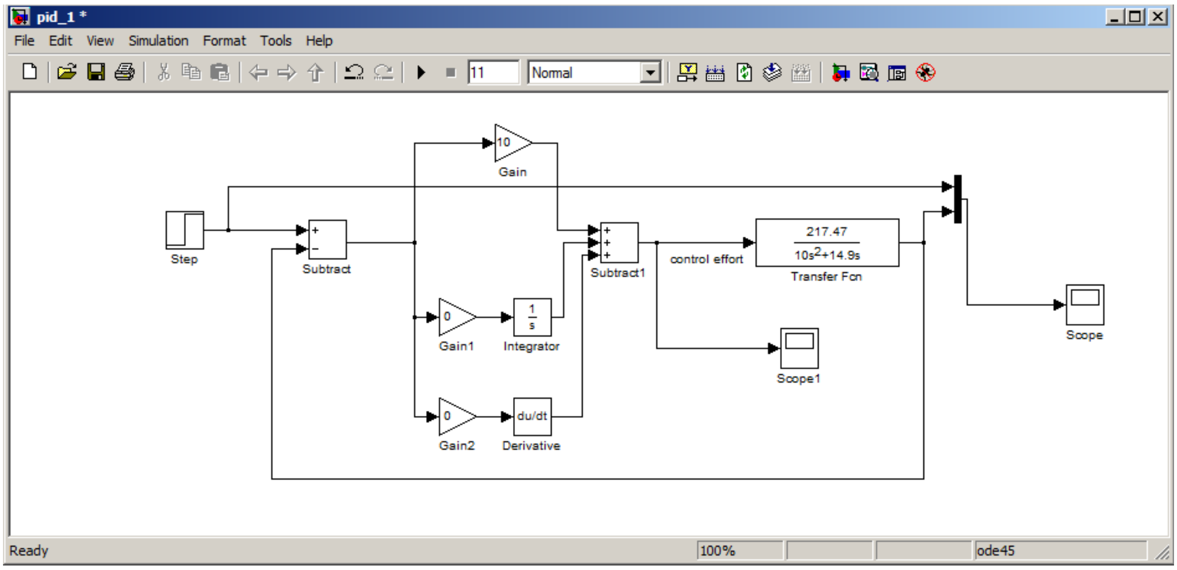
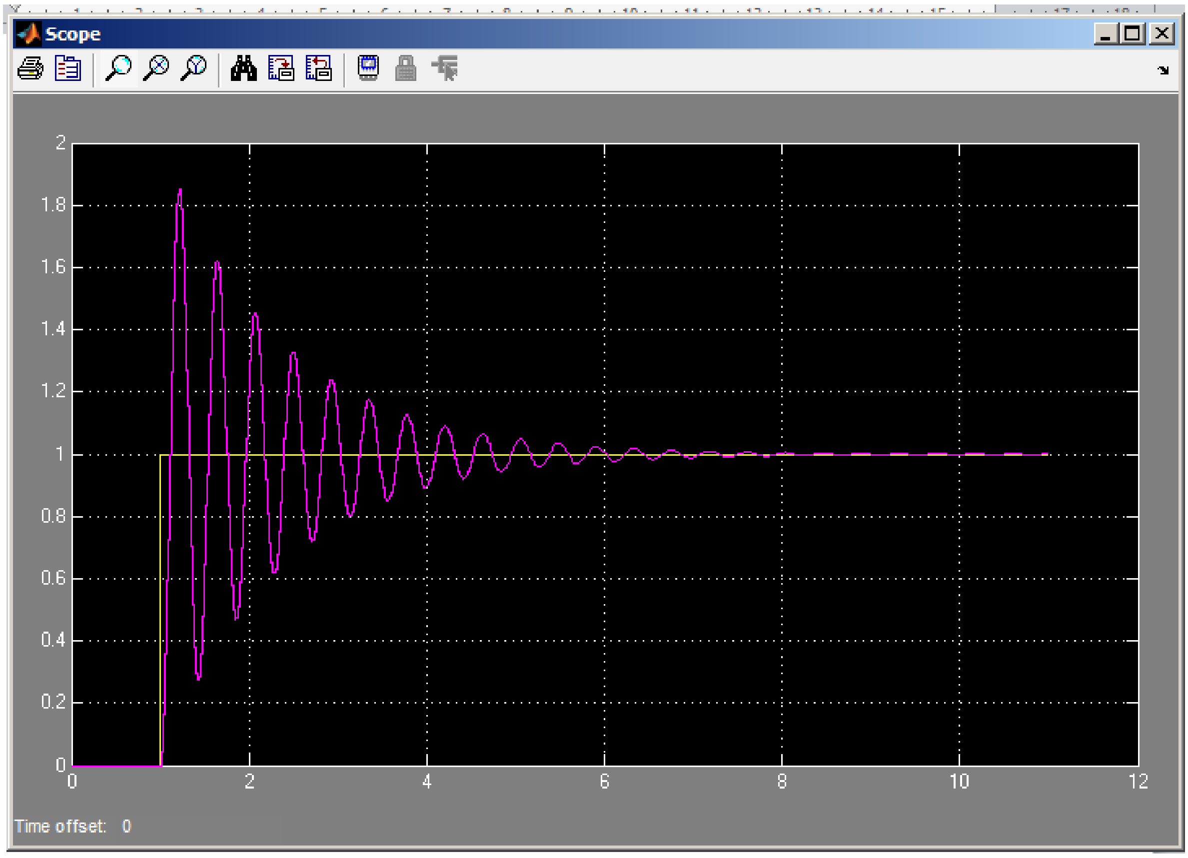
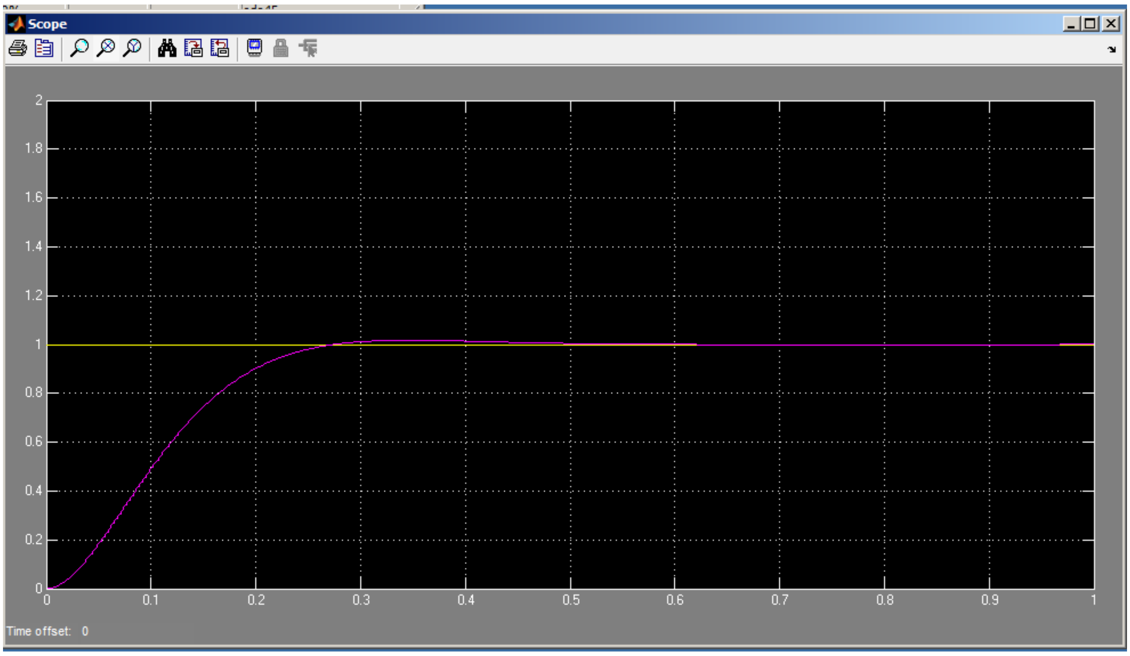


