Chapter 2-1
Resonators and Impedance Matching with Lumped Elements

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Resonators

- **Lump Resonators**

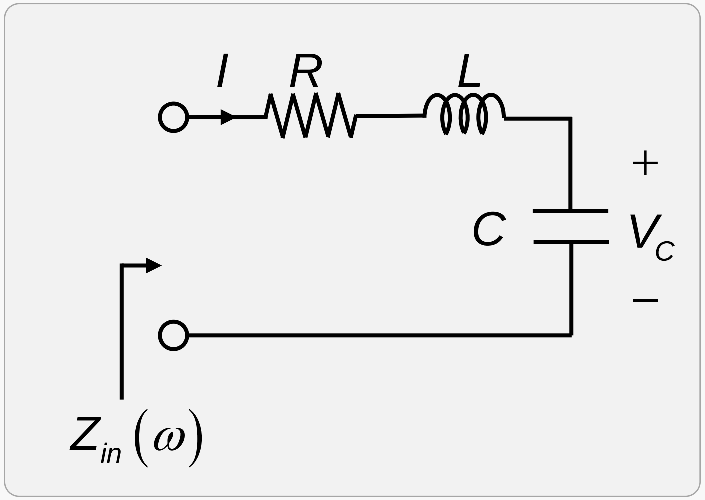
➢ **Series resonant circuit**

$$Z_{in}(\omega) = R + j\omega L + \frac{1}{j\omega C}$$

$$P_R = \frac{1}{2}|I|^2 R$$

$$W_m = \frac{1}{4}|I|^2 L$$

$$W_e = \frac{1}{4}|V_C|^2 C = \frac{1}{4}|I|^2 \frac{C}{\omega^2 C^2} = \frac{1}{4}|I|^2 \frac{1}{\omega^2 C}$$



At resonant frequency $\omega_0 = 2\pi f_0$

$$W_m = W_e \quad \Rightarrow \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$Z_{in}(\omega_0) = R$$

Resonance occurs when the average stored magnetic and electric energies are equal, and the input impedance is a purely real impedance.

Series Resonant Circuit

- **Quality Factor:**

$$Q \triangleq \omega \frac{W_m + W_e}{P_R} = \omega \frac{\text{average energy stored}}{\text{energy loss/sec}}$$

Q is a measure of the loss of a resonance circuit, and lower loss implies higher Q.

At resonance $\omega = \omega_0$

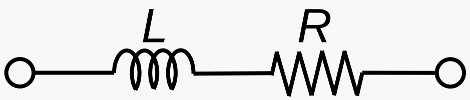
$$Q = \omega_0 \frac{2W_m}{P_R} = \omega_0 \frac{2 \cdot \frac{1}{4}|I|^2 L}{\frac{1}{2}|I|^2 R} = \frac{\omega_0 L}{R}$$

$$Q = \omega_0 \frac{2W_e}{P_R} = \omega_0 \frac{2 \cdot \frac{1}{4}|I|^2 \frac{1}{\omega_0^2 C}}{\frac{1}{2}|I|^2 R} = \frac{1}{\omega_0 CR}$$

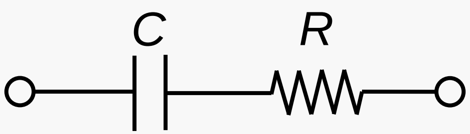
$Q = \dfrac{|X|}{R}$ **where** $|X| = \omega_0 L$ or $\dfrac{1}{\omega_0 C}$

Comparisons

- For series RL and series RC Circuits



$$Q \triangleq \frac{|X|}{R} = \frac{\omega L}{R}$$

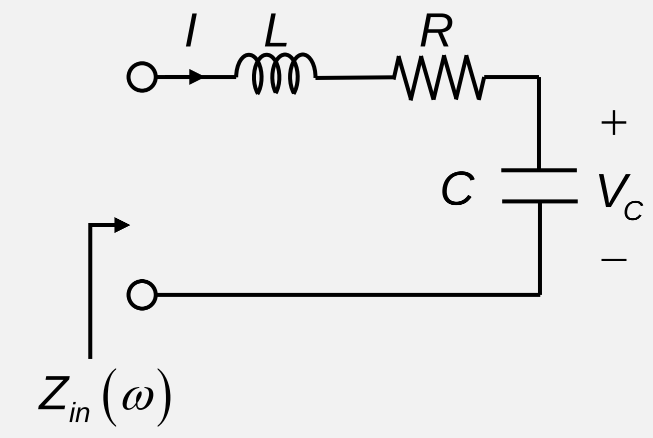


$$Q \triangleq \frac{|X|}{R} = \frac{1}{\omega C R}$$

Although the Q is defined, but the resonant frequency can’t be defined (they are not resonators).

Define the Bandwidth

- For series resonant circuits, as $\omega \to \omega_0$



$$Z_{in}(\omega) = R + j\omega L + \frac{1}{j\omega C}$$

Let $\omega = \omega_0 + \Delta\omega \quad \Delta\omega \to 0 \qquad \omega_0 = \frac{1}{\sqrt{LC}}$

$$Z_{in}(\omega) = R + j\omega L\left(1 - \frac{1}{\omega^2 LC}\right) = R + j\omega L\left(1 - \frac{1}{\omega^2 / \omega_0^2}\right) = R + j\omega L\left(\frac{\omega^2 - \omega_0^2}{\omega^2}\right)$$

$$= R + j\omega L \frac{(\omega + \omega_0)(\omega - \omega_0)}{\omega^2} \simeq R + jL\frac{2\omega_0 \cdot \Delta\omega}{\omega_0}$$

$$= R + 2j\Delta\omega L = R + 2j\Delta\omega\left(\frac{QR}{\omega_0}\right)$$ **where** $$Q = \frac{\omega_0 L}{R}$$

3-dB Bandwidth

- **Define the bandwidth** $BW \triangleq \dfrac{2\Delta\omega}{\omega_0}$

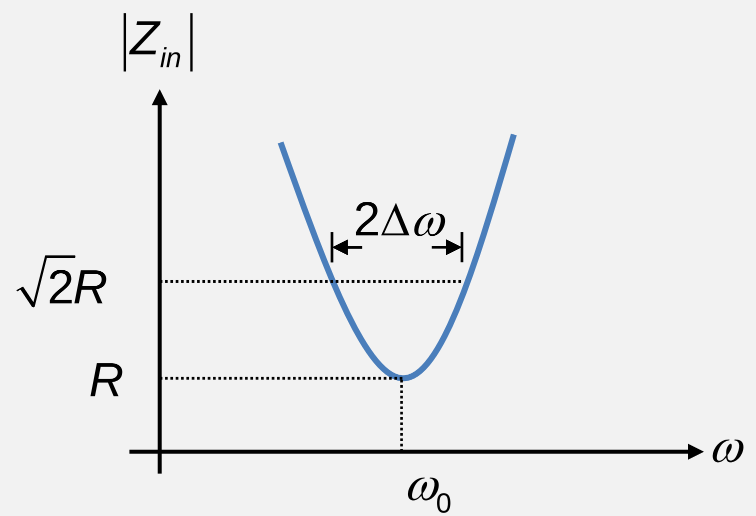
$$Z_{in} = R + 2j\frac{BW \cdot \omega_0}{2}\left(\frac{QR}{\omega_0}\right) = R + jBW \cdot QR$$

when $BW = \dfrac{1}{Q}$

$$|Z_{in}| = |R + jR| = \sqrt{2}R$$

$$\text{3-dB Bandwidth} = BW \cdot \omega_0 = \frac{\omega_0}{Q}$$

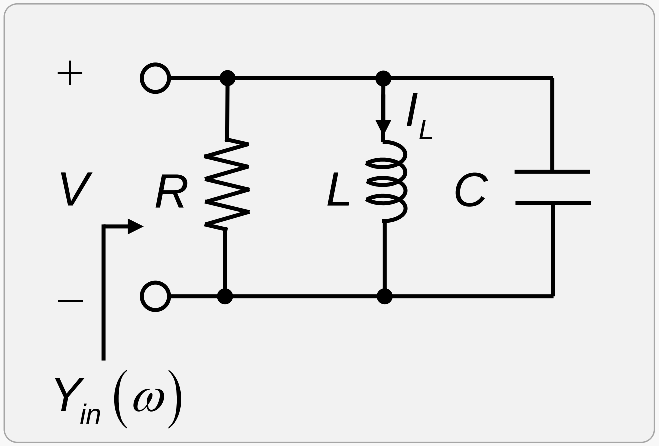
$$\text{3-dB Bandwidth in \%} = BW = \frac{1}{Q}$$



When resonance occurs, the absolute value of impedance is minimum in the series resonant circuit.

Parallel Resonant Circuit

- Parallel resonant circuit



$$Y_{in}(\omega) = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C$$

$$P_R = \frac{|V|^2}{2R}$$

$$W_e = \frac{1}{4}|V|^2 C$$

$$W_m = \frac{1}{4}|I_L|^2 L = \frac{1}{4}\frac{|V|^2}{\omega^2 L^2} L = \frac{1}{4}\frac{|V|^2}{\omega^2 L}$$

At resonance, the angular resonant frequency $\omega_0 = 2\pi f_0$

$$W_m = W_e \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$Y_{in}(\omega_0) = \frac{1}{R}$$

Parallel Resonant Circuit

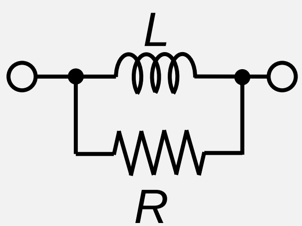
- **Quality Factor:**

$$Q \triangleq \omega \frac{W_m + W_e}{P_R} = \omega \frac{\text{average energy stored}}{\text{energy loss/sec}}$$

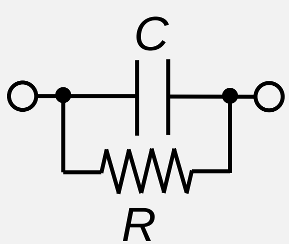
At resonance $\omega = \omega_0$

$$Q = \frac{|B|}{G} \quad \textbf{where} \quad G = \frac{1}{R} \quad \textbf{and} \quad |B| = \omega_0 C \text{ or } \frac{1}{\omega_0 L}$$

Comparisons:



$$Q \triangleq \frac{|B|}{G} = \frac{R}{\omega L}$$



$$Q \triangleq \frac{|B|}{G} = \omega CR$$

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3dB Bandwidth

- **For parallel resonant circuits, as** $\omega \to \omega_0$

$$Y_{in}(\omega) = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C \simeq \frac{1}{R} + 2jC\Delta\omega$$

Define $BW \triangleq \dfrac{2\Delta\omega}{\omega_0}$

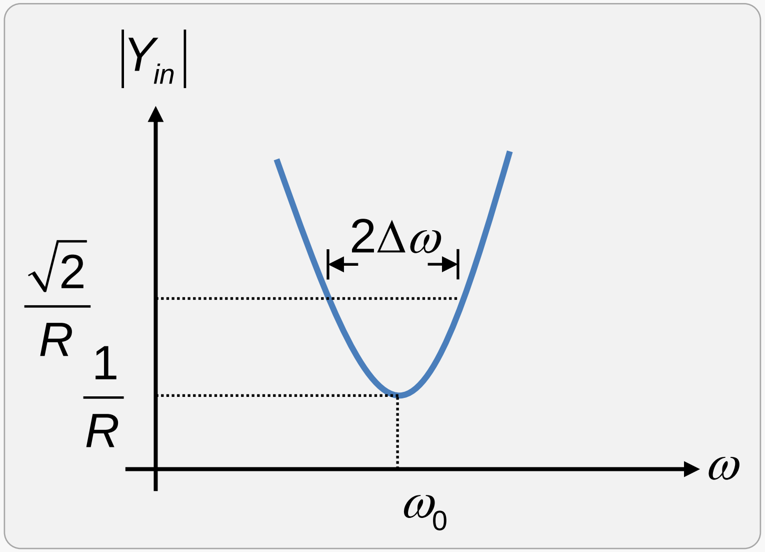
$$Y_{in}(\omega) = \frac{1}{R} + 2j\frac{BW \cdot \omega_0}{2}\frac{Q}{\omega_0 R} = \frac{1}{R} + jBW\frac{Q}{R}$$

when $BW = \dfrac{1}{Q}$

$$|Y_{in}| = \left|\frac{1}{R} + j\frac{1}{R}\right| = \frac{\sqrt{2}}{R}$$

$$\text{3-dB Bandwidth} = BW \cdot \omega_0 = \frac{\omega_0}{Q}$$

$$\text{3-dB Bandwidth in \%} = BW = \frac{1}{Q}$$



When resonance occurs, the absolute value of admittance is minimum (maximum impedance) in the parallel resonant circuit.

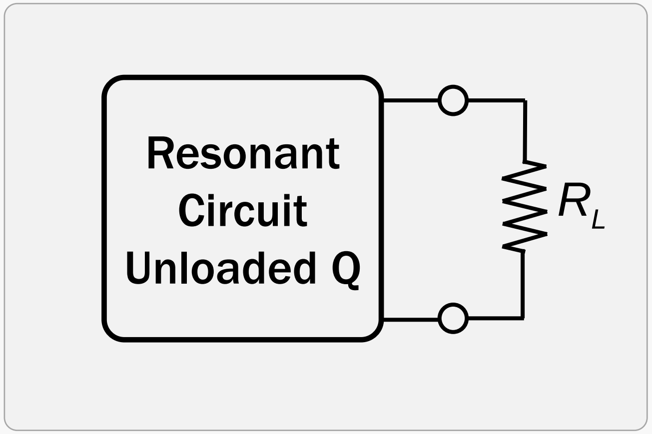
Loaded and Unloaded Q

- **For series RLC circuit**

$$Q_L = \frac{\omega_0 L}{R + R_L} = \frac{1}{\omega_0 C\left(R + R_L\right)}$$

- **For parallel RLC circuit**

$$Q_L = \frac{R \,//\, R_L}{\omega_0 L} = \omega_0 C\left(R \,//\, R_L\right)$$



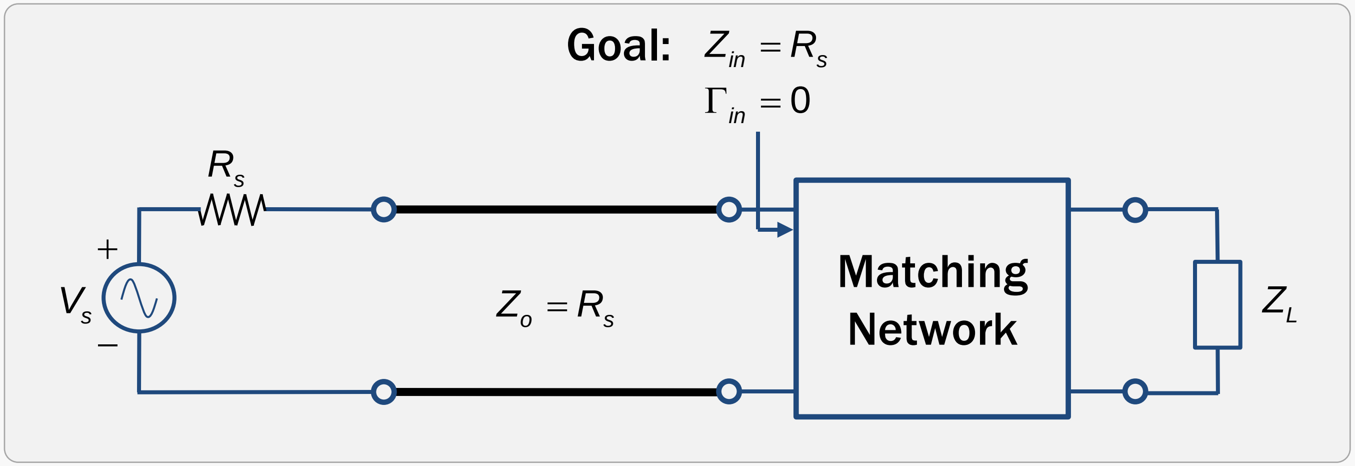
- **Define the external Q** $\left(Q_e\right)$

