EE 451 PROJECT FINAL REPORT
(ALAMOUTI SPACE TIME BLOCK CODING)

Mert TAN

IZMIR INSTITUTE OF TECHNOLOGY (IZTECH)

ABSTRACT

In this project our purpose is that, peformance analysis of Alamouti STBC and also comparing with SISO performance. Article includes definition of SISO system,STBC, after that Alamouti STBC theory and its mathematical expressions. Also we define process of project and lastly we give our results for each of SISO, 2x1 and 2x2 Alamouti STBC.

INTRODUCTION

1) SISO(Single Input Single Output) SYSTEM

In SISO system, we have just one Transmitter and one Receiver so we have just one channel and that means that there is no diversity. In other words, data is transmitted just one channel and if the channel damage data, received data is different from transmitted data so, communication can not be provided normally. All in all Bit-Error-Rate can be high which is not acceptable in communication.
Figure 2 shows that working principle of SISO system.

(Figure 2 is in "PROCESS OF PROJECT" which we discuss detailed)

2) SPACE-TIME BLOCK CODING

Space–time block coding is a technique used in wireless communications to transmit multiple copies of a data stream across a number of antennas and to exploit the various received versions of the data to improve the reliability of data-transfer. In other words, if we compare with SISO, we have more than one channel between Transmitters and Receivers so, there are diversity. Thanks to this diversity, data and its copies is send on several channels and the Bit-Error-Rate is less than SISO system.

3) ALAMOUTI SPACE-TIME BLOCK CODING

Alamouti STBC is a complex space-time diversity technique that can be used in 2x1 MISO mode or in a 2x2 MIMO mode. It is the only STBC that can achivie its full diversty gain without needing to sacrifice its data rate. this property usually gives Alamouti's code a significant advantage over the higher-order STBCs even though they achieve a better error-rate performance.

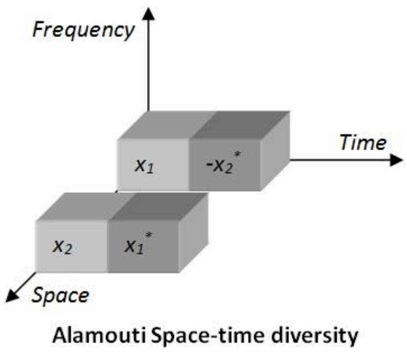

Figure 1

Figure 1 shows that two symbols and their conjugate are sent, in two time slots, which brings a diversity gain without having to compromise on the data rate. Over the air, the transmitted symbols will suffer from channel fading and noise at the receiver, their sum will be received. Working principles of 2x1 and 2x2 Alamouti STBC is indicated Figure 3 and Figure 4 respectively.

(Figures are in "PROCESS OF PROJECT" which we discuss detailed)

PROCESS OF PROJECT

A) Mathematical Expressions

1) Single Input Single Output (SISO)

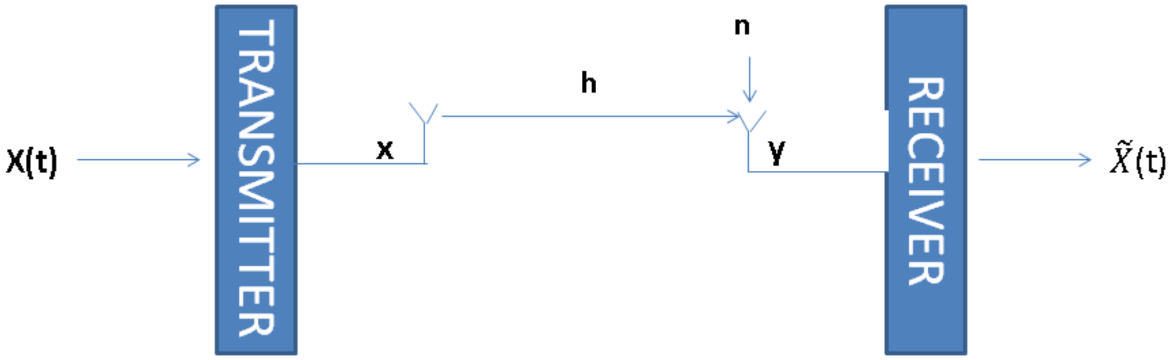

Figure 2

$y = xh + n$ (Eq 1.1)

x is the transmitted data
h is the Rayleigh channel
n is the AWGN noise
y is the received data