Proportional Gain Gc=10



Proportional Gain Input Step response

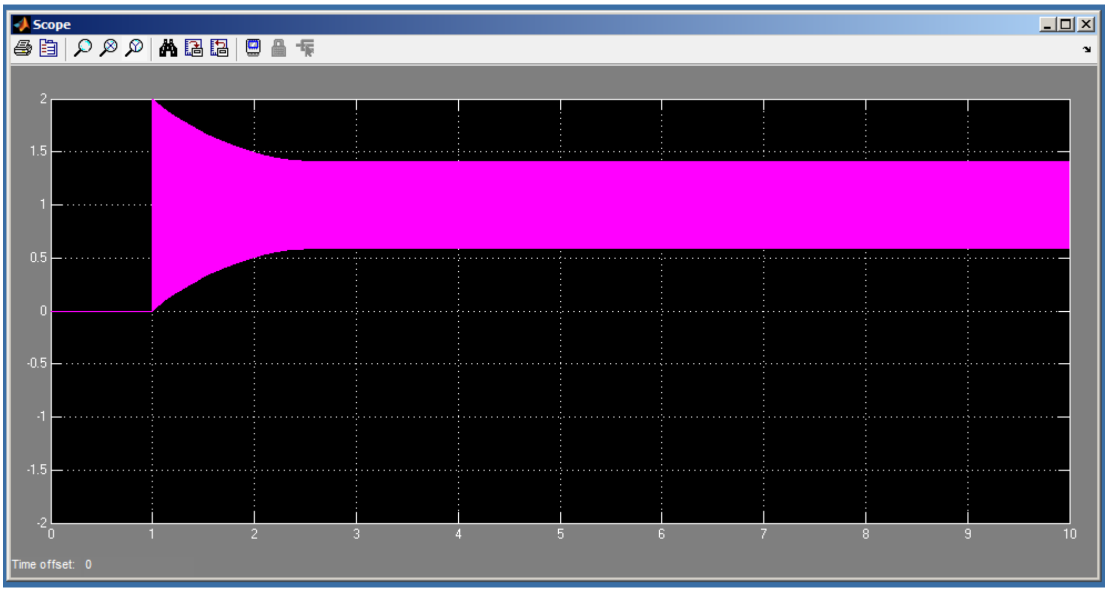
As one can see above, proportional control on its own results in quite a significant overshoot and a long settling time. As can be noticed, there is not steady state error due to the components in the denominator of the transfer function since from the derivation the ω_n^2 was cancelled out. This saves us from having to include a proportional integral control. This is also confirmed if one would readthe rise time which in fact is less than the 0.25 seconds required;



The proportional integral control would decrease the rise time which is unnecessary here.