$$Q_e = \frac{\omega_0 L}{R_L} = \frac{1}{\omega_0 R_L C} \quad \text{for series RLC}$$

$$Q_e = \frac{R_L}{\omega_0 L} = \omega_0 R_L C \quad \text{for parallel RLC}$$

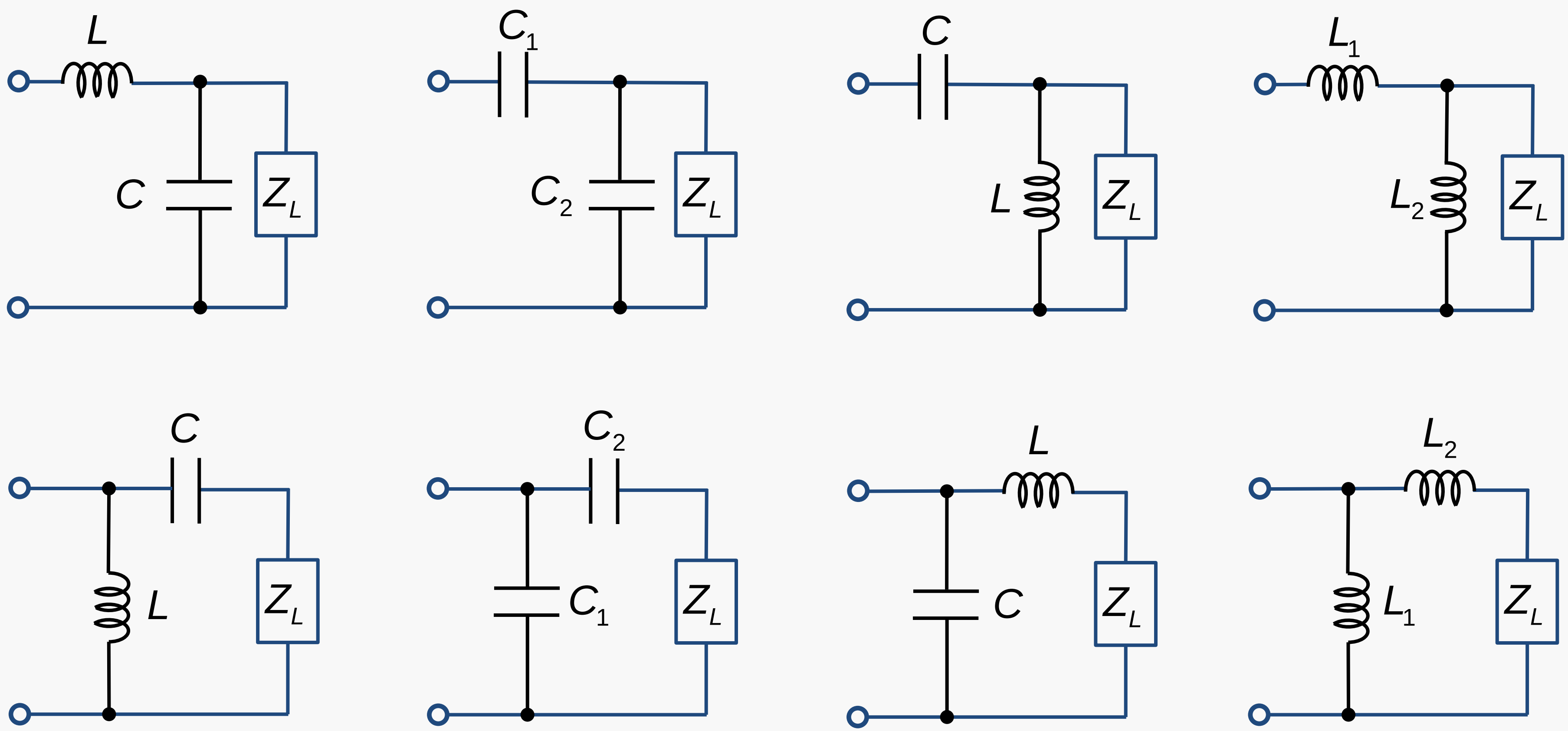
$$\frac{1}{Q_L} = \frac{1}{Q} + \frac{1}{Q_e} \quad \text{for both cases}$$

Impedance Matching



- The matching network is ideally lossless, to avoid unnecessary loss of power, and is usually designed so that the impedance seen looking into the matching network is Z_o (matched with the transmission line) or R_s (matched with the source impedance when no line connected).
- Maximum power is delivered when the load is matched to the line (assuming the generator is matched), and power los in the feed line is minimized.
- Impedance matching may be used to improve the signal-to-noise ratio or amplitude/phase errors in some applications.

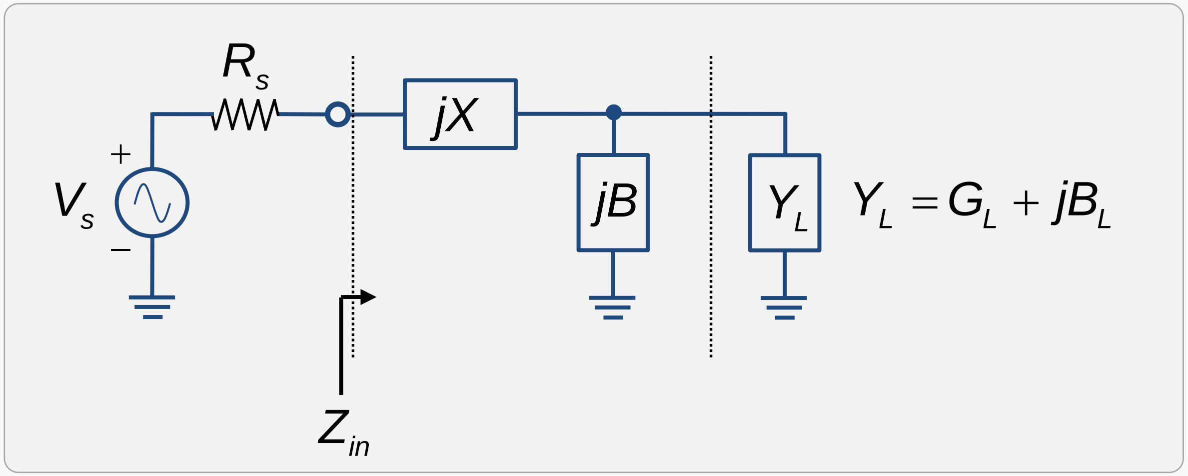
Eight Ell Matching Sections (L-Shape)



Lump Element Matching

- **Two element (L-shape) matching**

➢ **Case (a)** $R_s < \dfrac{1}{G_L}$ (or $R_s < R_L$)



Goal: $Z_{in} = R_s$

$$\Gamma = \frac{Z_{in} - R_s}{Z_{in} + R_s} = 0$$

X : matching reactance

B : matching susceptance

We want to find X and B

L-Shape Matching – Case (a) $R_s < R_L$

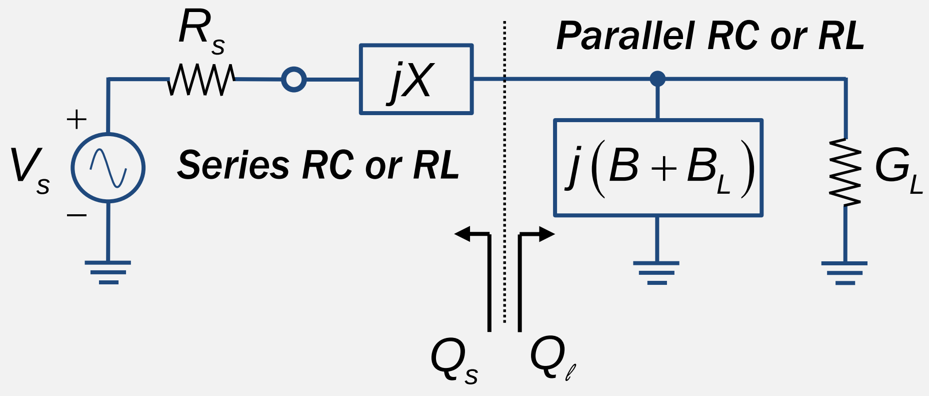
$$Z_{in} = jX + \frac{1}{(G_L + jB_L) + jB} = R_s$$

Real part: $R_s G_L + X(B + B_L) = 1$

Imaginary part: $R_s(B + B_L) - XG_L = 0$

When the impedances are matched, the imaginary part of the input impedance looking into the matching network is equal to zero.

Use the resonator concept:



$$Q_s = \frac{X}{R_s} \qquad Q_\ell = \frac{B + B_L}{G_L}$$

L-Shape Matching – Case (a) $R_s < R_L$

Imaginary part: $R_s\left(B+B_L\right)-XG_L=0$

Real part: $R_sG_L+X\left(B+B_L\right)=1$ **and** $Q_s=\dfrac{X}{R_s}=Q_\ell=\dfrac{B+B_L}{G_L}$

$$\Rightarrow R_sG_L\left(Q^2+1\right)=1 \Rightarrow Q=\pm\sqrt{\frac{1}{R_sG_L}-1}\quad\left(\text{or } \pm\sqrt{\frac{R_L}{R_s}-1}\right)$$

Choose $Q=Q_s=Q_\ell=+\sqrt{\dfrac{1}{R_sG_L}-1}$

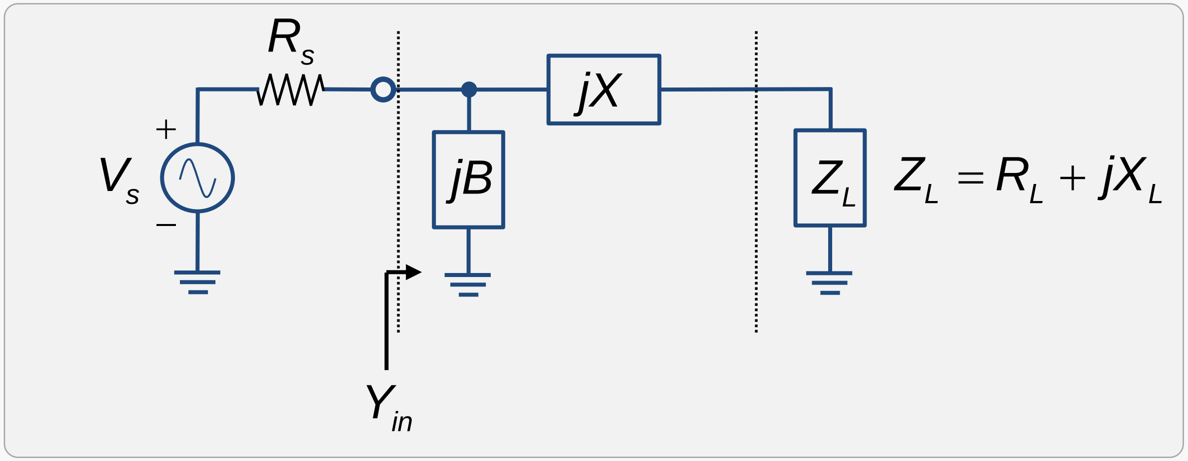
When Q is chosen, X and B are also decided.

$X=R_sQ=R_s\sqrt{\dfrac{1}{R_sG_L}-1}$ **(>0, inductance)**

$B=G_LQ-B_L=G_L\sqrt{\dfrac{1}{R_sG_L}-1}-B_L$ **(>0, capacitance)** **(<0, inductance)**

L-Shape Matching – Case (b) $R_s > R_L$

- Case (b) $R_s > R_L$



X : matching reactance

B : matching susceptance

Again, we want to find X and B

Goal:

$$Y_{in} = \frac{1}{R_s} \qquad \Gamma = \frac{\frac{1}{R_s} - Y_{in}}{\frac{1}{R_s} + Y_{in}} = 0$$

L-Shape Matching – Case (b) $R_s > R_L$

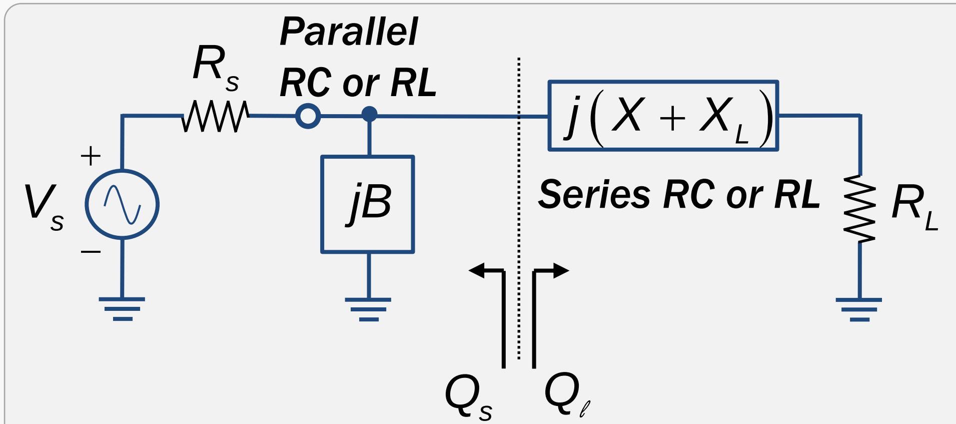
$$Y_{in} = jB + \frac{1}{(R_L + jX_L) + jX} = \frac{1}{R_s}$$

Real part: $BR_s(X + X_L) = R_s - R_L$

Imaginary part: $(X + X_L) - BR_sR_L = 0$

When the impedances are matched, the imaginary part of the input admittance looking into the matching network is equal to zero.

Use resonator concept:



$$Q_s = BR_s \qquad Q_\ell = \frac{X + X_L}{R_L}$$

L-Shape Matching – Case (b) $R_s > R_L$

Imaginary part: $(X + X_L) - BR_s R_L = 0$ $\frac{X + X_L}{R_L} = Q_\ell = BR_s = Q_s = Q$

Real part: $BR_s (X + X_L) = R_s - R_L$

➡ $Q^2 R_L = R_s - R_L$ $Q = \pm\sqrt{\frac{R_s}{R_L} - 1}$

Choose $Q = Q_s = Q_\ell = +\sqrt{\frac{R_s}{R_L} - 1}$

When Q is chosen, X and B are also decided.

