Optimal Transmission Power Allocation for Two-Way Relay Channel Using Analog Network Coding

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***Abstract*—Considering a system model of three nodes two-way relay channel with analog network coding, this paper discusses the optimal transmission power allocation problem to maximize the system achievable sum-rate. We propose an optimal transmission power allocation solution and show that our solution performs better than the uniform power allocation by numerical results.**

Keywords-Optimal Power Allocation; Analog Network Coding; Two-Way Relay Channel

I. INTRODUCTION

The analysis of wireless two-way relay channel (TWRC) has attracted a lot of interests. In a three nodes TWRC system, two nodes communicate with each other with the help of one relay node. There are several different strategies used in wireless relay, such as decode-and-forward (DF), compress-and-forward (CF) and amplify-and-forward (AF). In [1], Knopp studied the different strategies for relaying in TWRC, and analyzed the achievable rate of DF relay channel. And in [2], [3], Rankov respectively considered a three nodes TWRC system with full-duplex and half-duplex terminals, and analyzed the achievable rate for DF, CF and AF strategies.

However, the relay node simply forwards the information, which can't achieve the maximum flow of the network transmission. Since network coding was introduced by Ahlswedee firstly in [4], it has been studied intensively for improving the performance of wireless networks. Considering the wireless mesh networks, Katti proposed a new architecture using the XOR coding protocol in [5]; and considering the wireless ad hoc networks in [6], S. Zhang thought that network coding can be applied at the physical layer, and proposed a physical-layer network coding (PNC) scheme; in [7], Katti showed how to push network coding to the analog domain, as analog network coding (ANC); additionally in [8], Katti compared the performance of ANC to the pure routing approach of two-way relay channel in both full-duplex mode and half-duplex mode. But in [9], Sharma introduced an important concept of ANC noise, which can diminish the advantage of ANC in cooperative communications. And in [10], Maric demonstrated that ANC's performance was limited by propagated noise, and only performs well in high SNR.

Though the study of cooperative relaying with network coding for wireless transmission has gained plenty of progress, some key problems still need to be discussed such as transmission power allocation. The early studies in this flied assume that all nodes' power are equal including the relay nodes, but it's not the optimal scheme obviously. In [11], Q. Zhang had found the optimal power allocation for DF relay in the one-way relay system. In [12], J. Zhang investigated the optimal power allocation scheme for AF wireless one-way relay system. In [13], Larsson showed the power allocation scheme maximized the sum-rate of XOR-based network coding. And in [14], Shin also proposed an optimal power allocation in TWRC system employing the PNC protocol. In this paper, we propose an optimal power allocation scheme in TWRC system using ANC protocol.

II. SYSTEM MODEL AND PROTOCOL

In this section, we describe a three nodes half-duplex TWRC system using ANC protocol, where including two communication nodes and one AF relay node. As the nodes worked in half-duplex mode, we assume that there is no direct connection between the source node and the destination node. Different from the traditional pure routing approach in TWRC system with four time slots, the TWRC system using ANC requires only two time slots (see Fig. 1).

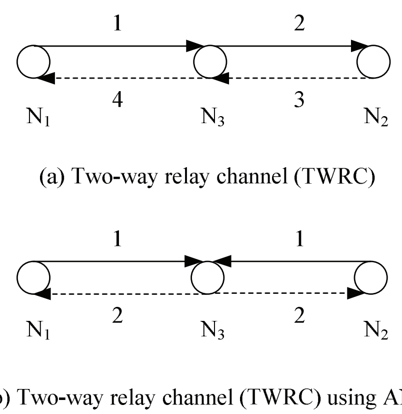

Figure 1. (a) Two-way relay channel (TWRC). (b) Two-way relay channel (TWRC) using ANC

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As described in Fig. 1 (a), nodes N_1 and N_2 exchange information through relay node N_3 without network coding needs 4 timeslots. When ANC is operated at the relay node, it accomplishes the exchange only in 2 timeslots, which improves the network throughput greatly.