Applying a derivative gain of 1 enabled the system to settle within less than 1 second in the +/-5% settling criterion less than 5% overshoot. These values are also practical when considering the highest gain is that of 10 for proportional gain since it can easily achieved without saturation as in the experiment.

ii) The point of sustained oscillations were at a critical gain of 5.45E6 with proportional integral and proportional derivative set to zero. The waveform is shown below;

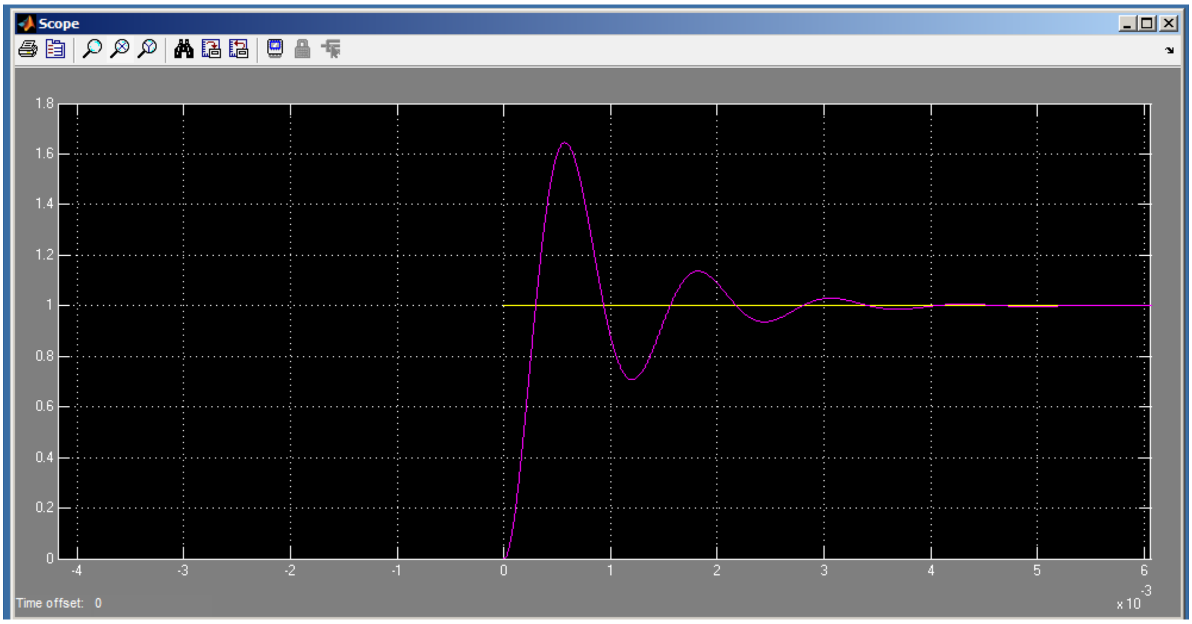


Sustained oscillations

As seen from the zoomed waveform below T_c is equal to the period of the signal when the system reaches the critical gain K_c. T_c=0.725ms-0.15ms = 0.575ms

Trying the PID for some overshoot;

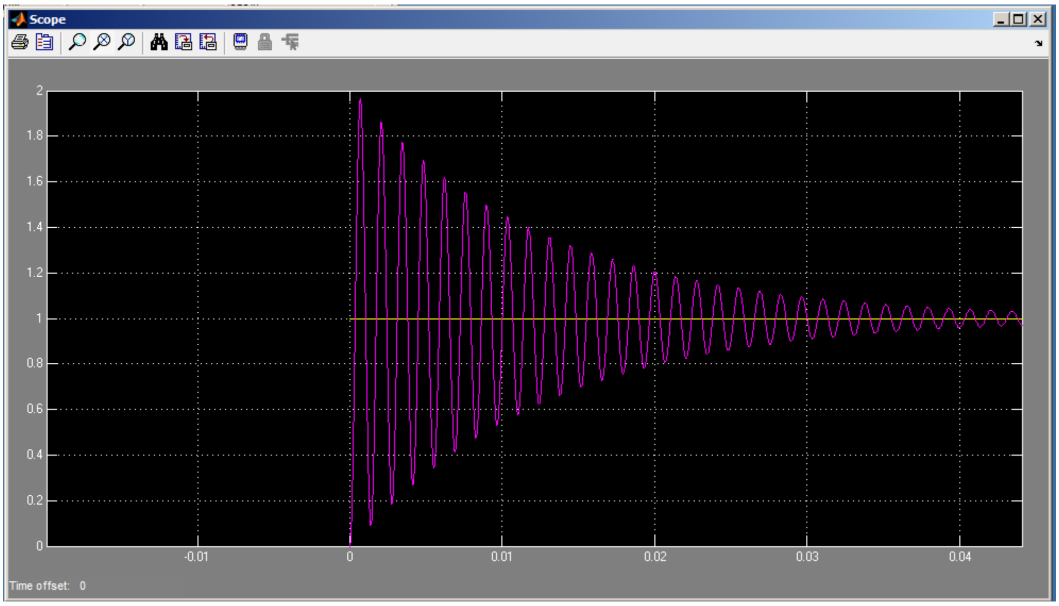
Control	kp	Ti	Td
PID some overshoot	0.33Kc	0.5 Tc	0.33Tc



Response for PID designed with Ziegler-Nicholson method

As can be notice from the figure above, the PID with some overshoot chosen from the table was not suitable as the overshoot climbed to nearly 70% which is way too far from the specifications.