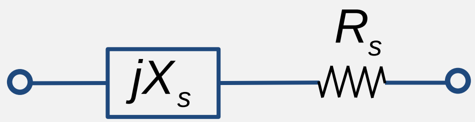
$B = \frac{Q}{R_s} = \frac{1}{R_s}\sqrt{\frac{R_s}{R_L} - 1}$ (>0, capacitance)

$X = R_L Q - X_L = R_L\sqrt{\frac{R_s}{R_L} - 1} - X_L$ (>0, inductance)

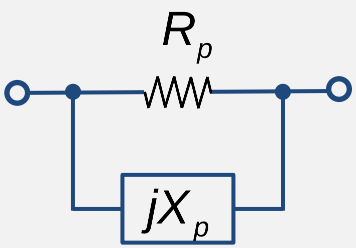
(<0, capacitance)

Series-to-Parallel Transformation

- Series-to-Parallel transformation



$$Q_s = \frac{X_s}{R_s} \qquad Z_s = R_s + jX_s$$

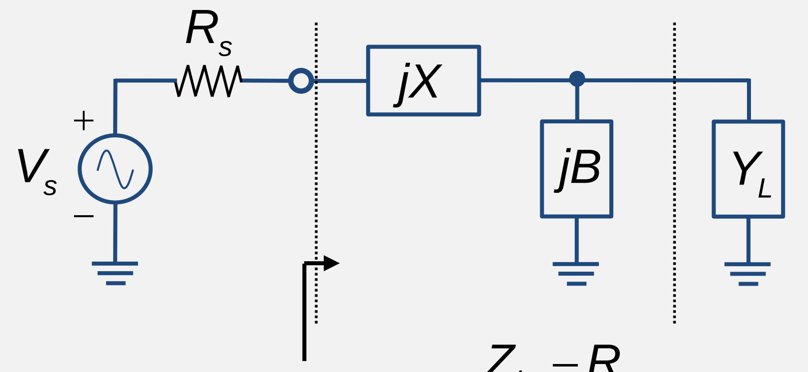


$$Q_p = \frac{R_p}{X_p} \qquad Z_p = \frac{R_p \cdot jX_p}{R_p + jX_p}$$

$$\left.\begin{aligned} Q_s &= Q_p = Q = \frac{X_s}{R_s} = \frac{R_p}{X_p} \\ Z_s &= Z_p = R_s + jX_s = \frac{R_p \cdot jX_p}{R_p + jX_p} \end{aligned}\right\} \quad R_s\left(1+Q^2\right) = R_p$$

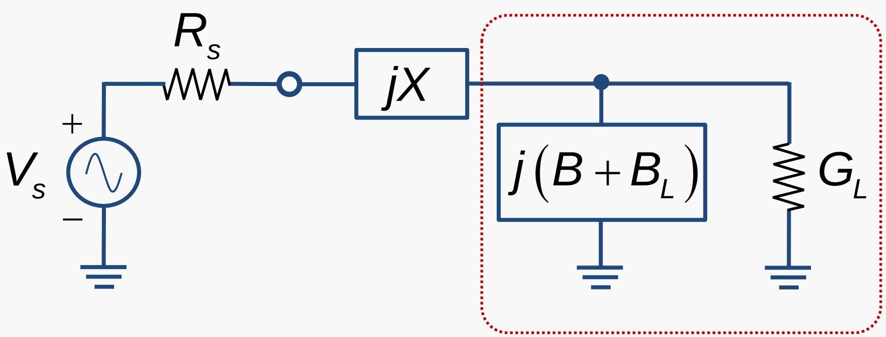
Matching Bandwidth (I)

➢ **Case (a)** $R_s < \dfrac{1}{G_L}$ (or $R_s < R_L$)



$Y_L = G_L + jB_L$

$\Gamma = \dfrac{Z_{in} - R_s}{Z_{in} + R_s}$



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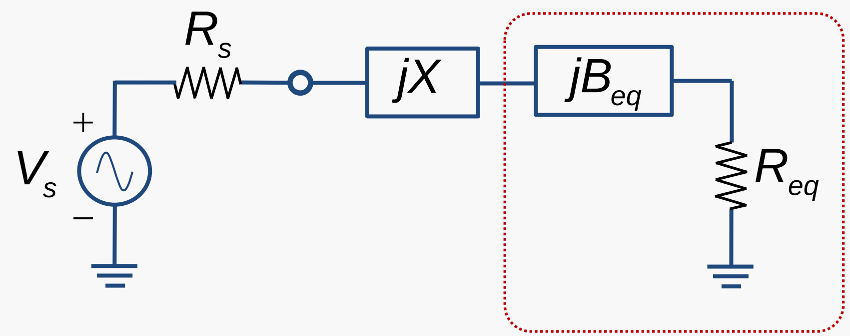
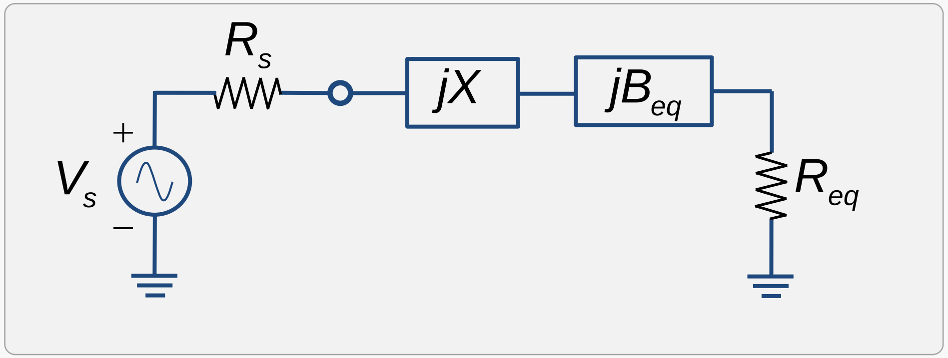


$$R_{eq} = \frac{1}{G_L}\frac{1}{1+Q^2} \qquad B_{eq} = \frac{1}{QR_{eq}} = \frac{G_L\left(1+Q^2\right)}{Q}$$

Matching Bandwidth (II)



Series resonant circuit

Impedance matching at $\omega = \omega_0$

$$Z_{in}\left(\omega = \omega_0\right) = jX + \frac{1}{jB_{eq}} + R_{eq} = R_s$$

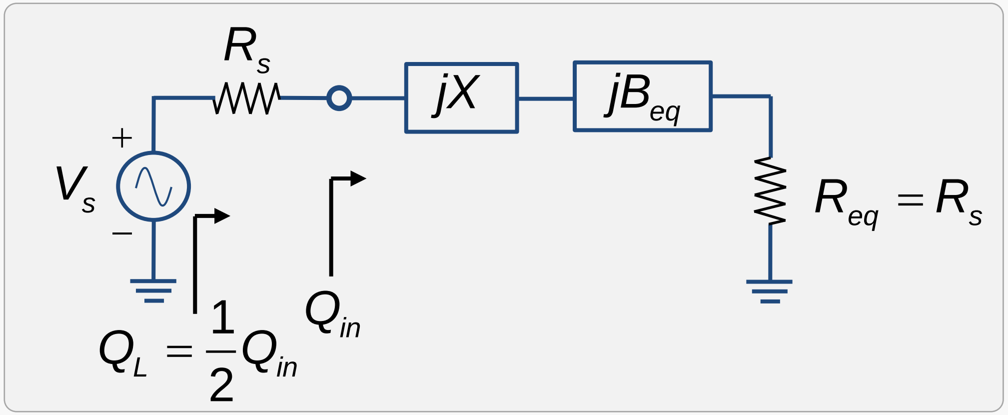
Imaginary part: $jX + \dfrac{1}{jB_{eq}} = 0 \;\Rightarrow\; XB_{eq} = 1$

Let $X = \omega_0 L$ **and** $B_{eq} = \omega_0 C$ **,where** $\omega_0 = \dfrac{1}{\sqrt{LC}}$

Real part: $R_{eq} = R_s \;\Rightarrow\; \dfrac{1}{G_L}\dfrac{1}{1+Q^2} = R_s$ **and** $Q = \pm\sqrt{\dfrac{1}{G_L R_s} - 1}$

Matching Bandwidth (III)

- Define Q_L and Q_{in} for RLC resonator



$$Q_{in} = \frac{X}{R_{eq}} = R_s \cdot |Q| \cdot G_L \left(1 + Q^2\right) = |Q|$$

➢ **Find 3-dB bandwidth for $|\Gamma|$**

Let $X = \omega L$ **and** $B_{eq} = \omega C$

as $\omega \to \omega_0$ **,let** $\omega = \omega_0 + \Delta\omega$ **where** $\Delta\omega \to 0$ **and** $\omega_0 = \frac{1}{\sqrt{LC}}$

$$\Gamma = \frac{Z_{in} - R_s}{Z_{in} + R_s} = \frac{jX + \frac{1}{jB_{eq}}}{jX + \frac{1}{jB_{eq}} + 2R_s} \simeq \frac{2jL\Delta\omega}{2jL\Delta\omega + 2R_s}$$

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Matching Bandwidth (IV)

- 3-dB bandwidth for $|\Gamma|$ $(=2\Delta\omega)$ occurs when

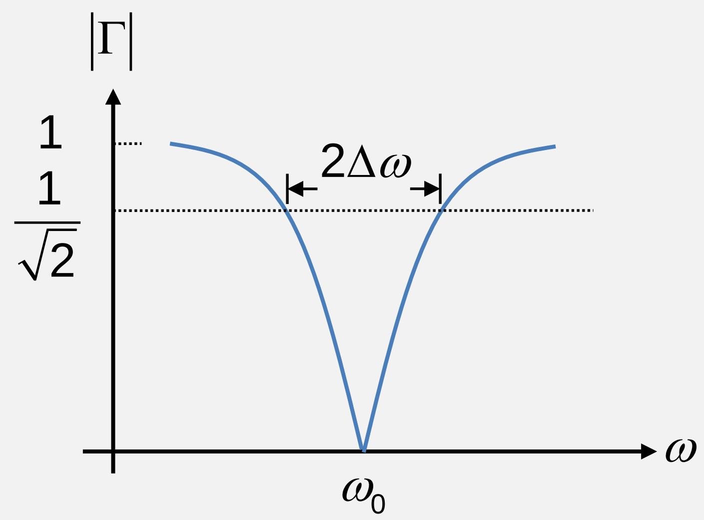
$$2L\Delta\omega = 2R_s \Rightarrow 2\Delta\omega = \frac{2R_s}{L} = \frac{2R_s\omega_0}{X}$$

3-dB Bandwidth in % $= BW$

$$BW = \frac{2\Delta\omega}{\omega_0} = \frac{2R_s}{X} = \frac{2}{|Q|} = \frac{1}{Q_L} = \frac{2}{\sqrt{\dfrac{1}{R_sG_L}-1}}$$

- Similarly, for case (b)

$$BW = \frac{1}{Q_L} = \frac{2}{|Q|} = \frac{2}{\sqrt{\dfrac{1}{R_sG_L}-1}}$$



- Case (a)

$$R_s < \frac{1}{G_L}$$

$$|Q| = \sqrt{\frac{1}{R_sG_L}-1}$$

- Case (b)

$$R_s > R_L$$

$$|Q| = \sqrt{\frac{R_s}{R_L}-1}$$

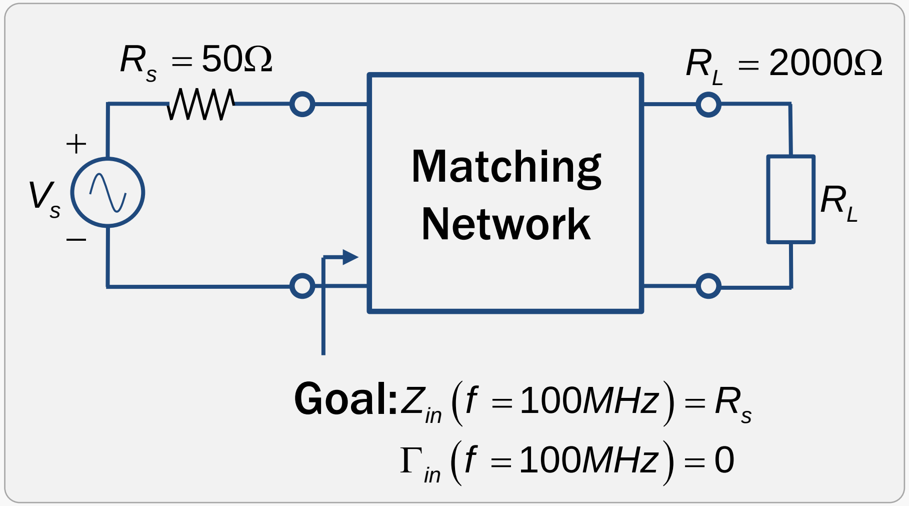
Example – L-Shape Matching (I)

- Find element values and 3-dB fractional bandwidth of the two-element matching network.

$$R_s < \frac{1}{G_L}, \quad \text{choose case (a)}$$

$$Q = \pm\sqrt{\frac{1}{R_s G_L} - 1} = \pm\sqrt{\frac{2000}{50} - 1}$$

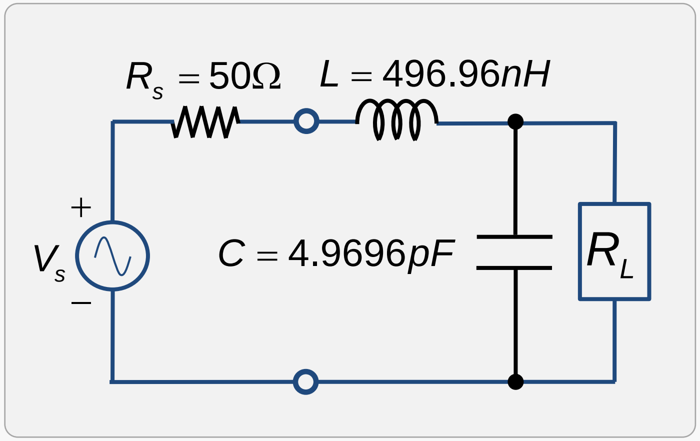
$$= \pm\ 6.245$$



➢ **1st Solution: choose** $Q = +6.245$

$$\begin{cases} X = R_s Q = 50 \cdot 6.245 = 2\pi \cdot 100 \times 10^6 \cdot L \\ B = G_L Q - B_L = 6.245/2000 - 0 = 2\pi \cdot 100 \times 10^6 \cdot C \end{cases}$$

$$\begin{cases} L = 496.96 \times 10^{-9} (H) = 496.96 \quad (nH) \\ C = 4.9696 \times 10^{-12} (F) = 4.9696 \quad (pF) \end{cases}$$



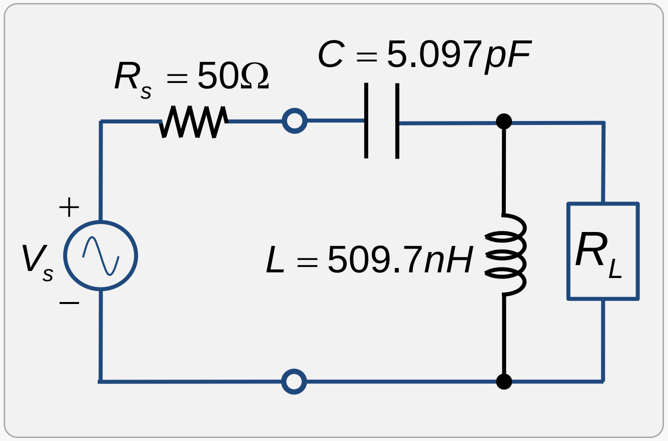
Example – L-Shape Matching (II)

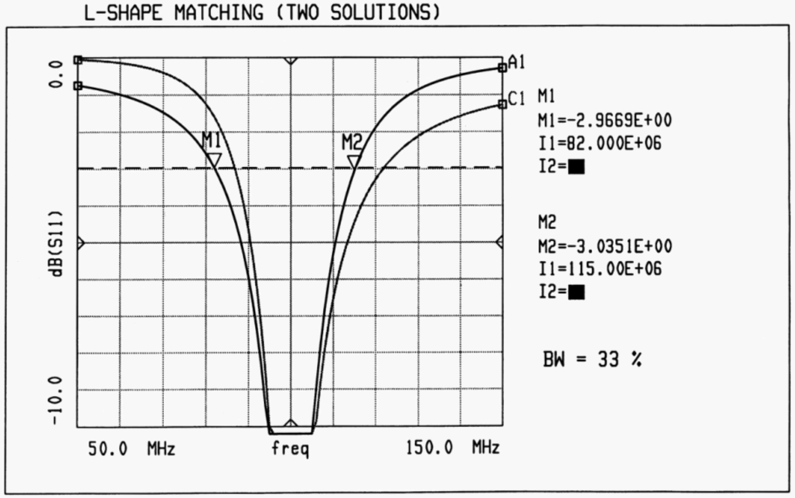
- **2nd Solution:** **choose** $Q = -6.245$

$$\Rightarrow \begin{cases} X = R_s Q = 50 \cdot (-6.245) = -1/(2\pi \cdot 100 \times 10^6 \cdot C) \\ B = G_L Q - B_L = (-6.245)/2000 - 0 = -1/(2\pi \cdot 100 \times 10^6 \cdot L) \end{cases}$$

$$\Rightarrow \begin{cases} C = 5.097 \times 10^{-12} (F) = 5.097 \quad (pF) \\ L = 509.7 \times 10^{-9} (H) = 509.7 \quad (nH) \end{cases}$$

