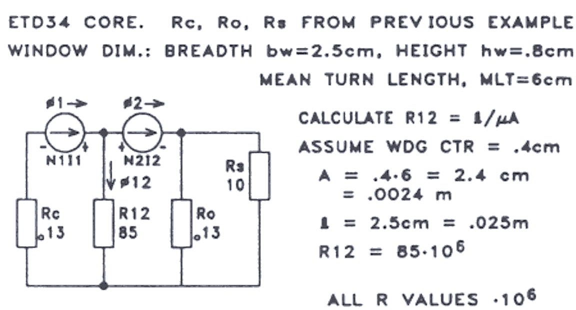

Fig 8. - Transformer Reluctance Model

Figure 9 goes through the duality process, and Figure 10 is the final result. In Fig. 10, an ideal transformer is added to one terminal pair to allow for a turns ratio other than 1:1, and to provide galvanic isolation.

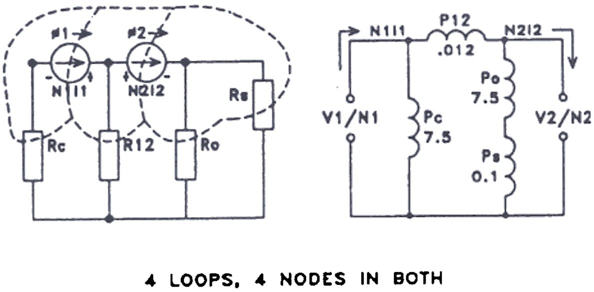

Fig 9. - The Transformer Electrical Dual

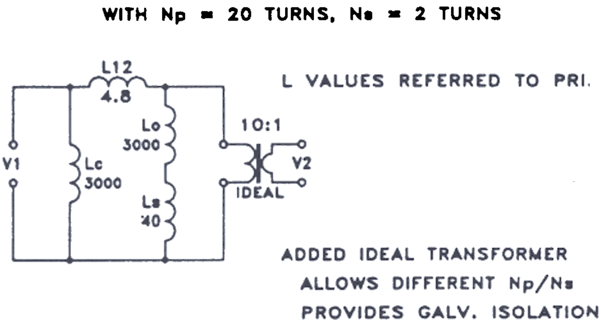

Fig 10. - Transformer Equivalent Circuit

A Three-Winding Transformer: Figure 11 is a three-winding transformer showing the physical structure and the equivalent electrical circuit. The reluctance model is not shown. The equivalent circuit model shows how the leakage inductances are distributed between windings and their magnitudes.

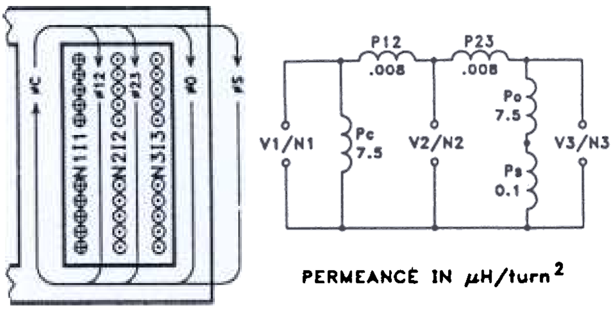

Fig 11. - Three-Winding Transformer

Coupled Inductor: Topic M7 in the Design Reference Section of the Seminar Manual describes the benefits of coupled filter inductors in multi-output buck regulators. Figure 12 shows the physical structure and resulting equivalent circuit which can provide insight into the design of this device.

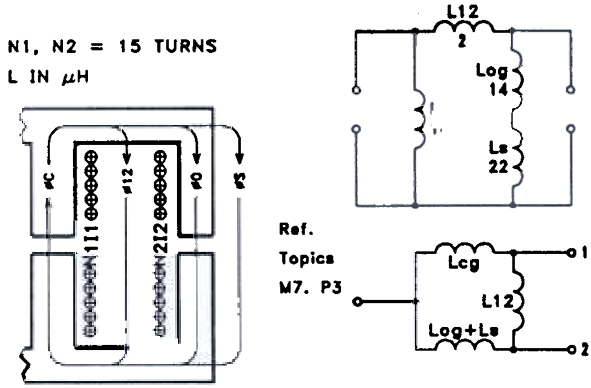

Fig 12. - Coupled Inductor

Fractional Turns: Transformers with fractional turns have been featured in previous Seminars. Figure 13 provides insight into the design and behavior of this device.