In order to get transmitted data at the receiver with noise and channel effect we divide **h** both side of **(Eq 1.1)** ;

$$\tilde{x} = y/h = x + n/h$$

2) 2x1 Alamouti STBC

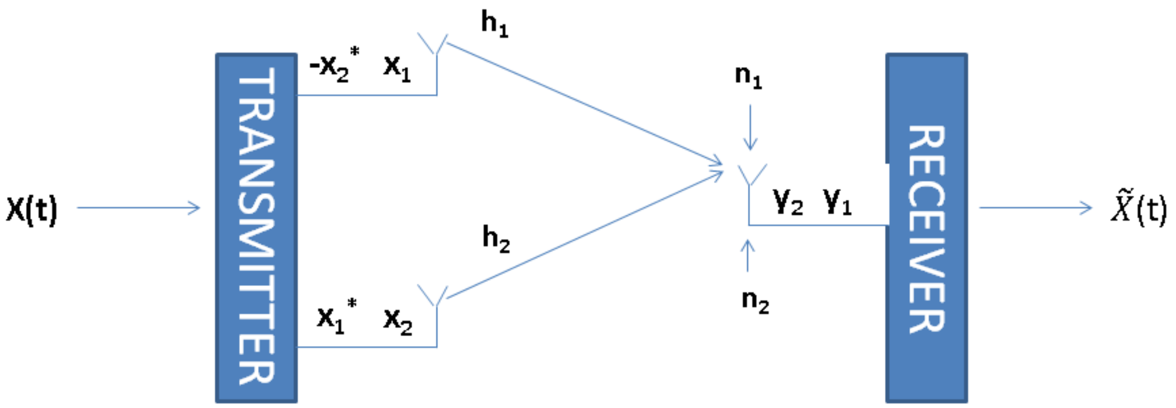

Figure 3

	Transmitter 1	Transmitter 2
Time t	X_1	X_2
Time t+T	$-x_2^*$	X_1^*

Table 1

Table 1 shows that working principle of 2x1 Alamouti STBC. At time t x_1 and x_2 are transmitted by Transmitter 1 and 2 and after that at time (t+T) their conjugates are transmitted.

From Table 1 and Figure 3 we can write receiving data;

$y_1=x_1h_1+ x_2h_2+ n_1$ (first time slot) (Eq 2.1)

$y_2=h_1 (-x_2^*)+ h_2 (x_1^*)+ n_2$ (second time slot) (Eq 2.2)

where;

- x_1 and x_2 transmitted data,
- y_1 and y_2 received data,
- h_1 and h_2 Rayleigh channel,
- n_1 and n_2 AWGN noise,

In matrix form of these **(Eq 2.1)** and **(Eq 2.2);**

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} \qquad \textit{(Eq 2.3)}$$

In order to get rid of conjugates of transmitted data at the receiver, we take conjugate of y_2;

$$y_1 = x_1 h_1 + x_2 h_2 + n_1 \qquad \text{(Eq 2.1)}$$

$$y_2^* = h_1^* (-x_2) + h_2^* (x_1) + n_2^* \qquad \text{(Eq 2.4)}$$

matrix form of **(Eq 2.1)** and **(Eq 2.4);**

$$\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2^* \end{bmatrix} \qquad \text{(Eq 2.5)}$$

Let’s define

$H=\begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix}$ and inverse of **H** is equal to $\mathbf{H^{-1}}$,

We multiply both side of **(Eq 2.5)** with H^{-1},then we get ;

$$H^{-1}\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = H^{-1}H\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + H^{-1}\begin{bmatrix} n_1 \\ n_2^* \end{bmatrix} \qquad \text{(Eq 2.6)}$$

From **(Eq 2.6)** we get;

$$H^{-1}\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2^* \end{bmatrix} = \tilde{X} \qquad \text{(Eq 2.7)}$$

From **(Eq 2.7)** we write $\tilde{\mathbf{X}}$ seperately;

$$\tilde{\mathbf{x}}_1 = \mathbf{x_1 + (h_1^* n_1 + h_2 n_2^*) / (|h_1|^2 + |h_2|^2)}$$

$$\tilde{\mathbf{x}}_2 = \mathbf{x_2 + (n_1 h_2^* - h_1 n_2^*) / (|h_1|^2 + |h_2|^2)}$$

Finally we get transmitted data with noise and channel effect. This process is called ***Equalization.***