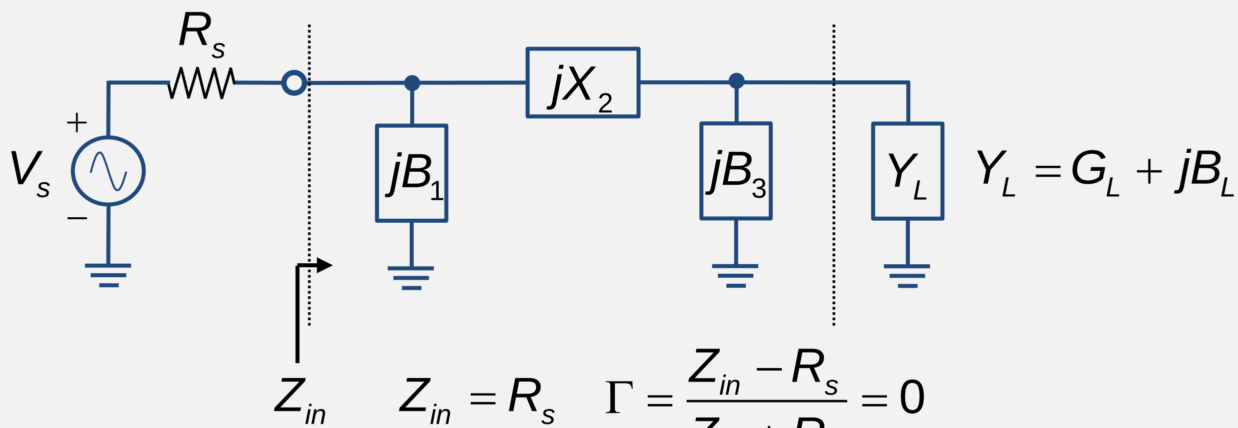
$BW_{3dB} = \frac{1}{Q_L} = \frac{2}{|Q|} = \frac{2}{6.245} = 32\%$ **and** $BW_{9dB} \approx 16\%,\ BW_{15dB} \approx 8\%$





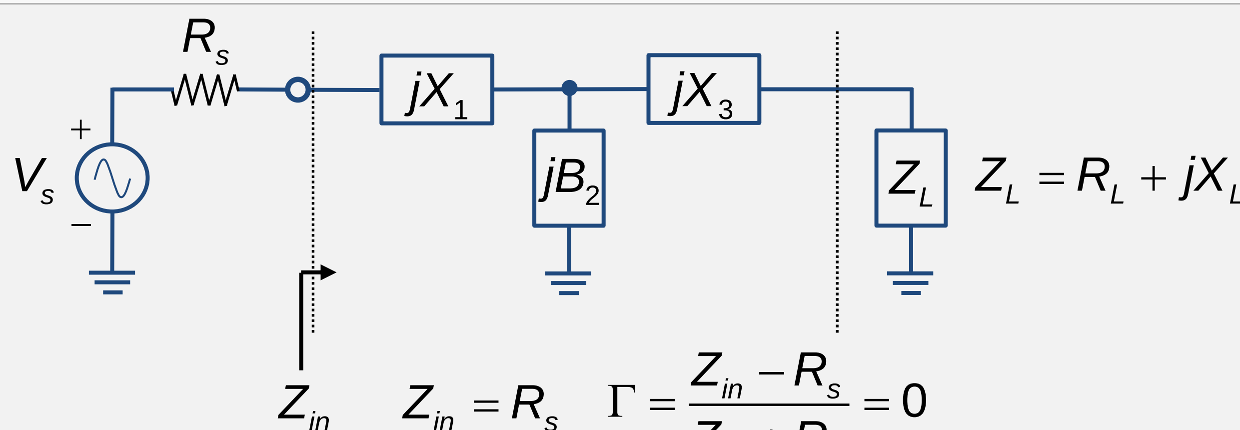
Three Elements Matching (High Q)

- Case (a) Pi-Shape Matching



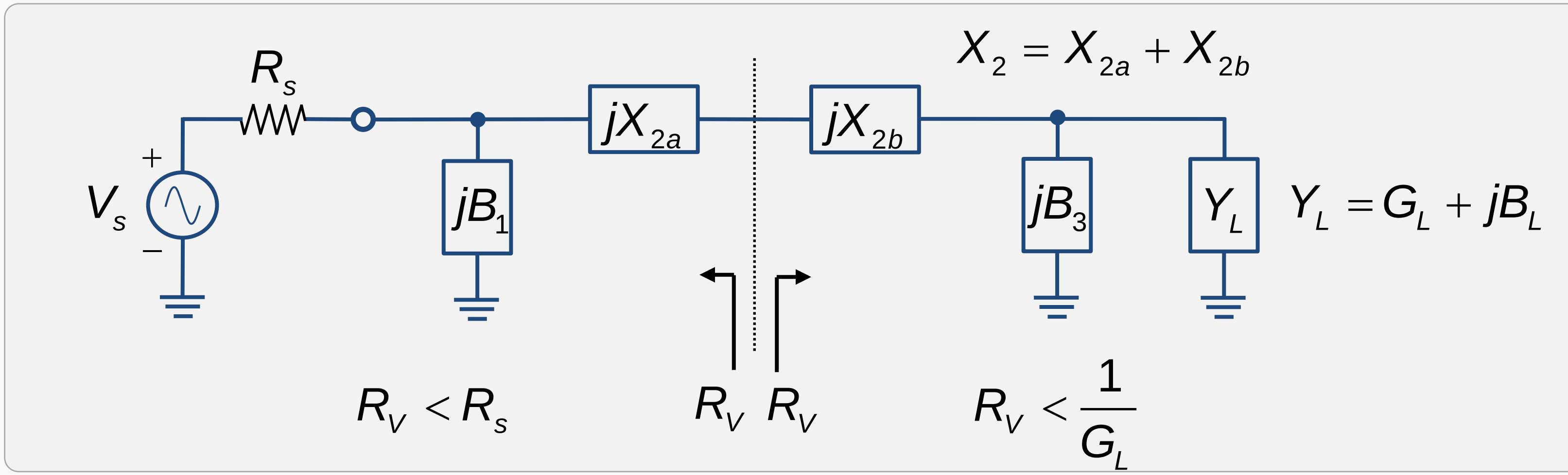
$Z_{in} = R_s$ $\quad \Gamma = \dfrac{Z_{in} - R_s}{Z_{in} + R_s} = 0$

- Case (b) T-Shape Matching



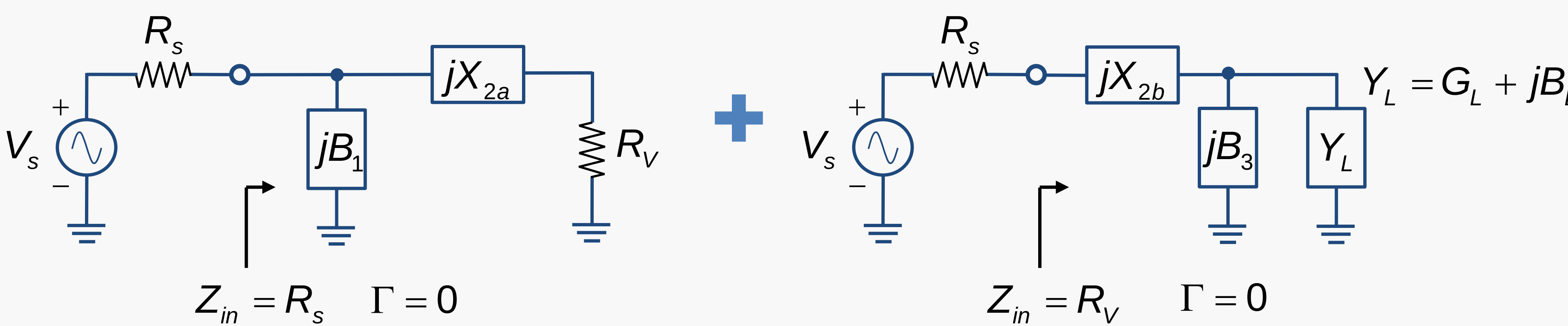
$Z_{in} = R_s$ $\quad \Gamma = \dfrac{Z_{in} - R_s}{Z_{in} + R_s} = 0$

Pi-Shape Matching

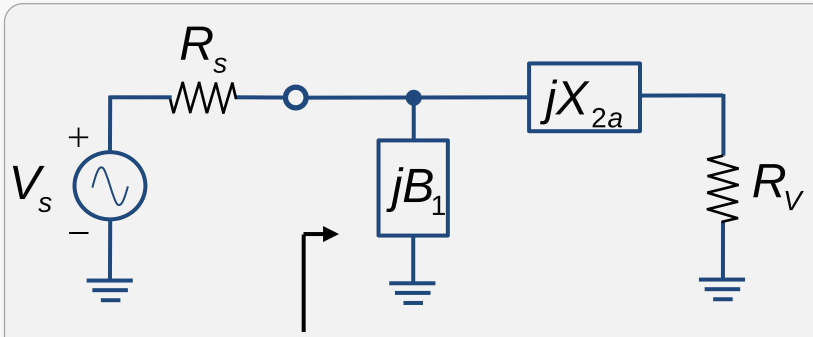


R_V : virtual resistance (designed by yourself)

Splitting into 2 "L-shape" matching networks

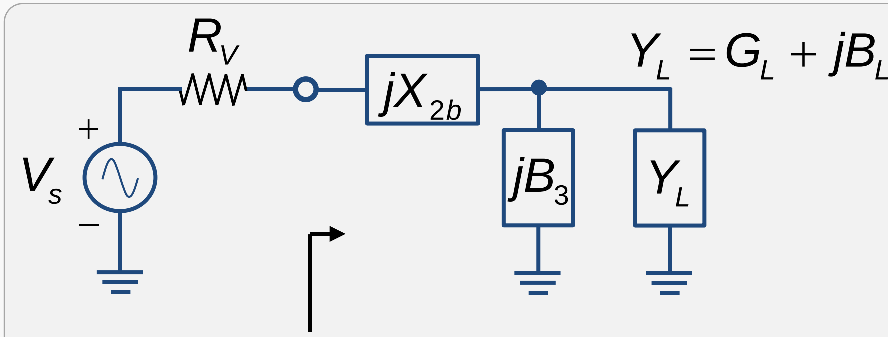


Pi-Shape Matching



$Z_{in} = R_s \qquad \Gamma = 0$

$$Q_1 = \pm\sqrt{\frac{R_s}{R_V} - 1}$$



$Z_{in} = R_V \qquad \Gamma = 0$

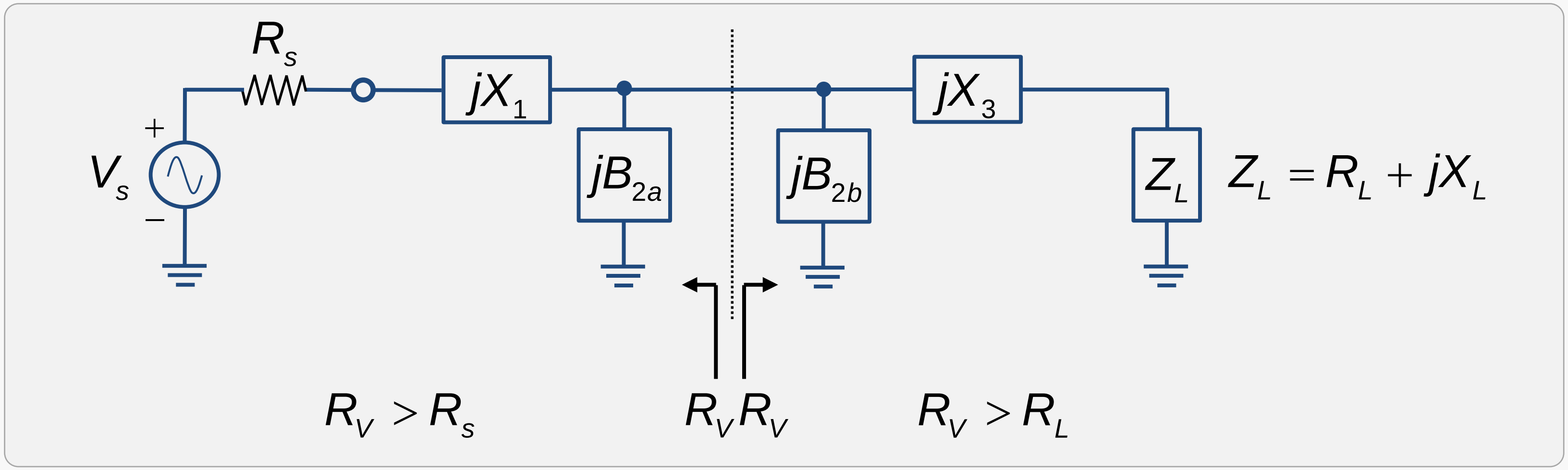
$$Q_2 = \pm\sqrt{\frac{1}{R_V G_L} - 1}$$

Since R_V **is small, you can get a high Q**

$$BW_1 = \frac{2}{|Q_1|} \qquad\qquad BW_2 = \frac{2}{|Q_2|}$$

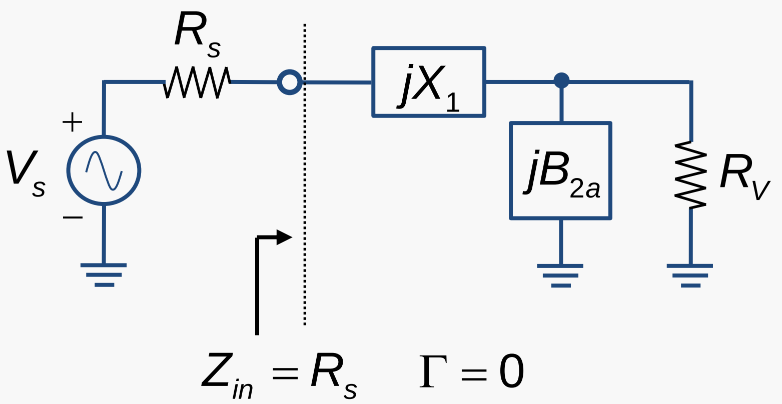
$$BW \simeq \min(BW_1, BW_2)$$

T-Shape Matching

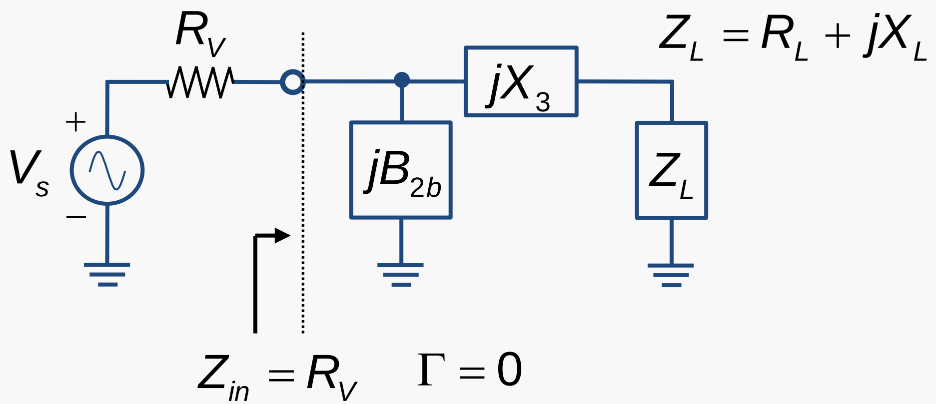


R_V : virtual resistance (designed by yourself)

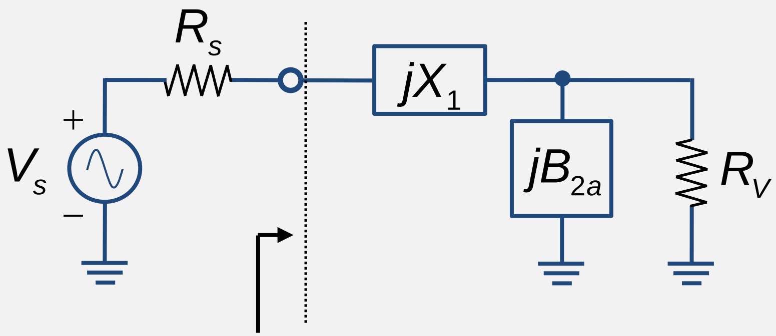
Splitting into 2 "L-shape" matching networks



+

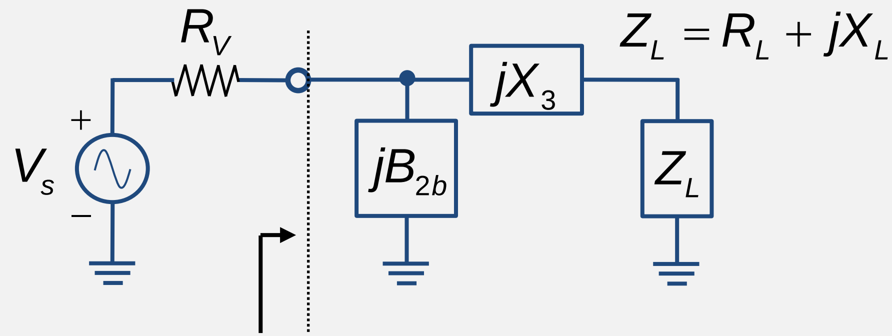


T-Shape Matching



$Z_{in} = R_s \quad \Gamma = 0$

$$Q_1 = \pm\sqrt{\frac{R_V}{R_s} - 1}$$



$Z_{in} = R_V \quad \Gamma = 0$

$$Q_2 = \pm\sqrt{\frac{R_V}{R_L} - 1}$$

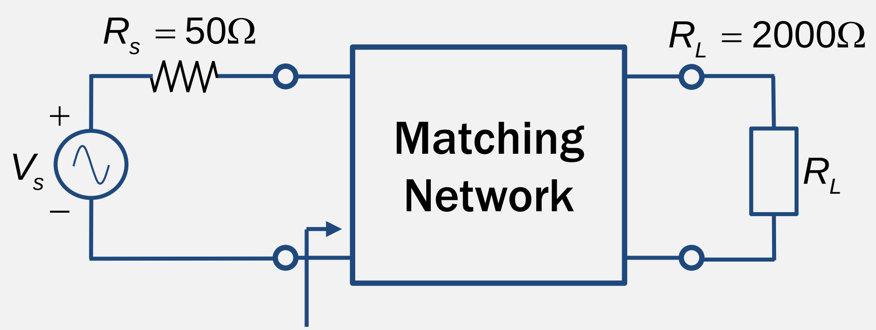
Since R_V is large, you can get a high Q

$$BW_1 = \frac{2}{|Q_1|} \qquad BW_2 = \frac{2}{|Q_2|}$$

$$BW \simeq \min(BW_1, BW_2)$$

Example – Pi and T Matching Networks

- Use Pi and T matching networks to achieve a matching BW<5%.



