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Brief paper

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An Accurate Formula For The Firing Angle Of The Phase Angle Control In Terms Of The Duty Cycle Of The Integral Cycle Control

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Abstract- This paper presents development of a simple and accurate formula for evaluation of the firing angle (α) of the phase angle control in terms of the duty cycle (K) of the integral cycle control when the same average output power transfer is required. The accuracy of the formula developed is verified by comparing the values obtain from the exact iterative solution. It has been shown that, the maximum error over the whole range of the entire firing angle (α) computed for any value of the exact value is found to be less than 2 degree. The formula presents a very simple way for the evaluation of the comparison of the harmonics properties of the phase angle control and integral cycle control for the same average power transfer. In addition, the developed formula reduces the computational time and also reduces time for manual classroom calculation.

Keywords: Phase angle control, Integral cycle control, Duty cycle, firing angle.

1. INTRODUCTION

It is well established that for a signal phase A.C. voltage regulator with resistor load shown in figure(1), there are two methods used normally for controlling the power transfer which are [1-12]:

1- ON-OFF control (Integral Cycle Control).
2- Phase-Angle Control.

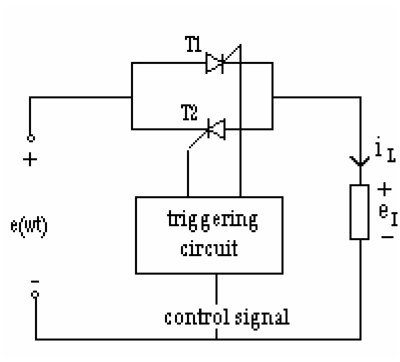

Figure 1: Single phase ac voltage regulator

In integral cycle control, thyristor switches (Triacs) connect the load to the ac source for few cycles of input voltage and then disconnect it for another few cycles as shown in figure 2. While, in phase angle control, thyristor switches (Triacs) connect the load to the ac source for a portion of each cycle of input voltage as shown in figure 3.

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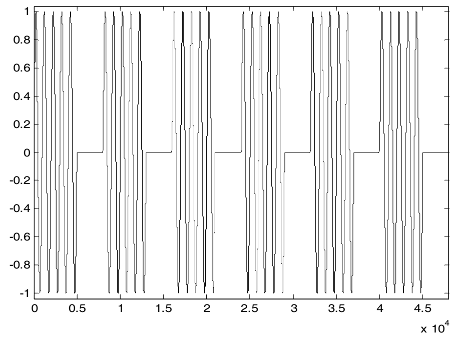


Figure 2: Output voltage waveform in case of integral cycle control.

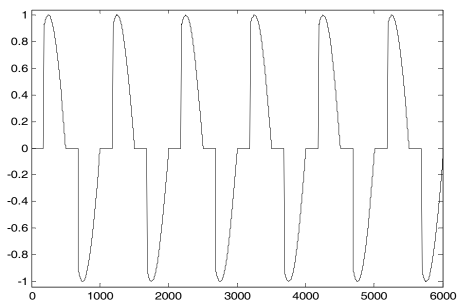


Figure 3: Output voltage waveform in case of phase angle control.

In many power electronics books evaluation of each method performance has been well published individually without comparing the power transfer of each method. Therefore, it has been the objective of this paper to develop a simple and accurate formula for the computation of the firing angle (α) in terms of the duty cycle (K) in case of same power transfer.

2. DEVELOPMENT OF THE FORMULA

In case of the integral cycle control, the output voltage waveform is shown in figure (2). If the input voltage [Vs=√2sinwt] is connected to load for N cycles and is disconnected for M cycles, then the rms output voltage can be found from:

$$V_o = \sqrt{\frac{N}{2\pi(N+M)} \int_0^{2\pi} 2V_s^2 \sin^2 wt\,dt} \qquad (1)$$

$$V_o = V_s \sqrt{\frac{N}{N+M}} \qquad (2)$$

$$V_o = V_s \sqrt{K} \qquad (3)$$

Where K=N/(N+M) and it is called the duty cycle.
Hence the output power (average power) can be found from:

$$P_o = \frac{V_o^2}{R} = K\frac{V_s^2}{R} \qquad (4)$$

On the other hand, in case of phase angle control (the output voltage waveform is shown in figure 3), if the delay angle (firing angle) is α (0º< α <180º), then the rms output voltage can be found from:

$$V_o = \sqrt{\frac{2}{2\pi}\int_{\alpha}^{\pi}(\sqrt{2}V_s)^2 \sin^2 wtdwt} \quad (5)$$

$$V_o = V_s\sqrt{\frac{1}{\pi}(\pi - \alpha + \frac{\sin 2\alpha}{2})} \quad (6)$$

And the output power can be found from:

$$P_o = \frac{V_s^2}{\pi R}[\pi - \alpha + \frac{\sin 2\alpha}{2}] \quad (7)$$

If it is required to produce same average output power from both methods, then from (4) and (7) the following equation can be found:

$$K = \frac{1}{\pi}[\pi - \alpha + \frac{1}{2}\sin(2\alpha)] \quad (8)$$

As it can be seen from equation (8) that the relation between the duty cycle (K) and the firing angle (α) is not linear. In case of if the firing angle (α) is given and it is required to find the value of the duty cycle (K) for the same average output power, equation (8) can be easily used. But in case of if it is required to find the firing angle (α) for a given duty cycle (K), it needs time consuming iterative solution. This difficulty has been solved by using the following new developed equation:

$$\alpha = -4.499K^3 + 6.79K^2 - 4.68K + 2.77 \quad (9)$$

To verify the validity of the above developed formula (equation (9)), the exact waveform for the relation between the firing angle (α) and the duty cycle (K) has been obtained from the equation (8) and compared with that obtained from the new developed formula. This comparison is shown in figure 4.