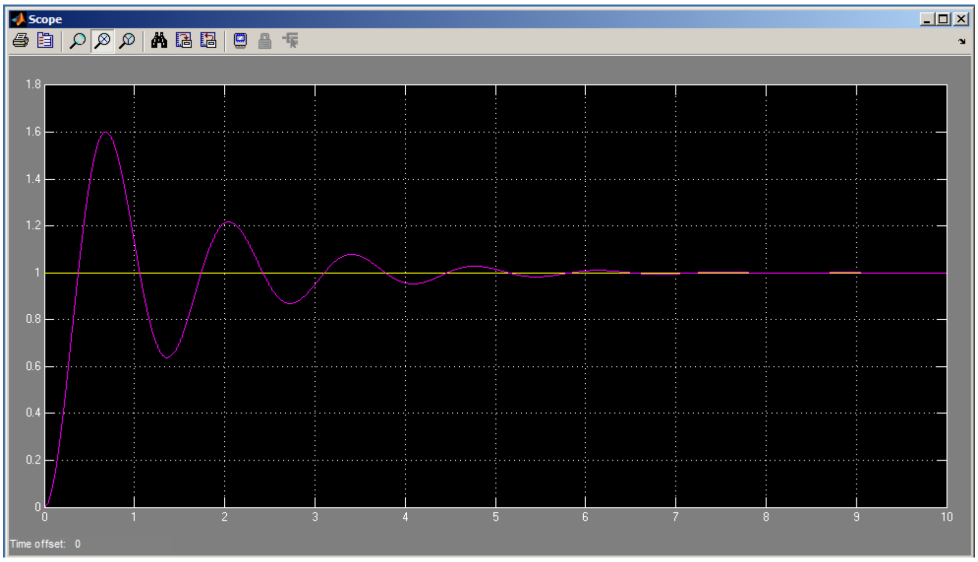
The PID no overshoot will be calculated;



This method of tuning it not perfect for every system as it is in this case. The results were far from suitable although the settling time was extremely short (approx. 40ms) which is a good thing, we have an overshoot of nearly 100% which is not practical.

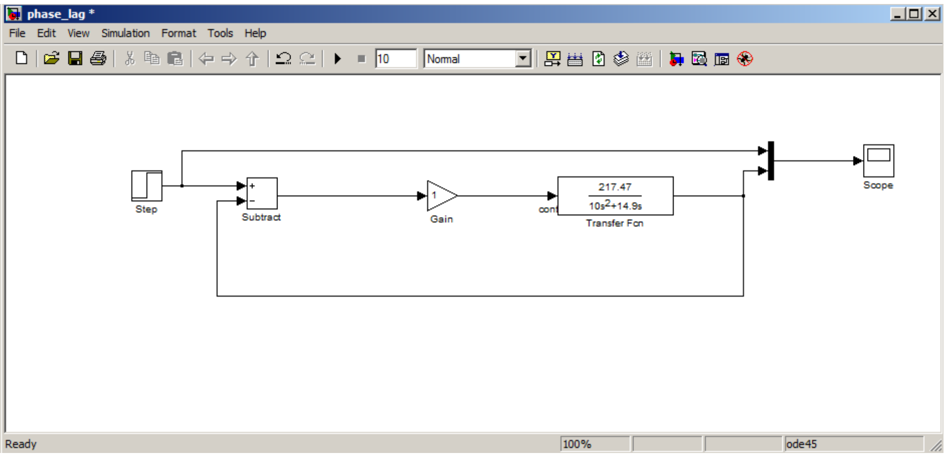
After this method of tuning, the values can be readjusted to one's requirements.