Goal: $Z_{in}\left(f = 100MHz\right) = R_s$

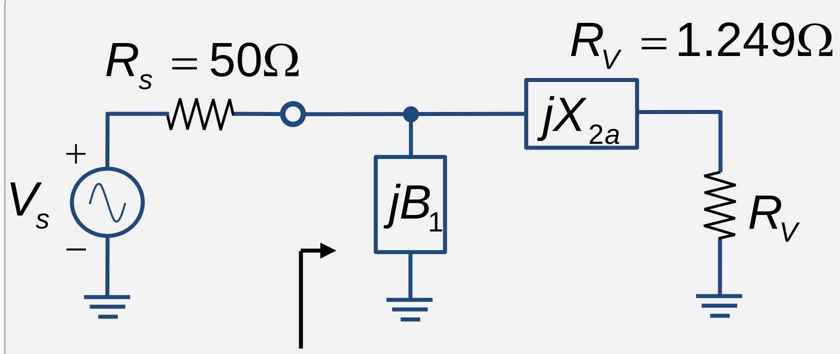
$\Gamma_{in}\left(f = 100MHz\right) = 0$

Example – Pi Matching Network (I)

- **Pi-section matching network**

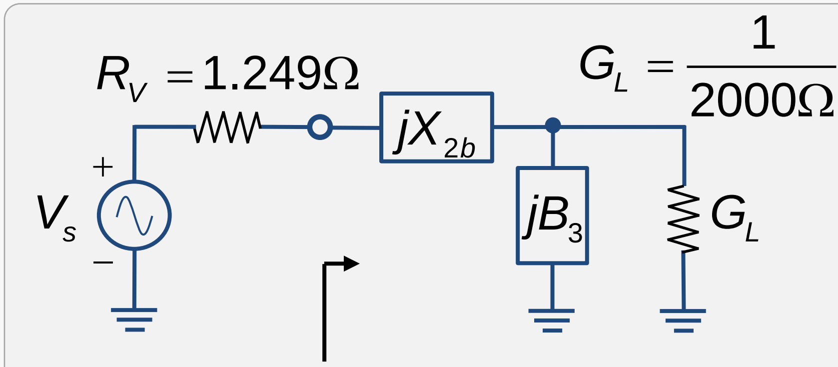
$$Q = \max(|Q_1|,|Q_2|) = \max\left(\sqrt{50/R_v - 1}, \sqrt{2000/R_v - 1}\right) = \sqrt{2000/R_v - 1}$$

$$BW = \frac{2}{\sqrt{(2000/R_v) - 1}} \le 5\% \Rightarrow R_v \le 1.249\ \Omega$$



$$Z_{in}(f = 100MHz) = 50\Omega$$

$$\Gamma(f = 100MHz) = 0$$



$$Z_{in}(f = 100MHz) = 1.2492\Omega$$

$$\Gamma(f = 100MHz) = 0$$

$$Q_1 = \pm\sqrt{\frac{R_s}{R_V} - 1} = \pm\sqrt{\frac{50}{1.2492} - 1} = \pm 6.247$$

$$Q_2 = \pm\sqrt{\frac{1}{R_V G_L} - 1} = \pm\sqrt{\frac{2000}{1.2492} - 1} = \pm 40$$

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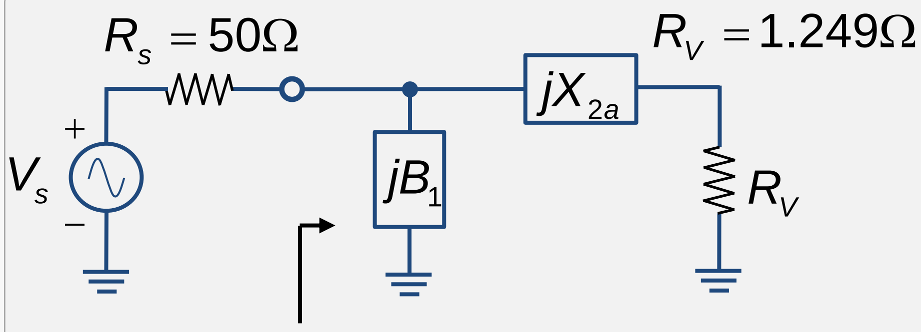
Example – Pi Matching Network (II)



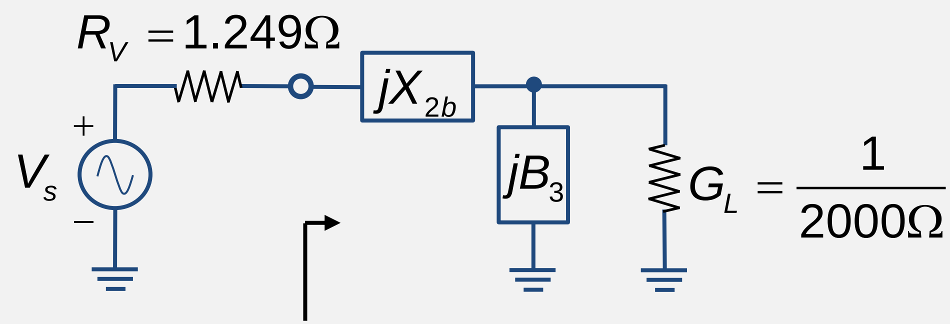
$$Z_{in}(f = 100MHz) = 50\Omega$$

$$\Gamma(f = 100MHz) = 0$$

$$Q_1 = +6.247 \Rightarrow \begin{cases} B_1 = \dfrac{Q_1}{R_s} = \omega_0 C & C = 198.85pF \\ X_{2a} = R_V Q_1 = \omega_0 L & L = 12.42nH \end{cases}$$

$$Q_1 = -6.247 \Rightarrow \begin{cases} B_1 = \dfrac{Q_1}{R_s} = \dfrac{-1}{\omega_0 L} & L = 12.74nH \\ X_{2a} = R_V Q_1 = \dfrac{-1}{\omega_0 C} & C = 203.95pF \end{cases}$$

Example – Pi Matching Network (III)



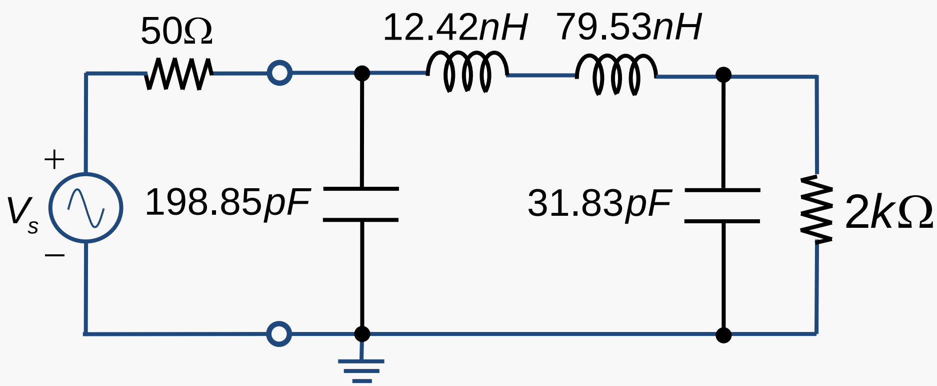
$$Z_{in}\left(f = 100MHz\right) = 1.2492\Omega$$

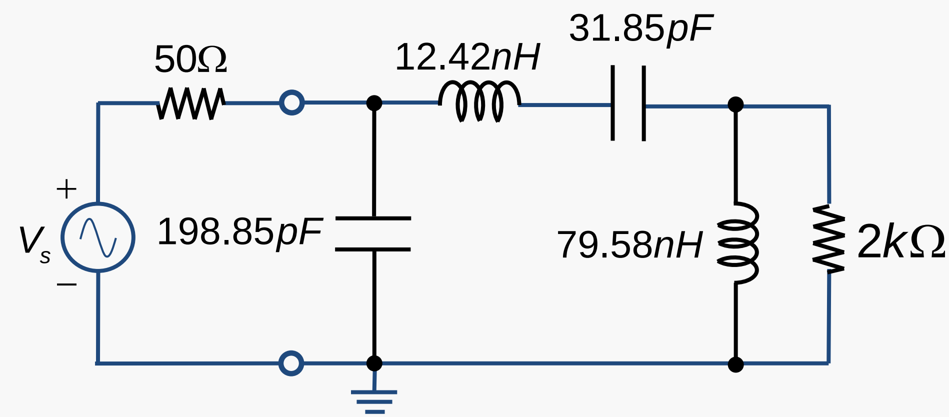
$$\Gamma\left(f = 100MHz\right) = 0$$

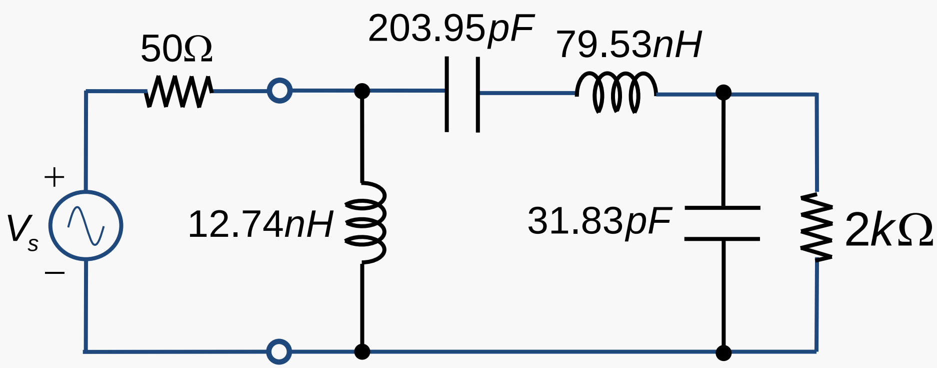
$$Q_2 = +40 \Rightarrow \begin{cases} B_3 = G_L Q_2 - B_L = \omega_0 C & C = 31.83pF \\ X_{2b} = R_V Q_2 = \omega_0 L & L = 79.53nH \end{cases}$$

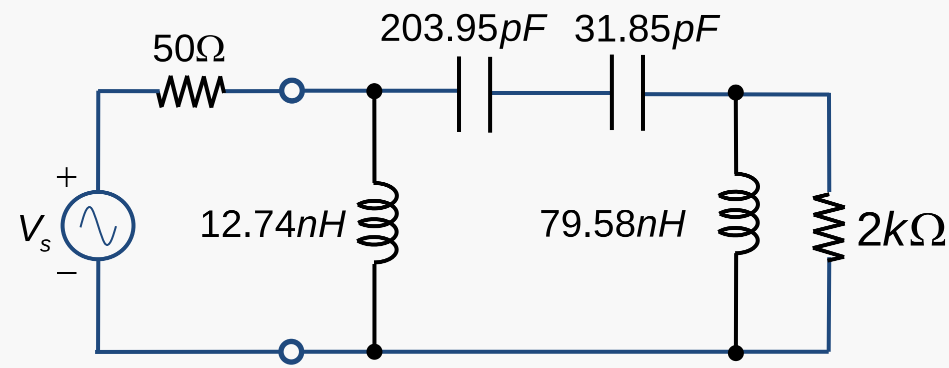
$$Q_2 = -40 \Rightarrow \begin{cases} B_3 = G_L Q_2 - B_L = \dfrac{-1}{\omega_0 L} & L = 79.58nH \\ X_{2b} = R_V Q_2 = \dfrac{-1}{\omega_0 C} & C = 31.85pF \end{cases}$$

Example – Pi Matching Network (IV)

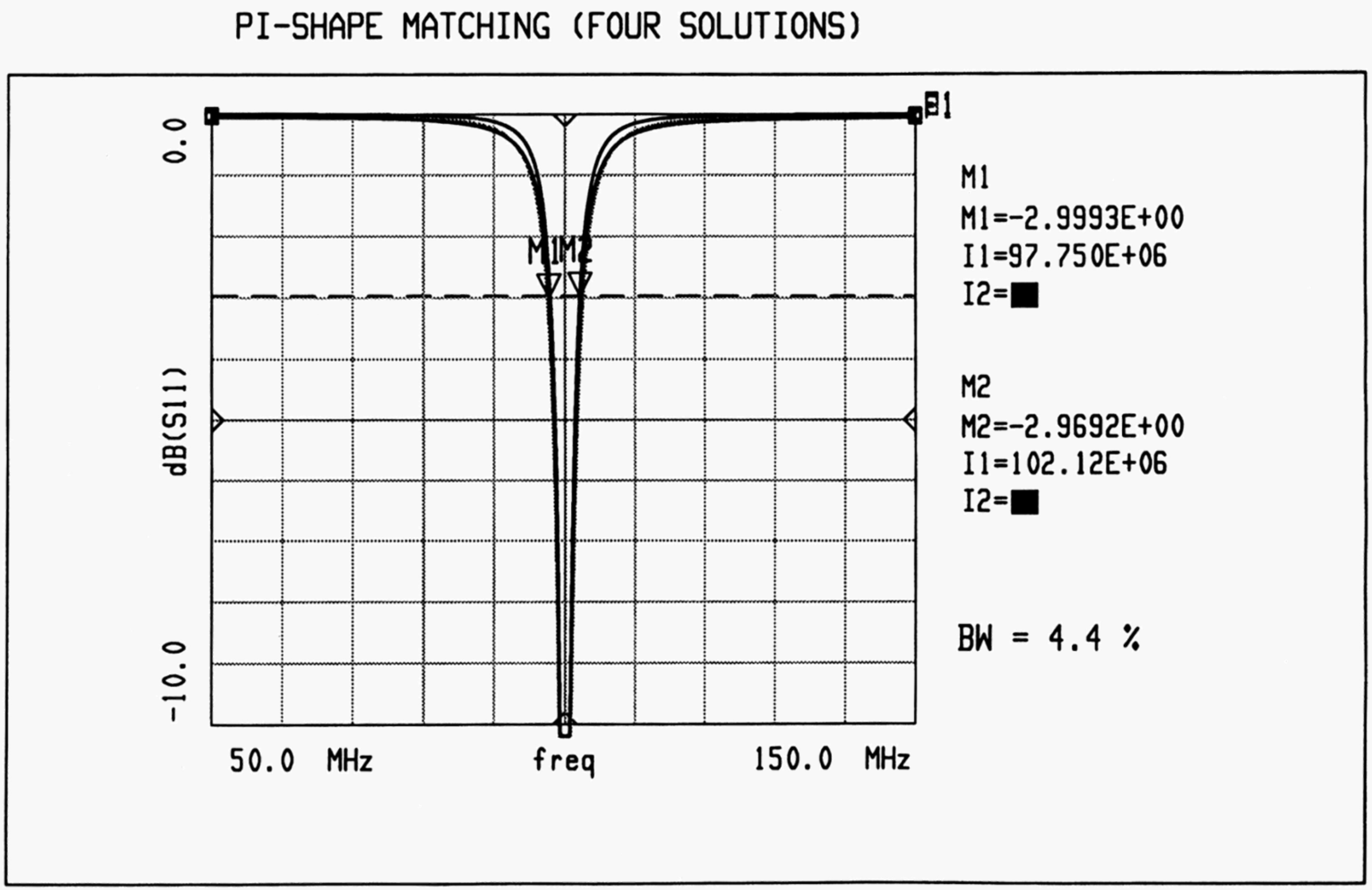








Example – Pi Matching Network (V)