See Fig. 1 (b), in the first time slot, the relay node N_3 receives the signals which N_1 and N_2 send simultaneously, and combines the two signals by simply superposition, with no need to decode the information. Then in the second timeslot, it amplifies and broadcasts the combined signal to both communication nodes. Since the receiving nodes know their own transmit signal, they can subtract their own signal from the received combining signals to extract the signal they wanted.

We denote that $x_1(n)$ and $x_2(n)$ are the signals that N_1 and N_2 send. The received and combined signal at relay node N_3 is given by

$$y_3(n) = h_1 x_1(n) + h_2 x_2(n) + z_3(n) \tag{1}$$

where h_1 is the channel gain between the node N_1 and N_3, h_2 is the channel gain between the node N_2 and N_3. And $z_i(n)$ is the white Gaussian noise with zero-mean and unit variance at the node N_i.

Then the relay node N_3 amplifies the signal $y_3(n)$, and broadcasts it. We have

$$x_3(n) = \alpha y_3(n) \tag{2}$$

where amplification factor

$$\alpha = \sqrt{\frac{p_3}{h_1^{\,2} p_1 + h_2^{\,2} p_2 + 1}} \tag{3}$$

and p_i is the transmission power of node N_i. The received signal at node N_1 is given by

$$\begin{aligned} y_1(n) &= h_1 x_3(n) + z_1(n) \\ &= h_1 \alpha y_3(n) + z_1(n) \\ &= \alpha h_1 [h_1 x_1(n) + h_2 x_2(n) + z_3(n)] + z_1(n) \end{aligned} \tag{4}$$

The formula (4) becomes

$$y_1(n) = \alpha h_1^2 x_1(n) + \alpha h_1 h_2 x_2(n) + \alpha h_1 z_3(n) + z_1(n) \tag{5}$$

Since N_1 knows the signal $x_1(n)$ it sent, and the channel state information, it can subtract this part from the received signal $y_1(n)$, and thus we get

$$y_1'(n) = \alpha h_1 h_2 x_2(n) + \alpha h_1 z_3(n) + z_1(n) \tag{6}$$

Similarly, we can get the received signal $y_2(n)$ at node N_2. After N_2 cancels its own part of signal, $y_2(n)$ becomes

$$y_2'(n) = \alpha h_1 h_2 x_1(n) + \alpha h_2 z_3(n) + z_2(n) \tag{7}$$

See from the formula (6), $y_1'(n)$ is the node N_1 extracted finally, but $x_2(n)$ is the signal it really wanted to receive from the node N_2. We find that the extracted signal $y_1'(n)$ contains two parts of noises, $\alpha h_1 z_3(n)$ and $z_1(n)$, which receives from the relay node N_3 and the destination node N_1. So the application of ANC in TWRC system brings noise propagation, which similar as "ANC noise" mentioned in [9].

Thus, the signal noise ratio (SNR) of the received signal at node N_1 and N_2 can be computed as:

$$SNR_1 = \frac{\alpha^2 |h_1|^2 |h_2|^2 p_2}{\alpha^2 |h_1|^2 + 1} \qquad SNR_2 = \frac{\alpha^2 |h_1|^2 |h_2|^2 p_1}{\alpha^2 |h_2|^2 + 1} \tag{8}$$

Therefore, the achievable sum-rate [3], [7] of a three nodes half-duplex TWRC system using ANC is:

$$R_{sum} = \frac{1}{2}\log(1 + SNR_1) + \frac{1}{2}\log(1 + SNR_2) \tag{9}$$

Using (8), the formula (9) can be re-written as:

$$R_{sum} = \frac{1}{2}\log(1+\frac{\alpha^2 |h_1|^2 |h_2|^2 p_2}{\alpha^2 |h_1|^2 + 1}) + \frac{1}{2}\log(1+\frac{\alpha^2 |h_1|^2 |h_2|^2 p_1}{\alpha^2 |h_2|^2 + 1}) \tag{10}$$

III. OPTIMAL POWER ALLOCATION SCHEME

Our goal to propose the optimal power allocation scheme is to maximize the quality of service (QOS) of the TWRC system using ANC. To power allocation problem, QOS can be expressed by several quality index, such as achievable sum-rate, outage probability, signal noise ratio (SNR), bit error rate (BER) etc. In our work, we choose achievable sum-rate as the target of maximizing, to study the optimal power allocation scheme.