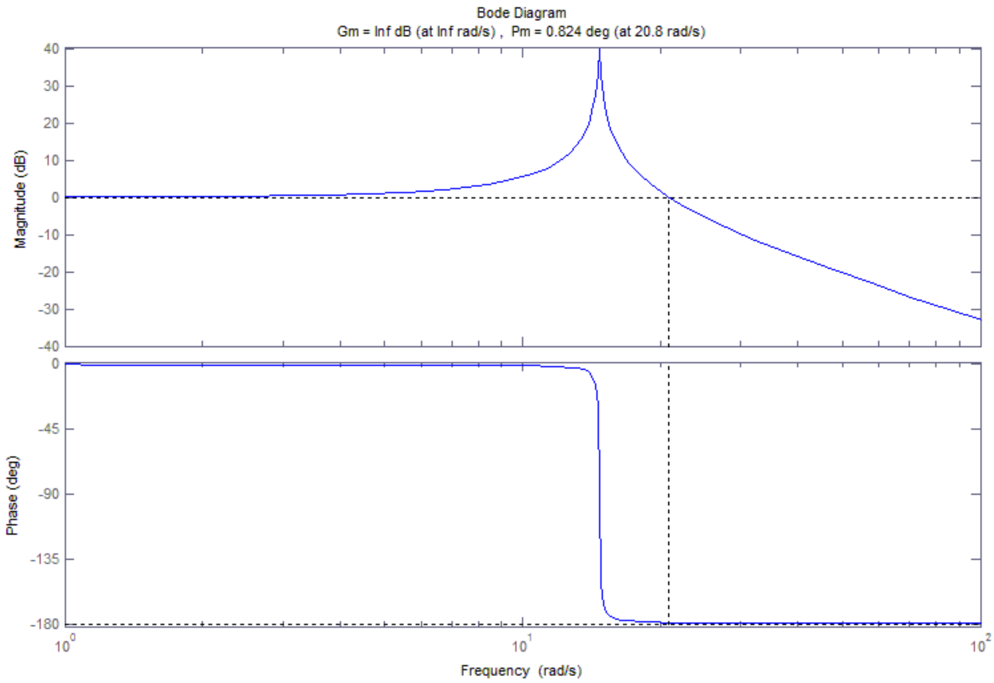
Phase Lag Compensator Design



K=1 time response

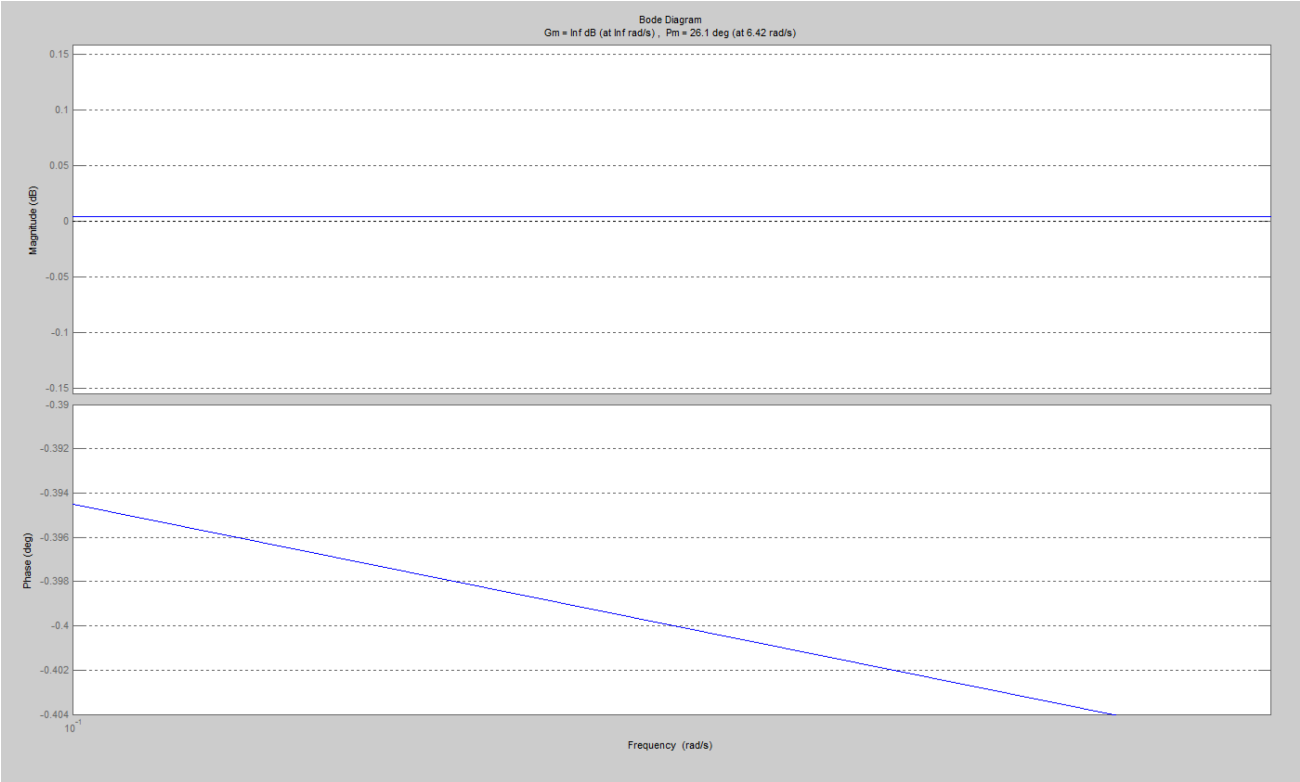
The system was only given a gain of one as one can see in the block diagram below since there was no steady state error in the time response graph.



