

It start with the curl Maxwell equations:

$$\begin{aligned}\frac{\partial \vec{B}}{\partial t} + \vec{\nabla} \wedge \vec{E} &= 0 \\ \frac{\partial \vec{D}}{\partial t} - \vec{\nabla} \wedge \vec{H} &= 0\end{aligned}$$

and the constitutive relations:

$$\begin{aligned}\vec{D} &= \epsilon_0 \vec{E} + \vec{P} & (1) \\ \vec{H} &= -\frac{1}{\mu_0} \vec{B} & (2)\end{aligned}$$

For linear and isotropic media that do not exhibit temporal and spatial dispersion, $\vec{P} = \epsilon_0 \chi \vec{E}$ and (1) becomes:

$$\vec{D} = \epsilon_0 \tilde{\epsilon}_r \vec{E} \quad (3)$$

$\tilde{\epsilon}_r$ and χ are complex scalar related by $\tilde{\epsilon}_r = 1 + \chi$. $\tilde{\epsilon}_r$ can be calculated from \tilde{n} , the complex refractive index of the material, via $\tilde{\epsilon}_r = \tilde{n}^2$. Using (2) and (3), Maxwell's curl equations become:

$$\begin{aligned}\mu_0 \frac{\partial \vec{H}}{\partial t} + \vec{\nabla} \wedge \vec{E} &= 0 \\ \epsilon_0 \tilde{\epsilon}_r \frac{\partial \vec{E}}{\partial t} - \vec{\nabla} \wedge \vec{H} &= 0\end{aligned} \quad (4)$$

Expressing (4) in the Fourier domain:

$$\begin{aligned}-\epsilon_0 (Re(\tilde{\epsilon}_r) + jIm(\tilde{\epsilon}_r)) j\omega \vec{E} &= \vec{\nabla} \wedge \vec{H} \\ -j\omega \epsilon_0 Re(\tilde{\epsilon}_r) \vec{E} + \epsilon_0 \omega Im(\tilde{\epsilon}_r) \vec{E} &= \vec{\nabla} \wedge \vec{H}\end{aligned}$$

Going back in the time domain:

$$\epsilon_0 Re(\tilde{\epsilon}_r) \frac{\partial \vec{E}}{\partial t} + \epsilon_0 \omega Im(\tilde{\epsilon}_r) \vec{E} = \vec{\nabla} \wedge \vec{H}$$

Using different notations:

$$\begin{aligned}\epsilon_r &= \text{Re}(\tilde{\epsilon}_r) \\ \sigma &= \epsilon_0 \omega \text{Im}(\tilde{\epsilon}_r)\end{aligned}$$

ϵ_r and σ are linked to $\text{Re}(\tilde{\epsilon}_r)$ and $\text{Im}(\tilde{\epsilon}_r)$. Since $\tilde{\epsilon}_r = \tilde{n}^2$ it is possible to deduce ϵ_r and σ from $\tilde{n} = n + jk$:

$$\begin{aligned}\epsilon_r &= n^2 - k^2 \\ \sigma &= 2\epsilon_0 \omega n k\end{aligned}$$

Maxwell's curl equations are finally:

$$\begin{aligned}\mu_0 \frac{\partial \vec{H}}{\partial t} + \vec{\nabla} \wedge \vec{E} &= 0 \\ \epsilon_0 \epsilon_r \frac{\partial \vec{E}}{\partial t} + \sigma \vec{E} &= \vec{\nabla} \wedge \vec{H}\end{aligned}$$