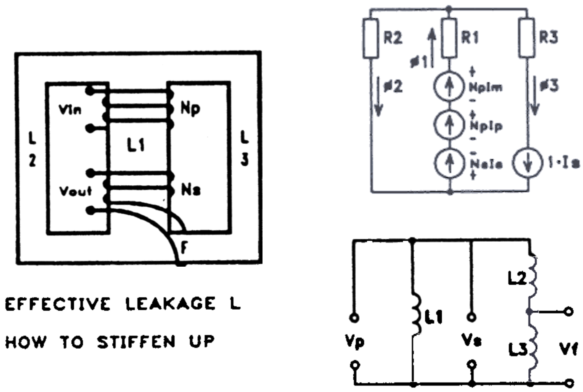

Fig 13. - Transformer With Fractional Turns

The Gyrator-Capacitor Approach:

Another method for defining an equivalent electrical circuit of a magnetic device structure completely avoids the Reluctance/Duality approach that has been discussed up to this point.[4,5] The equivalent circuit layout achieved with the Gyrator-Capacitor follows exactly the pattern of the magnetic structure. This makes it somewhat easier to relate electrical performance back to the magnetic elements. A major advantage of this method is that it does not require that the magnetic circuit be "planar", as the Reluctance/Dualitymethod does. Proponents of the Gyrator-Capacitor method cite these advantages.

However, the equivalent electrical circuit that results from this method looks nothing like a classical inductor or transformer. Inductive energy storage elements are replaced by capacitors (the electrical dual of inductance), and transformer windings are replaced by gyrators. (A gyrator is an ideal two-port element which one port reflects the reciprocal of the impedance at the other port, and scales the impedance according to a factor r^2.) In other words, the gyrators translate the *performance* of the equivalent circuit, employing capacitors, into its electrical dual, employing inductances, but one never actually sees the dualized equivalent circuit – the gyrators take care of it transparently. Figure 14 shows the equivalent circuit of a simple flyback converter modeled using the Gyrator-Capacitor method.

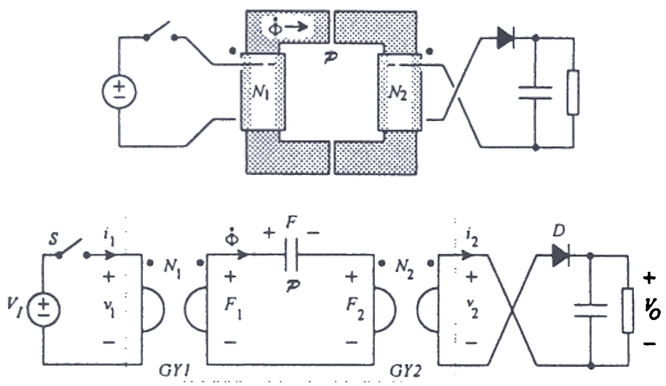

Fig 14. - Gyrator-Capacitor Model of a Forward Converter

So the choice is – do you prefer:
(1) A conventional electrical circuit whose magnetic elements include leakage and magnetizing inductances and transformer windings, etc. This facilitates intuitive and insightful circuit analysis, but it does *not* resemble the magnetic device physical structure, diminishing insight into the physical-electrical relationship, or
(2) An equivalent circuit with capacitos and gyrators replacing inductive elements whose layout closely resembles the structure of the magnetic device. This facilitates insight into the physical-electrical relationship, but severely diminishes insight into circuit analysis.

The author prefers to remain with the Reluctance/Duality method, but admits that it's probably a matter of what one is used to and more comfortable with.

References:

[1] R. Severns, "Finite Element Analysis in Power Converter Design," *IEEE APEC Proc. pp. 3-9*, Feb. 1994

[2] Dauhajre and Middlebrook, "Modelling and Estimation of Leakage Phenomena in Magnetic Circuits," *IEEE PESC Record, pp. 213-226*, 1986

[3] E. C. Cherry, "The Duality Between Electric and Magnetic Circuits and the Formation of Transformer Equivalent Circuits," *Proc. Physical Soc. London, Vol 62B pp.101-111*, Feb. 1949

[4] D.C. Hamill, "Lumped Equivalent Circuits of Magnetic Components: The Gyrator-Capacitor Approach," *IEEE Trans. on Power Electronics, vol. 8, no. 2, pp. 97-103*, April 1993

[5] D.C. Hamill, "Gyrator-Capacitor Modeling: A Better Way of Understanding Magnetic Components," *IEEE APEC Proc. pp. 326-332*, Feb. 1994

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