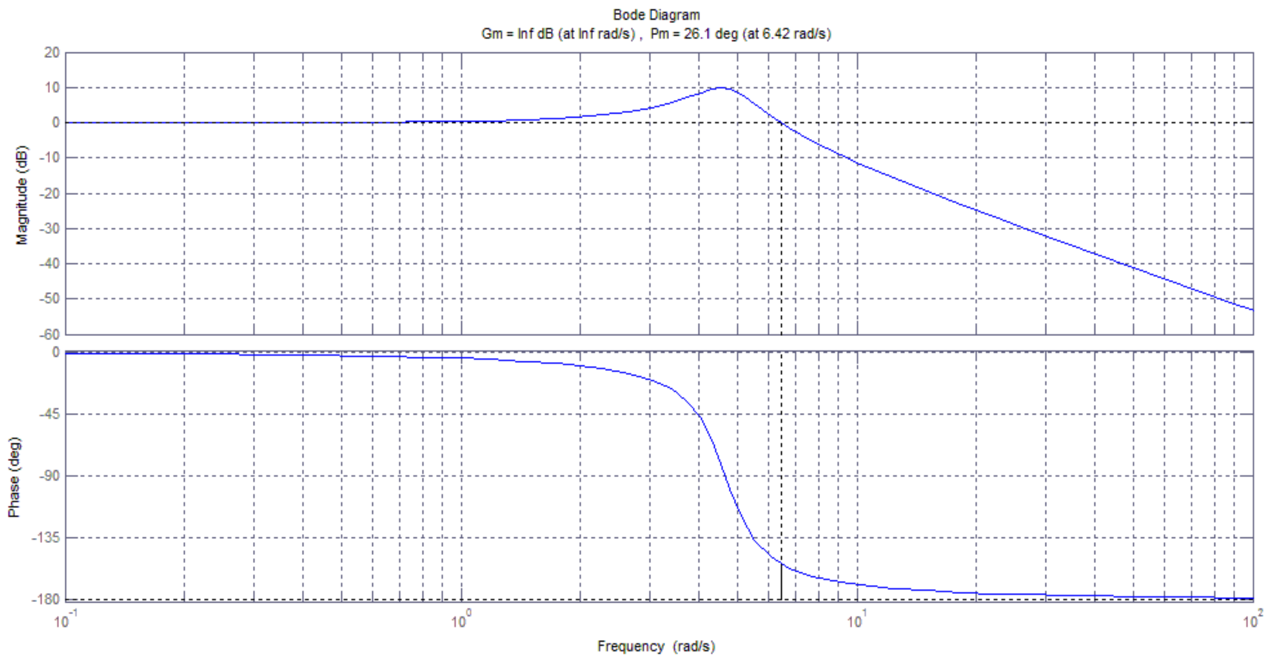


Bode Plots for CLTF with unity gain

As one can notice from the plots above, it would be sensible to modify the phase plot in order to achieve a safe phase margin. The phase margin here is 0.824 degrees which is very low giving proof for the oscillations in the time response graph.



Linear Gain showing steady state is kept way below 0.05



Bode Plot of uncompensated System with Unity Gain

By examining the bode plot, the phase margin specified of 50 degrees would be satisfied at 5.23 rad/sec with an extra 5 degrees for a safety margin. The zero of the compensator can be located at least one decade below this frequency (0.52 rad/s).

In order to make 5.23 rad/s the zero dB crossover point, an attenuation of 7.05dB is required.

Alpha can be found by;