3) 2x2 MIMO Alamouti STBC

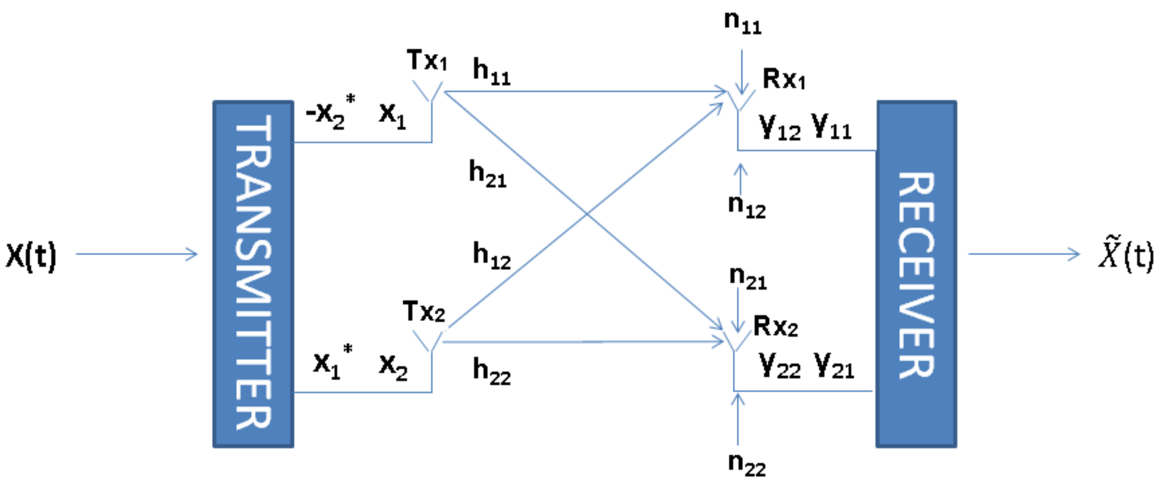

Figure 4

System is similar to 2x1 Alamouti but we have one more receiver. This system is consists of 2 Transmitter and 2 Receiver. Working principle is also similar to 2x1 Alamouti. In 2x1 Alamouti we send block codes to just one receiver but in this system we send block codes to 2 receiver.

$y_{11}= x_1h_{11}+ x_2h_{12}+ n_{11}$ (first time slot received data in Rx_1) (Eq 3.1)

$y_{12}= -x_2^*h_{11}+ x_1^*h_{12}+ n_{12}$ (second time slot received data in Rx_1) (Eq 3.2)

$y_{21}= x_1h_{21}+ x_2h_{22}+ n_{21}$ (first time slot received data in Rx_2) (Eq 3.3)

$y_{22}= -x_2^* h_{21}+ x_1^* h_{22}+ n_{22}$ (second time slot received data in Rx_2) (Eq 3.4)

where;
$\mathbf{x_1}$ and $\mathbf{x_2}$ transmitted data

$\mathbf{y_{11}}$ and $\mathbf{y_{21}}$ are received data (for first time slot)
$\mathbf{y_{12}}$ and $\mathbf{y_{22}}$ are received data in (for second time slot)

$\mathbf{h_{11}}$,$\mathbf{h_{12}}$,$\mathbf{h_{21}}$,$\mathbf{h_{22}}$ are Rayleigh channel

$\mathbf{n_{11}}$,$\mathbf{n_{12}}$, $\mathbf{n_{21}}$,$\mathbf{n_{22}}$ are AWGN noise

Now we take conjugates of **y_{12} ,y_{22}** to get rid of conjugates of transmitted data at the receiver;

$y_{12}^{*} = -x_2 h_{11}^{*} + x_1 h_{12}^{*} + n_{12}^{*}$ (Eq 3.5)

$y_{22}^{*} = -x_2 h_{21}^{*} + x_1 h_{22}^{*} + n_{22}^{*}$ (Eq 3.6)

we write **(Eq 3.1) ,(Eq 3.5), (Eq 3.3), (Eq 3.6)** together to getting matrix form;

$y_{11} = x_1 h_{11} + x_2 h_{12} + n_{11}$ (Eq 3.1)

$y_{12}^{*} = -x_2 h_{11}^{*} + x_1 h_{12}^{*} + n_{12}^{*}$ (Eq 3.5)

$y_{21} = x_1 h_{21} + x_2 h_{22} + n_{21}$ (Eq 3.3)

$y_{22}^{*} = -x_2 h_{21}^{*} + x_1 h_{22}^{*} + n_{22}^{*}$ (Eq 3.6)

In matrix form of equations;

$$\begin{bmatrix} y_{11} \\ y_{12}^{*} \\ y_{21} \\ y_{22}^{*} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{12}^{*} & -h_{11}^{*} \\ h_{21} & h_{22} \\ h_{22}^{*} & -h_{12}^{*} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_{11} \\ n_{12}^{*} \\ n_{21} \\ n_{22}^{*} \end{bmatrix} \quad \text{(Eq 3.7)}$$

As can be seen from the **(Eq 3.7)** we get rid of conjugates of transmitted data.