To maximize the achievable sum-rate of the TWRC system using ANC, the transmission powers p_1, p_2 and p_3 should be properly allocated to the communication nodes N_1, N_2, and the relay node N_3, under the total transmission power constraint ($p_1 + p_2 + p_3 \le p_{tot}$). The formulation of this optimal problem is represented as:

$$\max_{p_1, p_2, p_3} R_{sum} \quad \text{subject to } p_1 + p_2 + p_3 \le p_{tot} \tag{11}$$

Due to the characteristic of logarithmic function, we rewrite formula (9) as:

$$\begin{aligned} R_{sum} &= \frac{1}{2}\log(1+SNR_1) + \frac{1}{2}\log(1 + SNR_2) \\ &= \frac{1}{2}(\log(1+SNR_1) + \log(1 + SNR_2)) \\ &= \frac{1}{2}\log(1+SNR_1)(1 + SNR_2) \end{aligned} \tag{12}$$

Maximizing the achievable sum-rate as formula (12) is equivalent to maximize $(1+SNR_1)(1+SNR_2)$. And the computation of maximizing is converted by the lemma below.

Lemma: When $(1+x_1)(1+x_2)$ achieves the maximum, $\frac{1}{x_1} + \frac{1}{x_2}$ achieves the minimum.

Proof: first, we have

$$\begin{aligned} (1+x_1)(1+x_2) &= 1 + x_1 + x_2 + x_1 x_2 \\ &\le 1 + x_1 + x_2 + (\frac{x_1 + x_2}{2})^2 \\ &\le (1 + \frac{x_1 + x_2}{2})^2 \end{aligned} \tag{13}$$

From formula (13), we get that $(1+x_1)(1+x_2)$ achieves the upper bound, if and only if $x_1 = x_2$ .And when $x_1 + x_2$

achieves the maximum, the upper bound achieves the maximum too.

Then, we have

$$\frac{1}{x_1}+\frac{1}{x_2}=\frac{x_1+x_2}{x_1x_2} \geq \frac{x_1+x_2}{(\frac{x_1+x_2}{2})^2}=\frac{4}{x_1+x_2} \quad (14)$$

We also get from formula (14), that $\frac{1}{x_1}+\frac{1}{x_2}$ achieves the lower bound, if and only if $x_1=x_2$. And when x_1+x_2 achieves the maximum, the lower bound achieves the minimum.

So we proof that, when $\frac{1}{x_1}+\frac{1}{x_2}$ achieves the minimum lower bound, $(1+x_1)(1+x_2)$ achieves the maximum upper bound.

From the Lemma, the maximizing problem of the achievable sum-rate can be converted to a minimizing problem of $\frac{1}{SNR_1}+\frac{1}{SNR_2}$.

Using formula (8) and substitute the amplification factor as formula (3), we have,

$$\frac{1}{SNR_1}+\frac{1}{SNR_2} = \frac{\alpha^2|h_1|^2+1}{\alpha^2|h_1|^2|h_2|^2p_2}+\frac{\alpha^2|h_2|^2+1}{\alpha^2|h_1|^2|h_2|^2p_1} = \frac{|h_1|^2p_3+|h_1|^2p_1+|h_2|^2p_2+1}{|h_1|^2|h_2|^2p_2p_3}+\frac{|h_2|^2p_3+|h_1|^2p_1+|h_2|^2p_2+1}{|h_1|^2|h_2|^2p_1p_3} \quad (15)$$