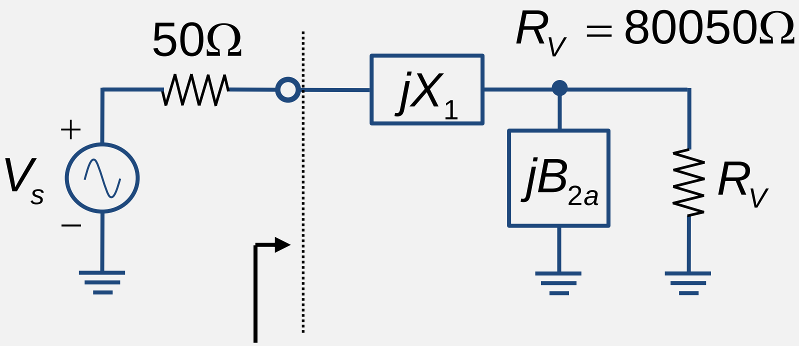


Example – T Matching Network (I)

- **T-section matching network**

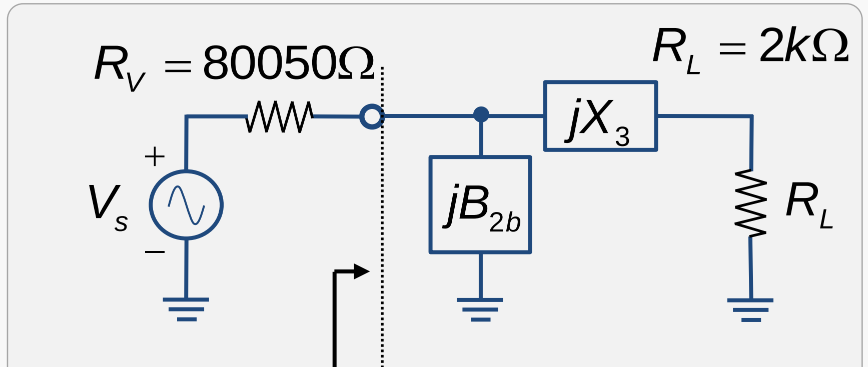
$$Q=\max(|Q_1|,|Q_2|)=\max\left(\sqrt{(R_v/50)-1},\sqrt{(R_v/2000)-1}\right)=\sqrt{(R_v/50)-1}$$

$$BW=\frac{2}{\sqrt{(R_v/50)-1}}\leq 5\%\Rightarrow R_v\geq 80050\ \Omega$$



$$Z_{in}(f=100MHz)=50\Omega$$

$$\Gamma(f=100MHz)=0$$



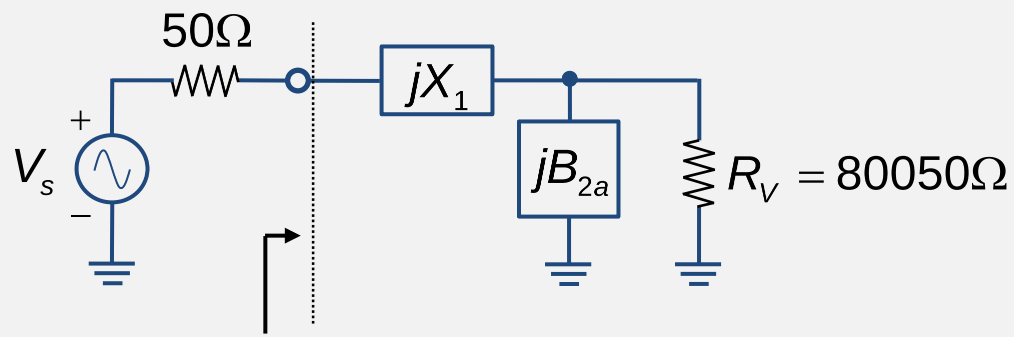
$$Z_{in}(f=100MHz)=80050\Omega$$

$$\Gamma(f=100MHz)=0$$

$$Q_1=\pm\sqrt{\frac{R_V}{R_s}-1}=\pm\sqrt{\frac{80050}{50}-1}=\pm 40$$

$$Q_2=\pm\sqrt{\frac{R_V}{R_L}-1}=\pm\sqrt{\frac{80050}{2000}-1}=\pm 6.247$$

Example – T Matching Network (II)



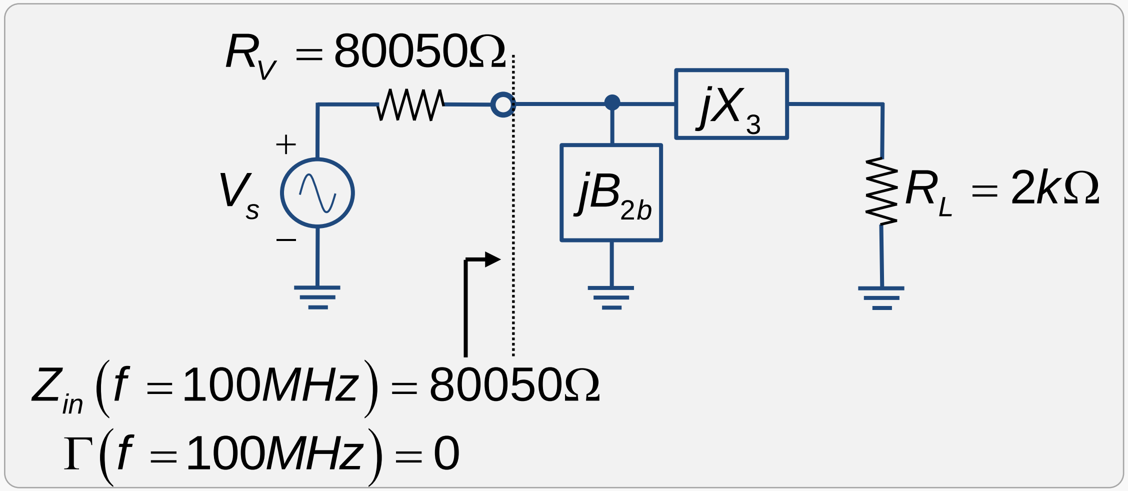
$$Z_{in}(f=100MHz)=50\Omega$$

$$\Gamma(f=100MHz)=0$$

$$Q_1=+40 \Rightarrow \begin{cases} X_1=R_sQ_1=\omega_0 L & L=3.183\mu H \\ B_{2a}=\dfrac{Q_1}{R_V}=\omega_0 C & C=0.795pF \end{cases}$$

$$Q_1=-40 \Rightarrow \begin{cases} X_1=R_sQ_1=\dfrac{-1}{\omega_0 C} & C=0.796pF \\ B_{2a}=\dfrac{Q_1}{R_V}=\dfrac{-1}{\omega_0 L} & L=3.185\mu H \end{cases}$$

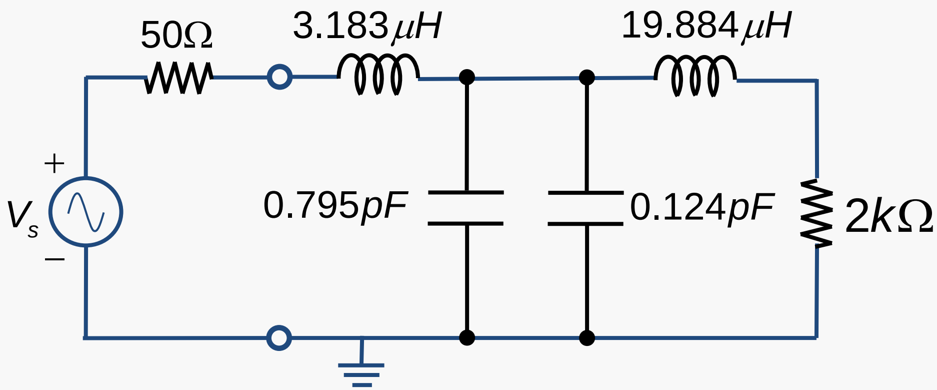
Example – T Matching Network (III)

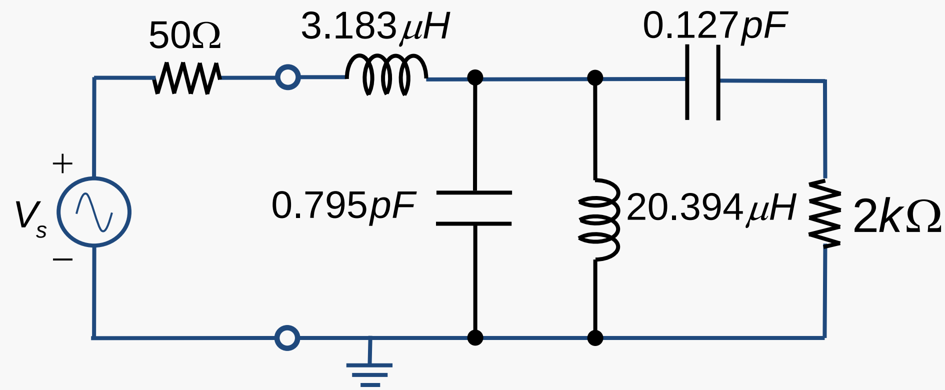


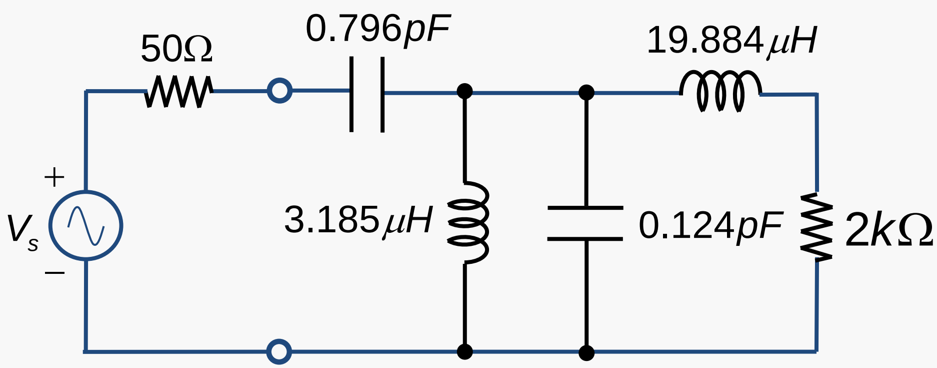
$$Q_2 = +6.247 \Rightarrow \begin{cases} B_{2b} = \dfrac{Q_2}{R_V} = \omega_0 C & C = 0.124pF \\ X_3 = R_L Q_2 - X_L = \omega_0 L & L = 19.884\mu H \end{cases}$$

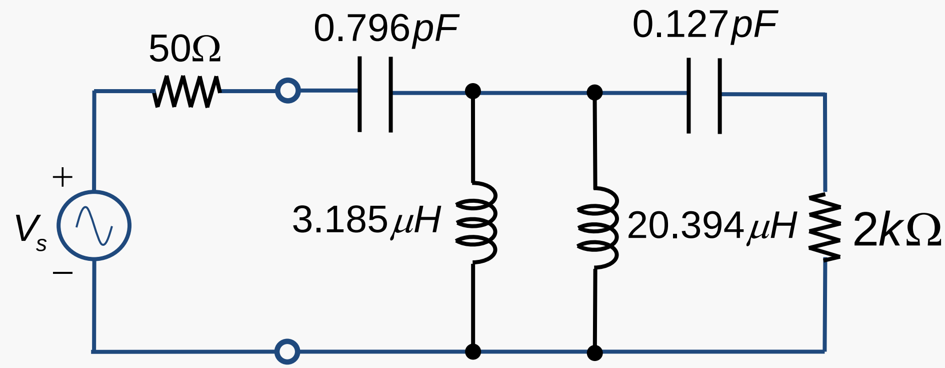
$$Q_2 = -6.247 \Rightarrow \begin{cases} B_{2b} = \dfrac{Q_2}{R_V} = \dfrac{-1}{\omega_0 L} & L = 20.394\mu H \\ X_3 = R_L Q_2 - X_L = \dfrac{-1}{\omega_0 C} & C = 0.127pF \end{cases}$$

Example – T Matching Network (IV)