We define X,Y,H,N matrices ;

$$X=\begin{bmatrix} x_1 \\ x_2 \end{bmatrix},\ Y=\begin{bmatrix} y_{11} \\ y_{12}^{*} \\ y_{21} \\ y_{22}^{*} \end{bmatrix},\ H=\begin{bmatrix} h_{11} & h_{12} \\ h_{12}^{*} & -h_{11}^{*} \\ h_{21} & h_{22} \\ h_{22}^{*} & -h_{12}^{*} \end{bmatrix},\ N=\begin{bmatrix} n_{11} \\ n_{12}^{*} \\ n_{21} \\ n_{22}^{*} \end{bmatrix}$$

We get;

Y=HX+N (Eq 3.8)

In order to get transmitted data at the receiver, we eliminate "***H*** " matrix as 2x1 Alamouti STBC, however this is not easy as before.Since, 2x1 Alamouti STBCs "***H"*** matrix is the square matrix and we eliminate multiply with its inverse easily. In 2x2 Alamouti STBCs H matrix is not square matrix so its inverse is not exists.Because of this reason we use Hermitian transposition, indicated with the symbol H. It is equivalent to transpose and do a complex conjugation of the matrix.

$$H^H=\begin{bmatrix} h_{11}^* & h_{12} & h_{21}^* & h_{22} \\ h_{12}^* & -h_{11} & h_{22}^* & -h_{21} \end{bmatrix}$$

$$H^H H=\begin{bmatrix} |h_{11}|^2+|h_{12}|^2+|h_{21}|^2+|h_{22}|^2 & 0 \\ 0 & |h_{11}|^2+|h_{12}|^2+|h_{21}|^2+|h_{22}|^2 \end{bmatrix}$$

We define d;

$$d=|h_{11}|^2+|h_{12}|^2+|h_{12}|^2+|h_{12}|^2$$

then;

$$H^H H=\begin{bmatrix} d & 0 \\ 0 & d \end{bmatrix}$$

In order to get identity matrix;

$$\frac{1}{d}\begin{bmatrix} d & 0 \\ 0 & d \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=I$$

We multiply both side of **(Eq 3.8)** with $\boldsymbol{\frac{1}{d}\mathbf{H}^{\mathbf{H}}}$;

$$\frac{1}{d}\ H^H Y=\frac{1}{d}H^H H X+\frac{1}{d}\ H^H N$$

$$=X+\ \frac{1}{d}\ H^H N=\tilde{X}$$

As a result we get transmitted data at the receiver with noise and channel effect;

$$\tilde{\mathbf{x}}_{\mathbf{1}}=\ \frac{1}{d}\left(\mathbf{h}_{11}^{*}\mathbf{y}_{11}+\mathbf{h}_{12}\mathbf{y}_{12}+\mathbf{h}_{12}^{*}\mathbf{y}_{21}+\mathbf{h}_{22}\mathbf{y}_{22}\right)$$

$$\tilde{\mathbf{x}}_{\mathbf{2}}=\ \frac{1}{d}\left(\mathbf{h}_{12}^{*}\mathbf{y}_{11}-\mathbf{h}_{11}\mathbf{y}_{12}+\mathbf{h}_{22}^{*}\mathbf{y}_{21}+\mathbf{h}_{21}\mathbf{y}_{22}\right)$$

B) Bit-Error-Rate Analysis on MATLAB

In our project we used QPSK modulation. **Figure5** shows how we do performance analysis of SISO, (2x1) and (2x2) Alamouti Space Time Coding with QPSK modulation with block diagrams step by step.