Applying Lagrange multiplier to minimize formula (15) in condition of $p_1+p_2+p_3=p_{tot}$, the optimal transmission power allocation p_1, p_2 and p_3 can be obtained. For clarity of disposition, detailed computation is given in the appendix and the result is summarized here.

$$p_1=\frac{p_{tot}}{2+2\sqrt{\frac{|h_1|^2p_{tot}+1}{|h_2|^2p_{tot}+1}}},\quad p_2=\frac{p_{tot}}{2+2\sqrt{\frac{|h_2|^2p_{tot}+1}{|h_1|^2p_{tot}+1}}},\quad p_3=\frac{p_{tot}}{2} \quad (16)$$

For the special case, when $|h_1|^2=|h_2|^2$, the optimal power allocation can be written as

$$p_1=\frac{1}{4}p_{tot},\quad p_2=\frac{1}{4}p_{tot},\quad p_3=\frac{1}{2}p_{tot} \quad (17)$$

IV. NUMBERICAL RESULTS

In this section, we demonstrate the achievable sum-rate performance of TWRC system using ANC with optimal power allocation compared to uniform power allocation via numerical computation. For the computation, we assume that d is the distance between N_1 and N_2 which normalized to 1, and the channel gain $|h_i|^2$ is modeled as $|d_i|^{-r}$, where d_1 is the distance between N_1 and N_3, $d_2=1-d_1$ is the distance between N_2 and N_3. The path loss exponent r is set to 4, and the variances of Gaussian noises at all nodes are normalized to 1.

Fig. 2 shows the achievable sum-rate as a function of SNR, where SNR varies from 0 to 30db, and $d_1=0.8$. See from Fig. 2, we can find that the scheme of ANC with optimal power allocation performs better than the uniform power allocation, which achieves a gain of 2-3dB. Fig. 3 shows the achievable sum-rate as a function of d_1, where d_1 varies from 0 to 1, and SNR=30db. See from Fig. 3, we can find that the scheme of ANC with optimal power allocation always performs better than the uniform power allocation, and both the schemes perform best at point $d_1=0.5$.

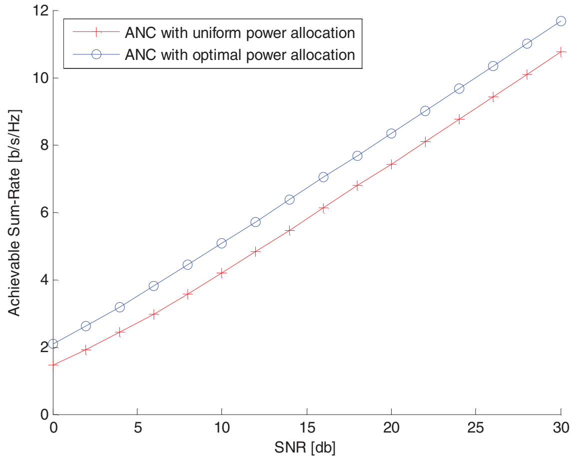

Figure 2. The Sum-rate of TWRC using ANC as a function of SNR.

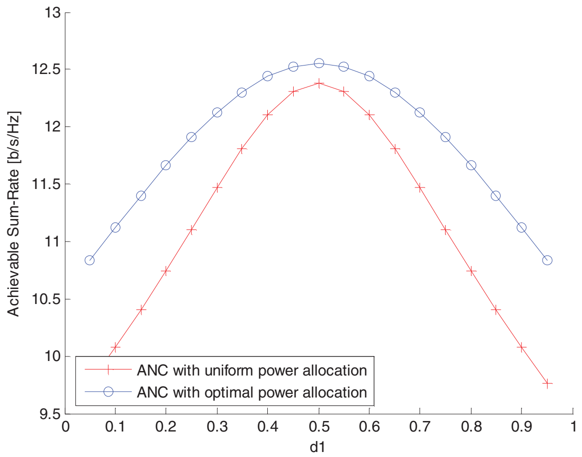

Figure 3. The Sum-rate of TWRC using ANC as a function of the distance between N_1 and N_3.