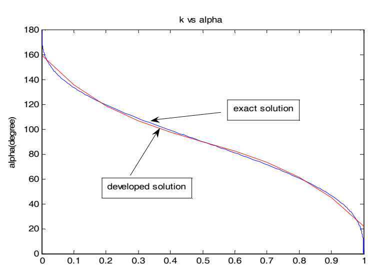


Figure 4: The comparison between the exact formula and developed formula.

Examination of the above figure clearly indicates that the developed formula gives reasonable accuracy results. The overall error over the entire firing angle (α) range computed for any value of the exact value was found to be less than 2 degree as shown in figure 5.

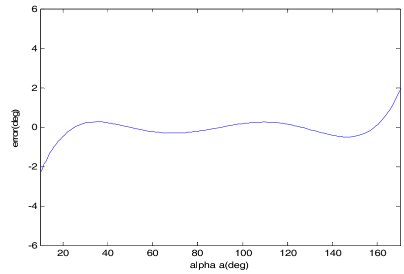
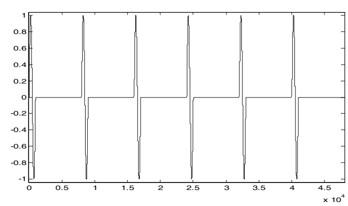


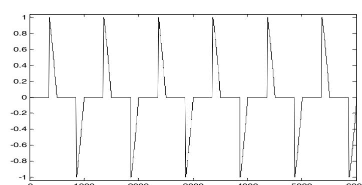
Figure 5: The overall error over the entire firing angle (α) range.

3. APPLICATION OF THE FORMULA

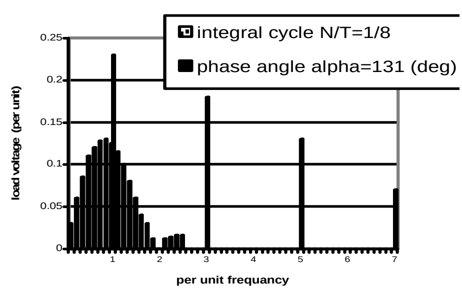
The main advantages of the developed formula are to facilitate the computation of the firing angle (α) for a given value of the duty cycle (K) for the classroom use and provide easy solution in computer aided or manual problems. In addition, one of the applications where this developed formula can be used is that, for comparing the harmonics properties of the integral cycle control and phase angle control for the same average output power. Theses comparisons are illustrated in figures 6 and 7.



(a)

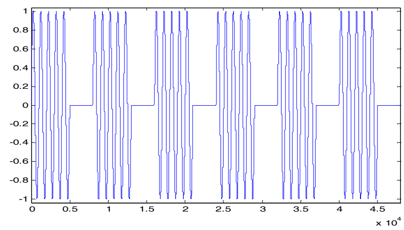


(b)

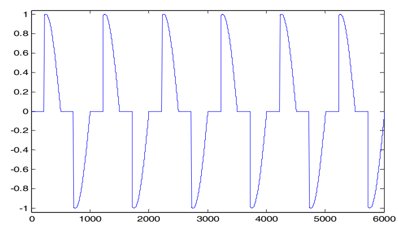


(c)

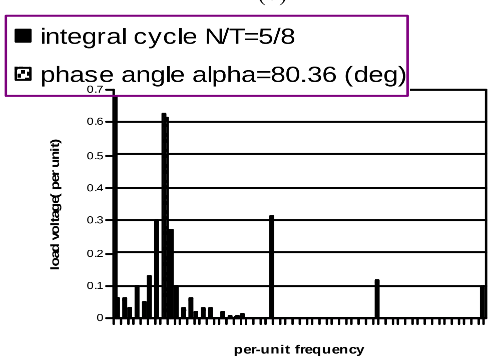
Figure 6: a) Output voltage waveform in case of integral cycle control (K=0.125)
b) Output voltage waveform in case of phase angle control (α= 130º)
c) Comparing the harmonics properties of the integral cycle control and phase angle control for the same average output power



(a)



(b)



(c)

Figure 7: a) Output voltage waveform in case of integral cycle control (K=0.625).
b) Output voltage waveform in case of phase angle control (α = 80º).
c) Comparing the harmonics properties of the integral cycle control and phase angle control for the same average output power.

4. CONCLUSIONS

A simple and accurate formula has been developed for evaluation of the firing angle (α) of the phase angle control in terms of the duty cycle (K) of the integral cycle control when the same average output power transfer is required. The accuracy of the formula developed verified by comparing the values obtain from the exact iterative solution. The maximum error over the whole range of the entire firing angle (α) computed for any value of the exact value was found to be less than 2 degree. The formula presents a very simple way for the evaluation of the comparison of the harmonics properties of the phase angle control and integral cycle control for the same average power transfer. In addition, the developed formula reduces the computational time and also reduces time for manual classroom calculation.

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