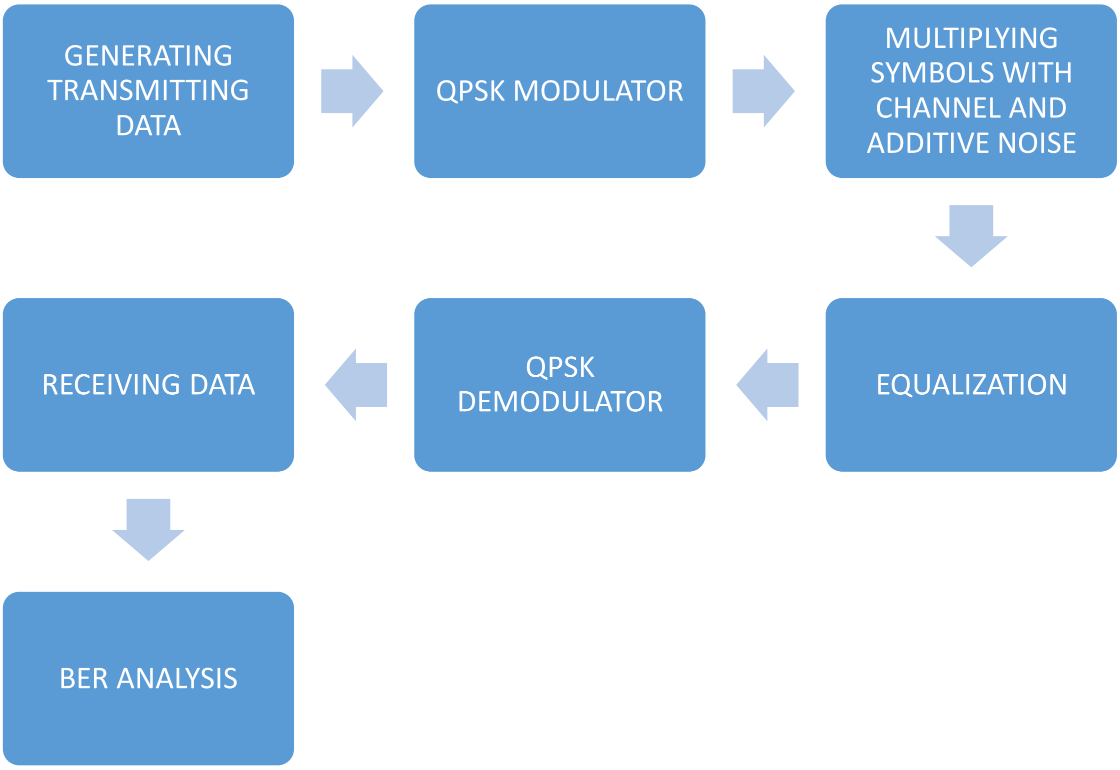

Figure5

Type of System / SNR(dB)	SISO(1x1)	2X1Alamouti	2x2Alamouti
5 dB	$6.4x10^{-2}$	$3.3x10^{-2}$	$3.7x10^{-3}$
15 dB	$7.8x10^{-3}$	$7.1x10^{-4}$	$3.0x10^{-6}$
21 dB	$1.9x10^{-3}$	$4.7x10^{-5}$	——
25 dB	$7.8x10^{-4}$	$7.0x10^{-6}$	——

Table 2

Figure 6 shows that graphical output of all systems and **Table 2** indicates BER values for some of SNR_dB values.

As can be seen from graph and table, 2x2 Alamouti STBC has the smallest BER values for all of SNR_dB values. This result is expected because 2x2 Alamouti system has one more receiver when compared with 2x1Alamouti system and thanks to this one more receiver it increases its diversity.
SISO system has the highest BER values for all of SNR_dB values because of no diversity.

CONCLUSION

In this project we have learned theory of Space-Time Block Coding, feature of Alamouti STBC and differences between SISO system. Also we have understood QPSK modulation specialities and have learned how can be made Bit-Error-Rate analysis. Furthermore thanks to this project we have developed MATLAB skill.

As a result we have finished our project successfully and our results nearly the same which were did by professional.

REFERENCES

1. **Shreedhar A Joshi , T S Rukmini and Mahesh H M** ”Space Time Block Coding for MIMO systems using Alamouti method with digital modulation techniques”
2. **Sebastian Caban, Christian MehlfÄuhrer, Arpad L. Scholtz, and Markus Rupp** “Indoor MIMO Transmissions with Alamouti Space -Time Block Codes”

3. http://spacetimecodes.blogspot.com/p/alamouti-code.html
4. http://nutaq.com/en/blog/alamouti-space-time-block-coding
5. http://en.wikipedia.org/wiki/Phase-shift_keying
6. http://en.wikipedia.org/wiki/Space%E2%80%93time_block_code