V. CONCLUSIONS

In this paper, we focus on the problem of transmission power allocation to maximize the achievable sum-rate of TWRC system using ANC. We compute and obtain an optimal power allocation solution which the sum-rate is highest. And we give the numerical results to demonstrate that our proposed scheme performs better than the uniform power allocation scheme. Our results offer new understanding of power allocation for TWRC system using ANC. As a future work, we will consider the extension of ANC protocol with diversity gain, and the multi-node scenarios.

APPENDIX

In this section, the detailed computation is given. Going back to formula (15), the problem is finding the solution of p_1, p_2 and p_3 which making formula (15) achieves the minimum in the constraints of $p_1 + p_2 + p_3 = p_{tot}$.

First, we simplify formula (15) as

$$\frac{(|h_1|^2 p_{tot}+1)p_1 + (|h_2|^2 p_{tot}+1)p_2}{|h_1|^2 |h_2|^2 p_1 p_2 p_3} \tag{16}$$

Applying Lagrange multiplier, first we construct the Lagrange function. Making formula (16) represent as function $f(p_1, p_2, p_3)$, and $g(p_1, p_2, p_3) = p_1 + p_2 + p_3 - p_{tot}$, we get

$$L(p_1, p_2, p_3, \lambda) = f(p_1, p_2, p_3) + \lambda g(p_1, p_2, p_3) \tag{17}$$

Then finding the stagnation point,

$$L_{p_1}(p_1, p_2, p_3, \lambda) = \frac{-(|h_2|^2 p_{tot}+1)}{|h_1|^2 |h_2|^2 p_1^2 p_3} + \lambda = 0 \tag{18a}$$

$$L_{p_2}(p_1, p_2, p_3, \lambda) = \frac{-(|h_1|^2 p_{tot}+1)}{|h_1|^2 |h_2|^2 p_2^2 p_3} + \lambda = 0 \tag{18b}$$

$$L_{p_3}(p_1, p_2, p_3, \lambda) = \frac{-(|h_2|^2 p_{tot}+1)}{|h_1|^2 |h_2|^2 p_1 p_3^2} + \frac{-(|h_1|^2 p_{tot}+1)}{|h_1|^2 |h_2|^2 p_2 p_3^2} + \lambda = 0 \tag{18c}$$

$$L_{\lambda}(p_1, p_2, p_3, \lambda) = p_1 + p_2 + p_3 - p_{tot} = 0 \tag{18d}$$

From formula (18a) (18b), we have

$$\frac{|h_2|^2 p_{tot}+1}{|h_1|^2 |h_2|^2 p_1^2 p_3} = \frac{|h_1|^2 p_{tot}+1}{|h_1|^2 |h_2|^2 p_2^2 p_3} \tag{19}$$

$$\frac{p_2}{p_1} = \sqrt{\frac{|h_1|^2 p_{tot}+1}{|h_2|^2 p_{tot}+1}} \tag{20}$$

From formula (18a) (18c), we have

$$\frac{|h_2|^2 p_{tot}+1}{|h_1|^2 |h_2|^2 p_1^2 p_3} = \frac{|h_2|^2 p_{tot}+1}{|h_1|^2 |h_2|^2 p_1 p_3^2} + \frac{|h_1|^2 p_{tot}+1}{|h_1|^2 |h_2|^2 p_2 p_3^2}$$
$$(|h_2|^2 p_{tot}+1)(p_3 - p_1)p_2 = (|h_1|^2 p_{tot}+1)p_1^2 \tag{21}$$
$$(p_3 - p_1)p_2 = \frac{|h_1|^2 p_{tot}+1}{|h_2|^2 p_{tot}+1} p_1^2$$

From formula (20), we have

$$p_3 = p_1 + p_2 \tag{22}$$

As a result from (18d) (20) (22), we find the solution of p_1, p_2 and p_3 mentioned above as formula (16).

ACKNOWLEDGMENT

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