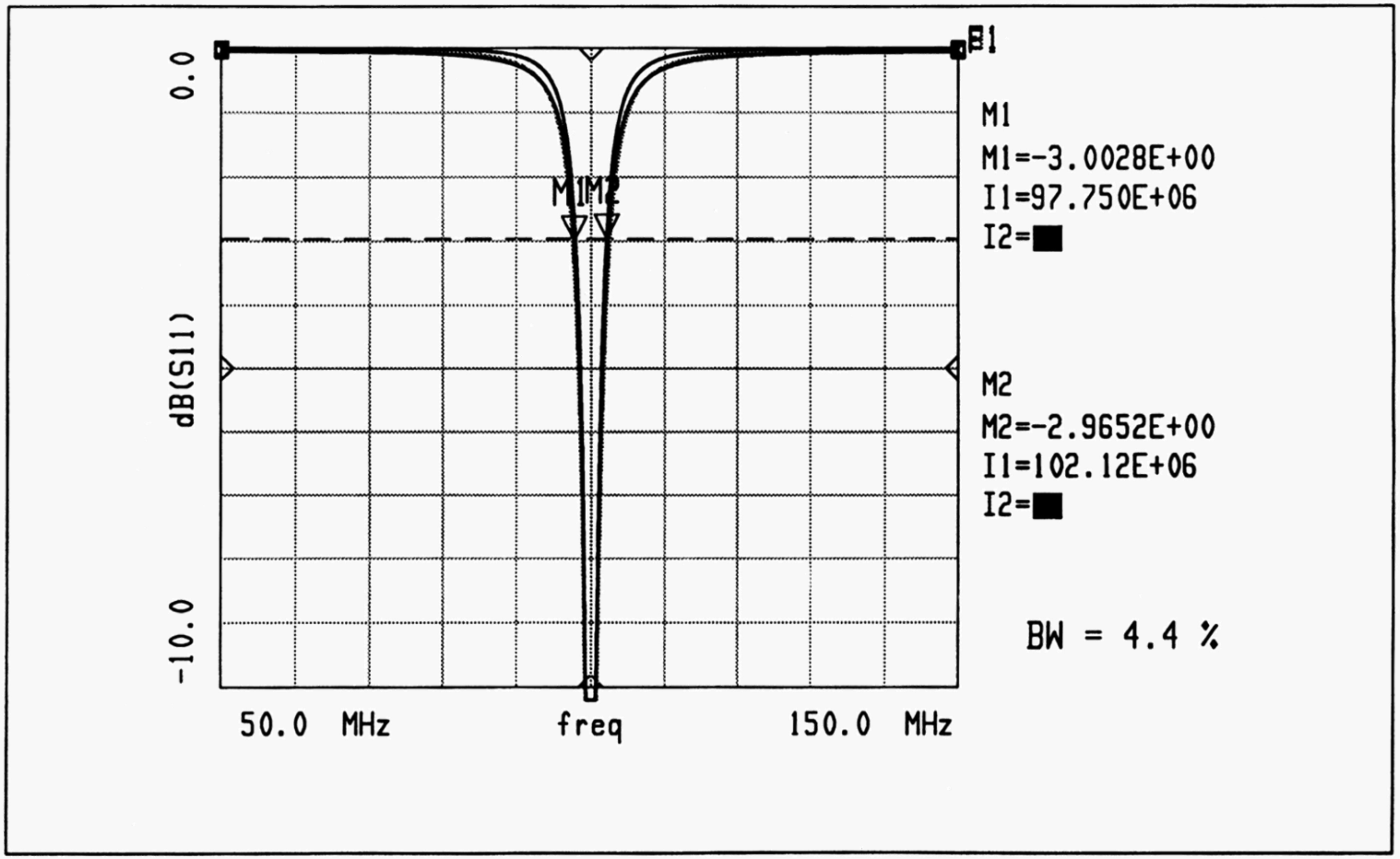






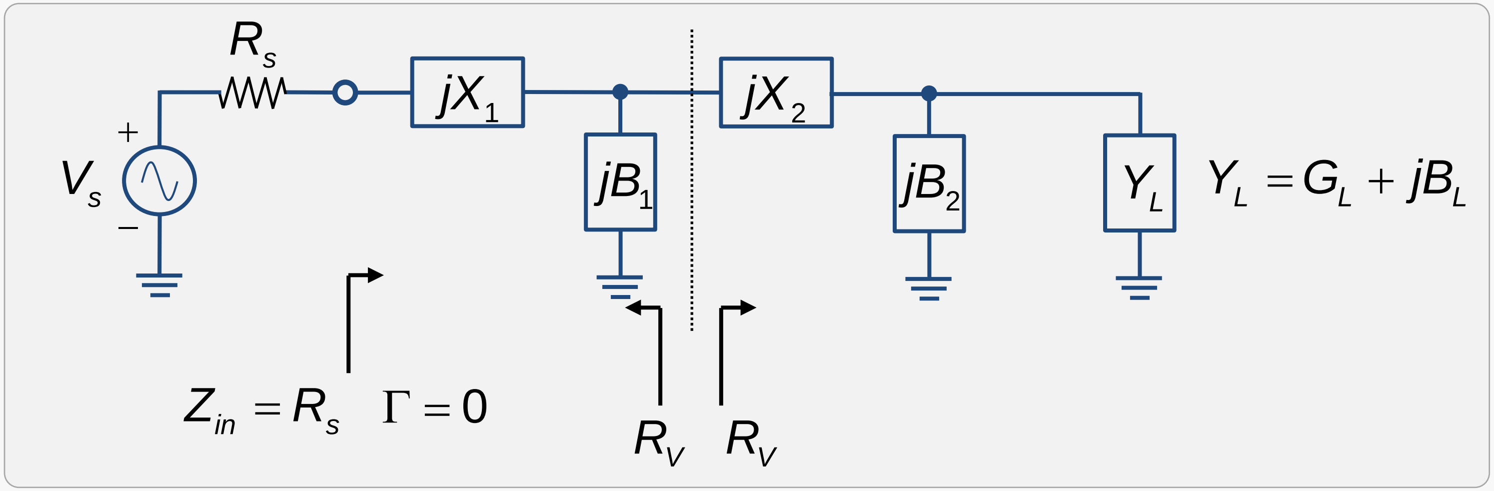
Example – T Matching Network (V)

T-SHAPE MATCHING (FOUR SOLUTIONS)

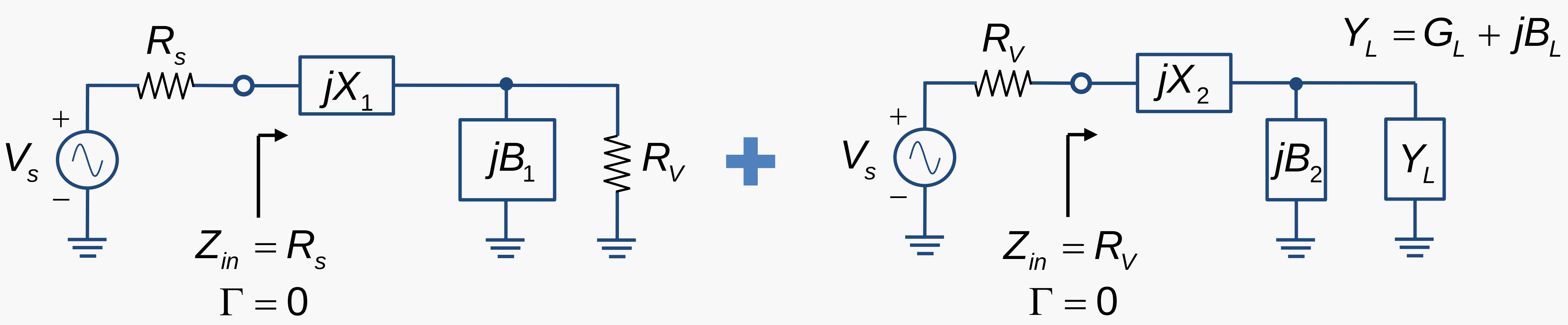


Cascaded L-Shape Matching (Low Q)

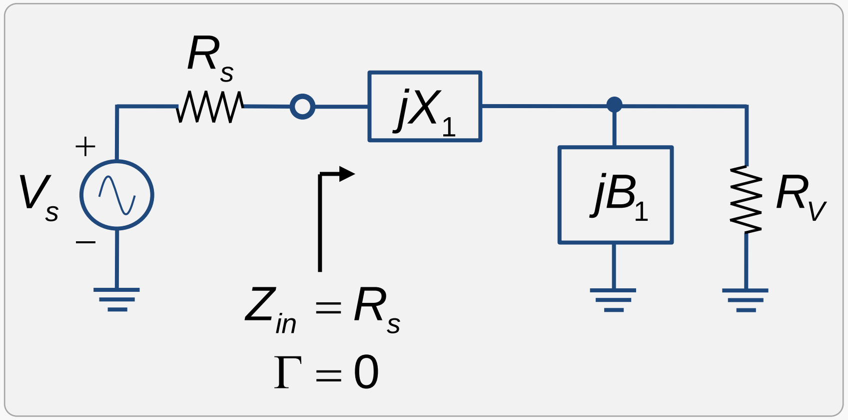
- Case (a) $R_s < R_V < \frac{1}{G_L}$

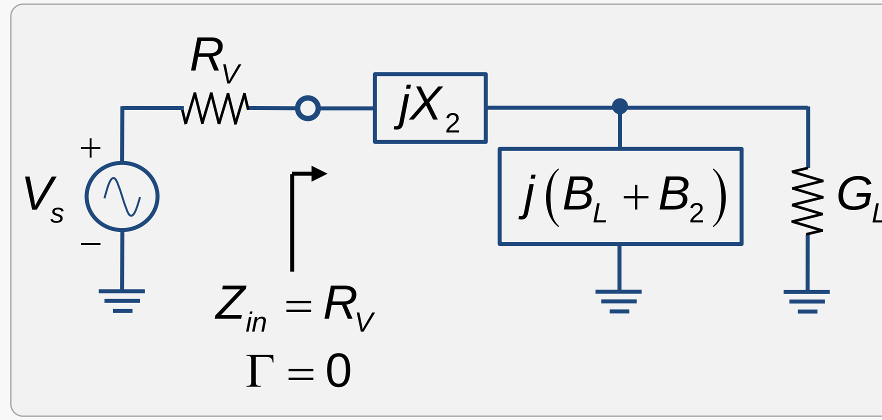


Splitting into 2 "L-shape" matching networks



Cascaded L-Shape Matching (Low Q)





$$Q_1 = \pm\sqrt{\frac{R_V}{R_s} - 1} \qquad Q_2 = \pm\sqrt{\frac{1}{G_L R_V} - 1}$$

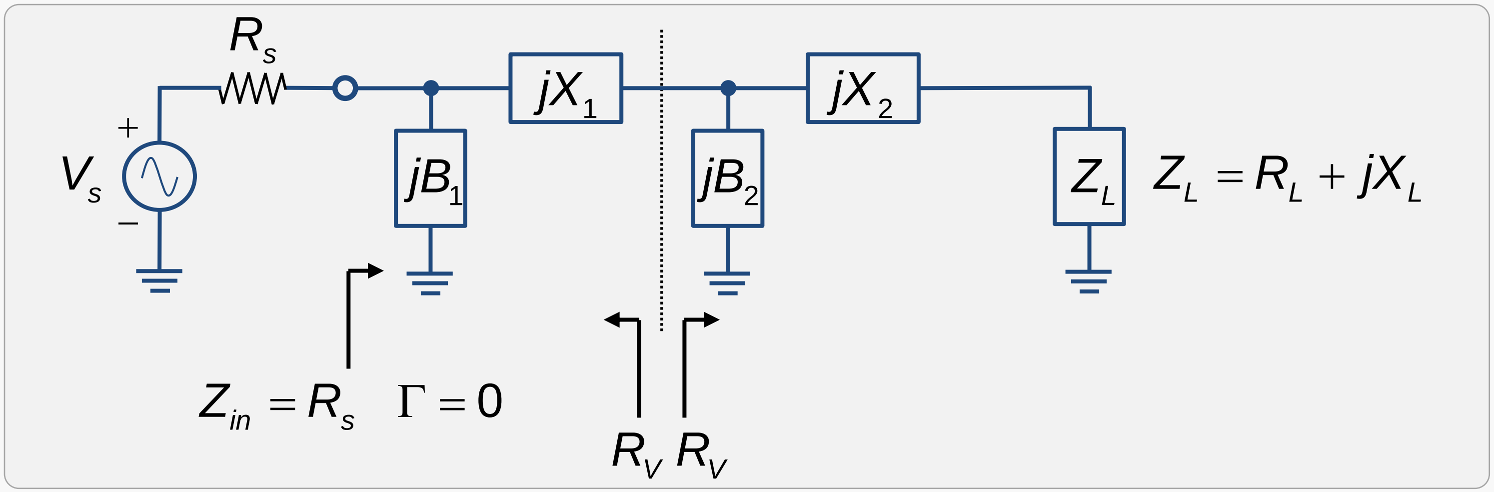
We want to have the maximum bandwidth (find minimum Q)

Let $|Q_1| = |Q_2| = Q \Rightarrow R_V = \sqrt{\dfrac{R_s}{G_L}}$

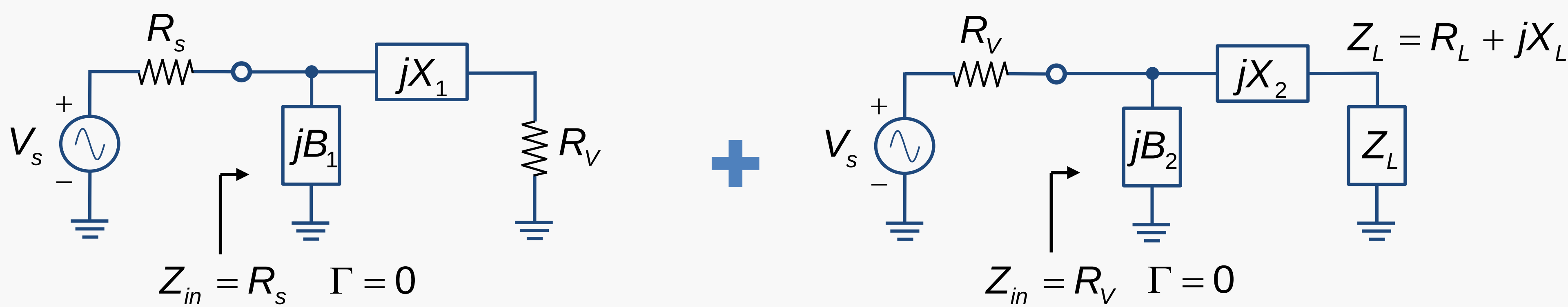
$$BW \simeq \frac{2}{Q\sqrt{2}} = \frac{\sqrt{2}}{Q} = \frac{\sqrt{2}}{\sqrt{\dfrac{1}{R_s G_L} - 1}}$$

Cascaded L-Shape Matching (Low Q)

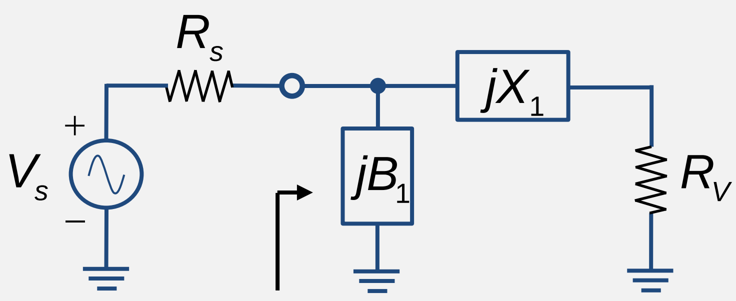
- **Case (b)** $R_s > R_V > R_L$



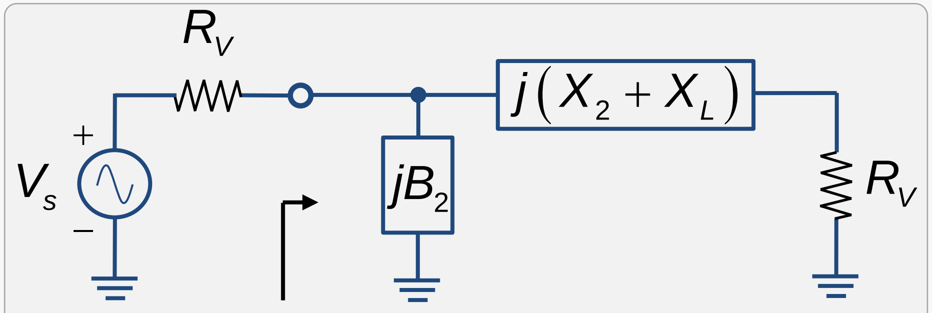
Splitting into 2 "L-shape" matching networks



Cascaded L-Shape Matching (Low Q)



$Z_{in} = R_s \quad \Gamma = 0$



$Z_{in} = R_V \quad \Gamma = 0$

$$Q_1 = \pm\sqrt{\frac{R_s}{R_V} - 1} \qquad Q_2 = \pm\sqrt{\frac{R_V}{R_L} - 1}$$

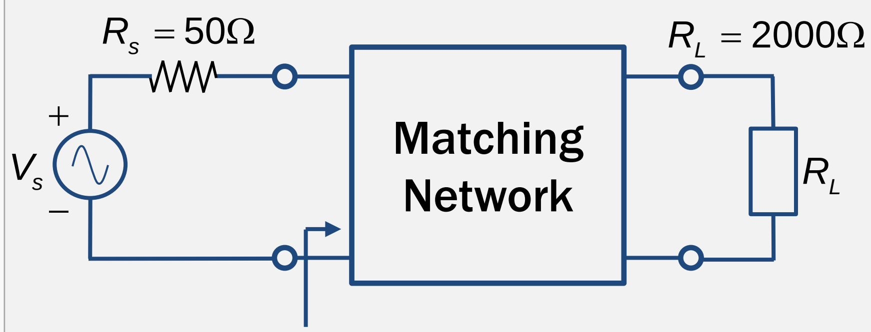
We want to have the maximum bandwidth (find minimum Q)

Let $|Q_1| = |Q_2| = Q$ $\Rightarrow$ $R_V = \sqrt{R_s R_L}$

$$BW \simeq \frac{2}{Q\sqrt{2}} = \frac{\sqrt{2}}{Q} = \frac{\sqrt{2}}{\sqrt{\frac{1}{\sqrt{R_s G_L}} - 1}}$$

Example – Cascaded L-Shape Matching (I)

- Use Cascaded L-shape matching networks to achieve BW>60%.