$$-7.05 = 20\log(\alpha)$$

$$\alpha = 0.444$$

$$\alpha = \frac{p}{z}$$

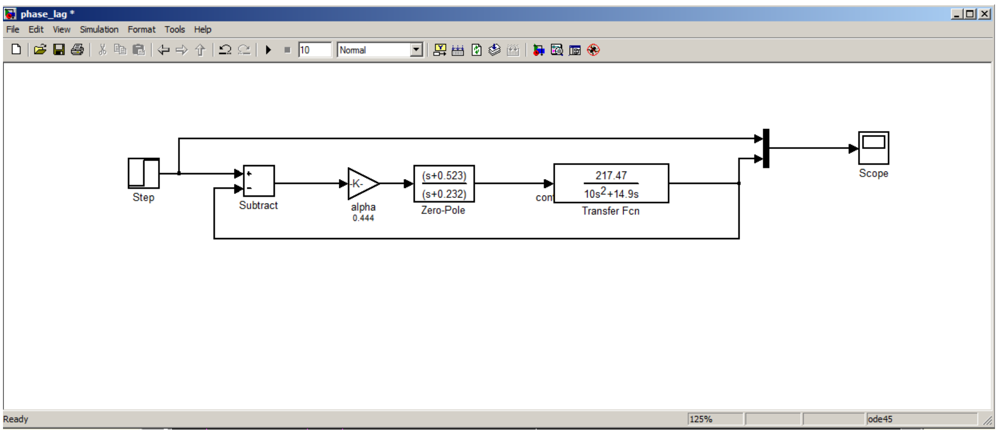
$$z = 0.523$$

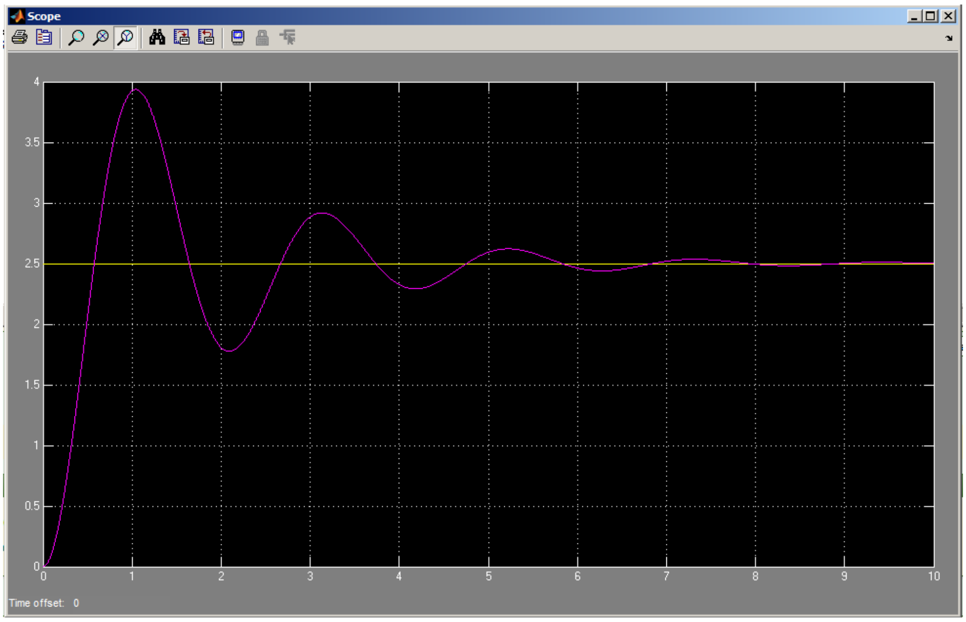
$$\therefore p = 0.523 \times 0.444$$

$$= 0.232$$

This describes the following compensator;

$$G_c(s) = 0.444\left(\frac{s + 0.523}{s + 0.232}\right)$$





Time Response of Compensated System

As it clearly seen from the above time response graph, the compensator only reduce the overshoot slightly and did not introduce any steady state error. The problem with this kind of controller is the long settling time of approximately 8 seconds.

<u>In order to check the achieved Phase Margin, an Open Loop bode plot is considered;</u>

<u>Frequency Response of Compensated Closed loop system;</u>

Phase-Lead Compensator

-Logarithmic mean frequency

$$\omega_m = \sqrt{|pz|}$$

-Maximum decrease in phase

$$\varphi_m = \sin^{-1}\left(\frac{\alpha - 1}{\alpha + 1}\right)$$

Loop gain of 1 was found earlier to be enough. Gain margin was found to be infinite while phase margin was a critical value of 26.1 degrees.

To give the desired phase margin, the minimum additional phase needed is the approximately 50-26.1= 23.9 degrees.

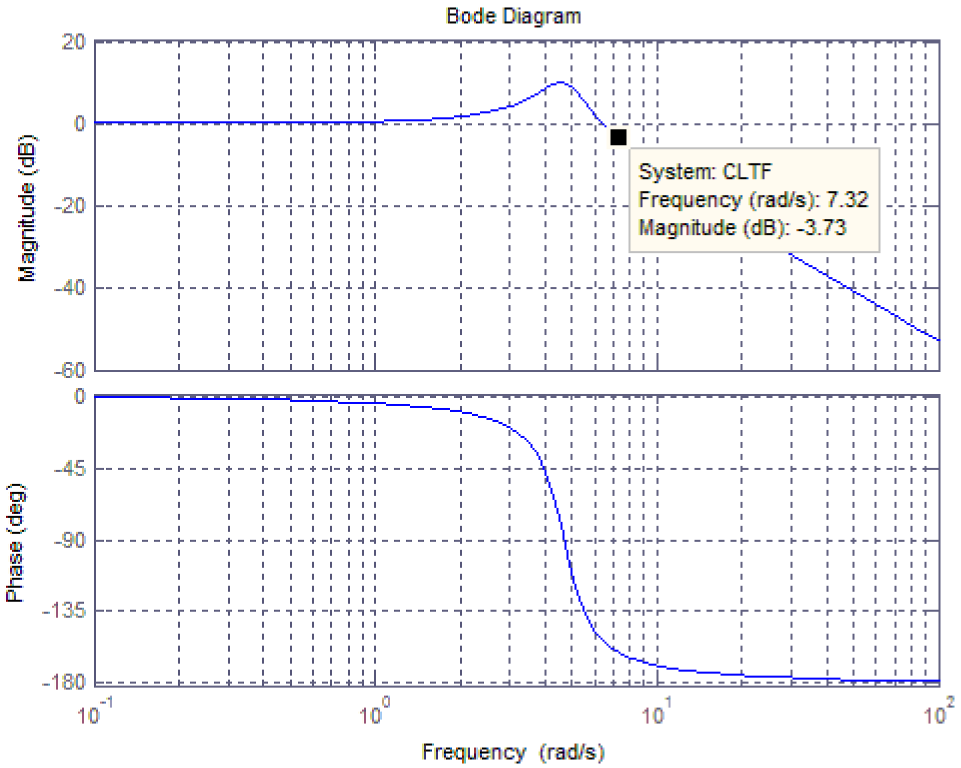
$$\alpha = \frac{1 + \sin \varphi_m}{1 - \sin \varphi_m}$$