Goal: $Z_{in}(f = 100MHz) = R_s$

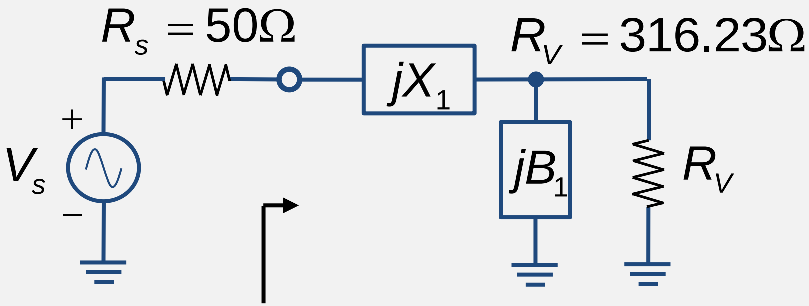
$\Gamma_{in}(f = 100MHz) = 0$

Choose R_V to let $R_s < R_V < R_L$

$$R_V = \sqrt{R_s R_L} = \sqrt{50 \cdot 2000} = 316.23$$

$$BW = \frac{\sqrt{2}}{\sqrt{\frac{1}{\sqrt{R_s G_L}} - 1}} = \frac{\sqrt{2}}{\sqrt{\sqrt{2000/50} - 1}} = 61.29\%$$

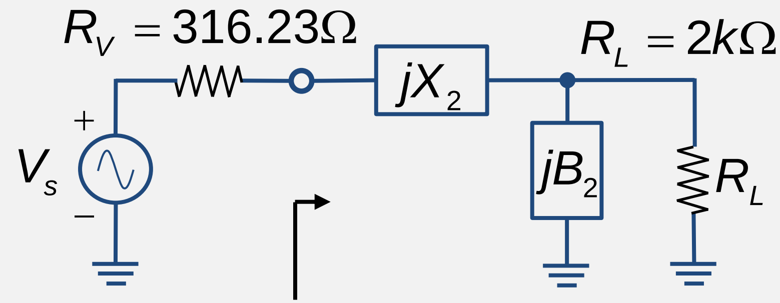
Example – Cascaded L-Shape Matching (II)



$$Z_{in}\left(f = 100MHz\right) = 50\Omega$$

$$\Gamma\left(f = 100MHz\right) = 0$$

+



$$Z_{in}\left(f = 100MHz\right) = 316.23\Omega$$

$$\Gamma\left(f = 100MHz\right) = 0$$

$$Q_1 = \pm\sqrt{\frac{R_V}{R_s} - 1} = \pm\sqrt{\frac{316.23}{50} - 1} = \pm 2.3075$$

$$Q_2 = \pm\sqrt{\frac{R_L}{R_V} - 1} = \pm\sqrt{\frac{2000}{316.23} - 1} = \pm 2.3075$$

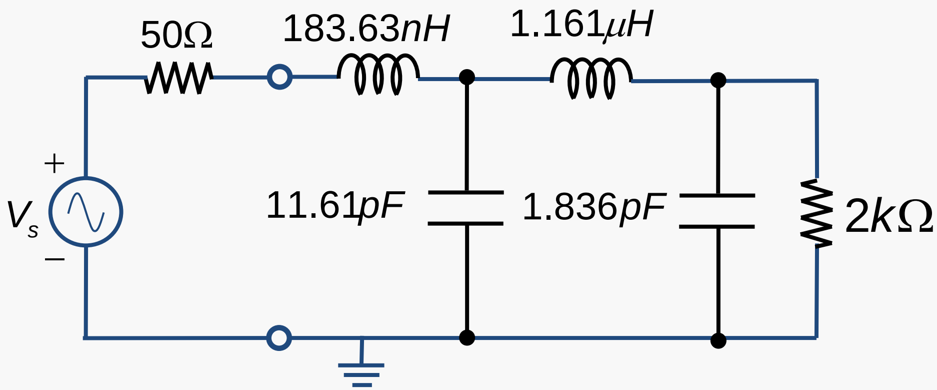
$$Q_1 = +2.3075 \Rightarrow \begin{cases} X_1 = R_s Q_1 = \omega_0 L & L = 183.63nH \\ B_1 = \dfrac{Q_1}{R_V} = \omega_0 C & C = 11.61pF \end{cases}$$

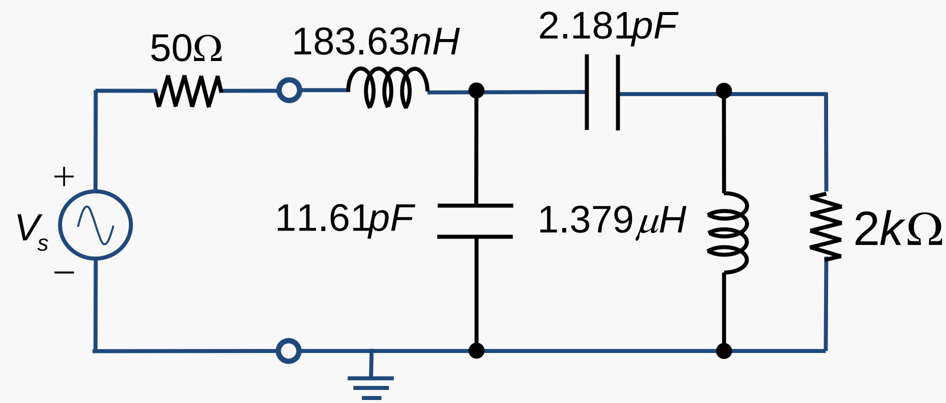
$$Q_2 = +2.3075 \Rightarrow \begin{cases} X_2 = R_V Q_2 = \omega_0 L & L = 1.161\mu H \\ B_2 = \dfrac{Q_2}{R_L} - B_L = \omega_0 C & C = 1.836pF \end{cases}$$

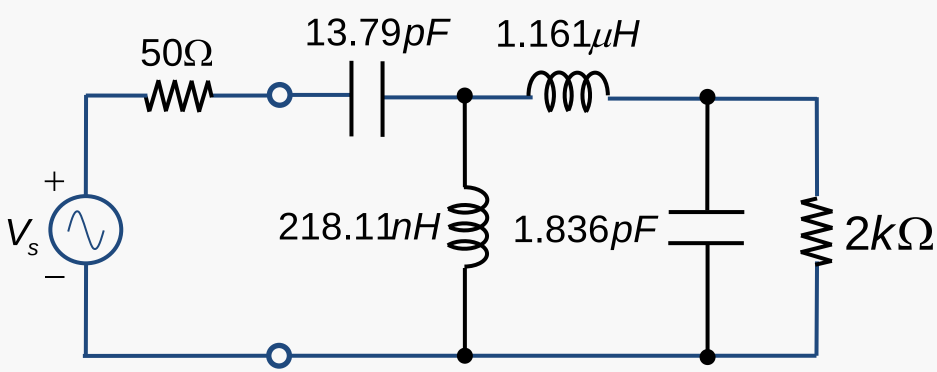
$$Q_1 = -2.3075 \Rightarrow \begin{cases} X_1 = R_s Q_1 = \dfrac{-1}{\omega_0 C} & C = 13.79pF \\ B_1 = \dfrac{Q_1}{R_V} = \dfrac{-1}{\omega_0 L} & L = 218.11nH \end{cases}$$

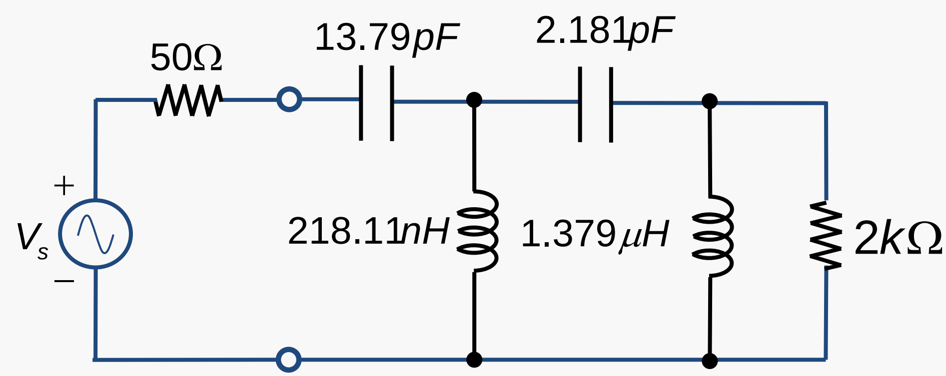
$$Q_2 = -2.3075 \Rightarrow \begin{cases} X_2 = R_V Q_2 = \dfrac{-1}{\omega_0 C} & C = 2.181pF \\ B_2 = \dfrac{Q_2}{R_L} - B_L = \dfrac{-1}{\omega_0 L} & L = 1.379\mu H \end{cases}$$

Example – Cascaded L-Shape Matching (III)

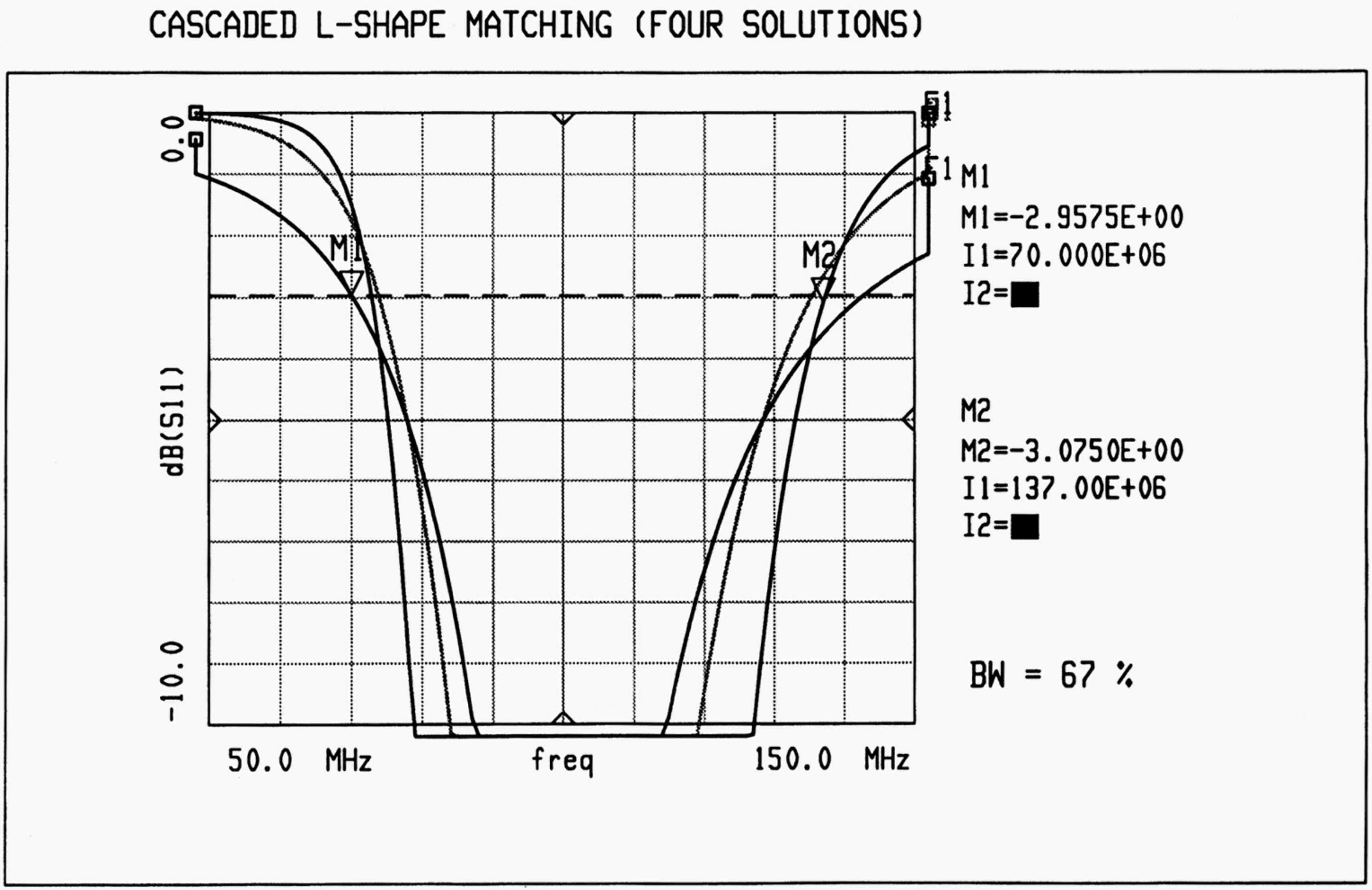






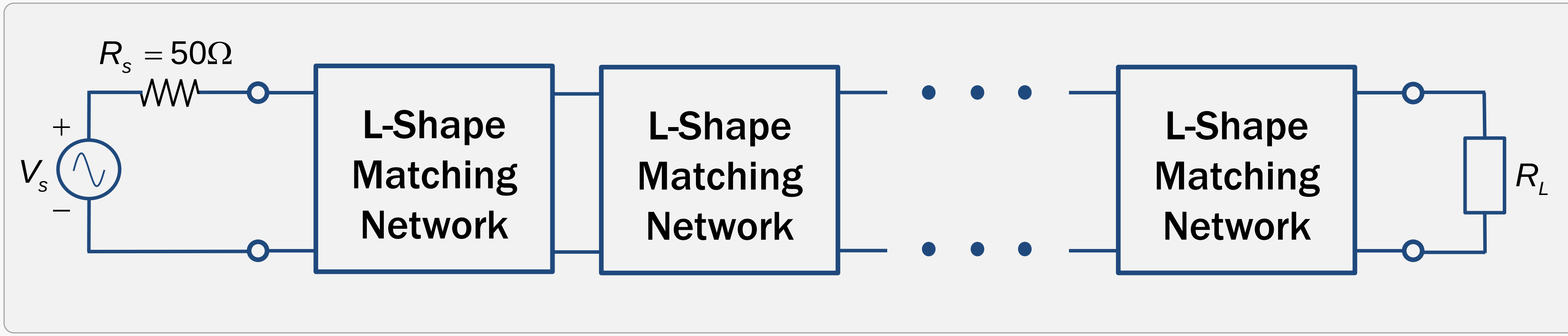


Example – Cascaded L-Shape Matching (IV)

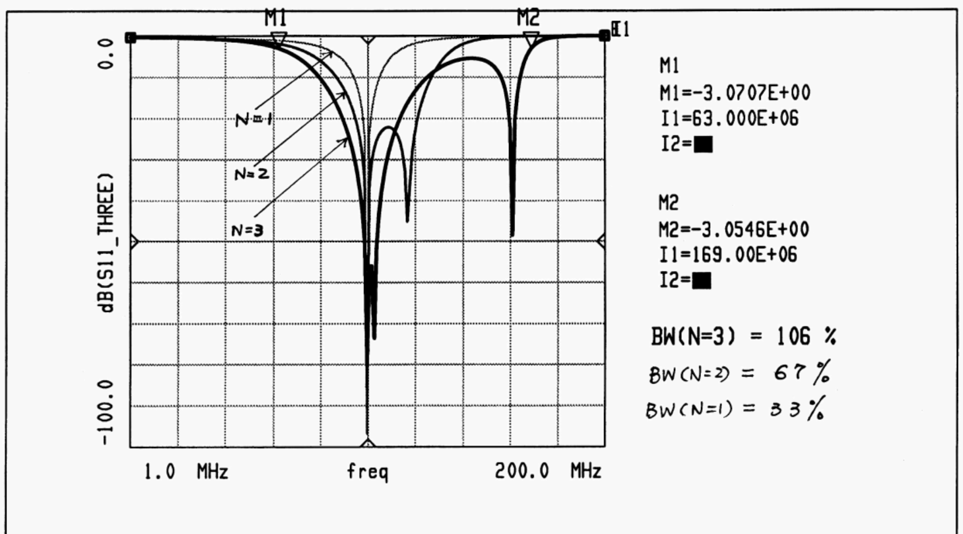


Matching For Larger Bandwidth

- Use multi-section L-shape matching networks to extend BW.



CASCADED L-SHAPE MATCHING



Summary

- Impedance matching is a procedure of transforming the complex load impedance Z_L to match with the source impedance Z_s or characteristics impedance Z_o of the transmission line (Generally, $Z_s = R_s = Z_o$).
- The lossless lumped element matching networks that are commonly used include L-section, Pi-section, T-section, and cascaded L-section. L-section has medium but not adjustable matching bandwidth. Pi and T sections have rather small and adjustable matching bandwidth. Cascaded L-section can increase bandwidth as the number of stages increases.