$$\alpha = \frac{1 + \sin 23.9}{1 - \sin 23.9}$$

$$\alpha = 2.36$$

$$M = -10log(\alpha)$$

$$M = -3.733$$



Logarithmic Mean Frequency @ Gain Margin of 5.57

$$\omega_m = 7.32rad/s$$

$$z = \frac{\omega_m}{\sqrt{\alpha}}$$

$$z = \frac{7.32}{\sqrt{2.36}}$$

$$z = 4.76$$

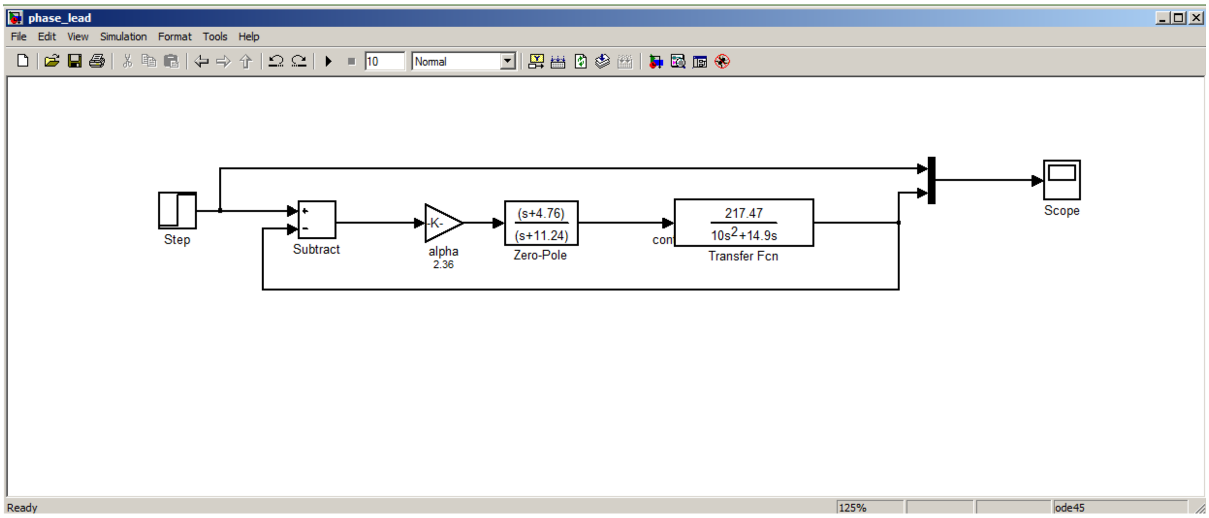
$$p = z\alpha$$

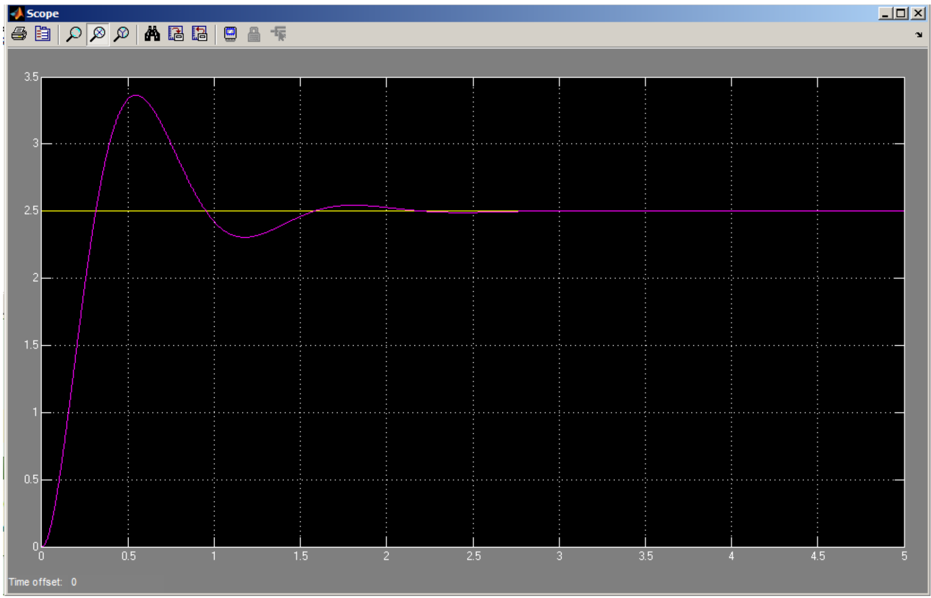
$$p = 4.76 \times 2.36$$

$$p = 11.24$$

$$\therefore\ G_c(s) = \alpha\left(\frac{s+z}{s+p}\right)$$

$$\therefore\ G_c(s) = 2.36\left(\frac{s+4.76}{s+11.24}\right)$$





CLTF time response graph for Phase Lead

In order to check the achieved Phase Margin, an Open Loop bode plot is considered;

Frequency Response of Compensated Closed loop system;

P1.4, P4.1,P4.2

Regarding the PID manual tuning, it is more time consuming, since values are adjusted and tried without any guidelines. Using the Ziegler Nichols tuning gave a rough idea of which components need to handle the control process. Using the ZN and even the manual tuning, one needs to take care of the Proportional, Integral and Derivative Parameters in the practical implementations, as the DC motor has finite power and also its supply. When analysing the bandwidth of the DC motor and flywheel, the bode plot revealed the resonant point of the system to be about 1.75Hz.

When designing phase compensators, it is obvious that a phase-lead compensator suited this system much better than the phase-lag. This could be noticed from the time response graph where there was a slight decrease in overshoot, the settling time is a lot shorter(below 1.5 second) and higher damping. Steady state error is also very satisfactory reaching the criteria at about 1.5 seconds.

The control effort was involving the correction of a type 1 system with a very low steady state error.