# Fall 2004: Phys4051
# Op Amp Notes

The notes here are intended to guide you in analyzing general op-amp circuits. Hopefully they will clear up some confusion about one of the biggest misconception in understanding op-amps, namely the application of the golden rules from Horowitz & Hill to all op-amp circuits. It cannot be emphasized enough that the golden rule only applies in certain specific circumstances and applying it to the wrong op-amp circuits often enhances misconceptions about op-amps. On the other hand, the "general rules" below are true for any op-amp circuit.

## 1. General Rules

The following two conditions define the rules that follow:

First, an amplifier "amplifies" the difference between its input voltages, $\Delta V_{in}$:

**1:** $V_{out} = A\ \Delta V_{in}$ **where** $\Delta V_{in} \equiv V_+ - V_-$

"A" is the "**open loop gain**" of the amplifier and for an ideal op amp is infinity and for a typical op amp is between $10^6$ to $10^8$.

Second since we are dealing with an actual physical device its output voltages can never exceed its supply voltages, $V_{--}$ and $V_{++}$.

**2:** $V_{--} < V_{out} < V_{++}$

Typical supply voltages are $V_{--}$ = -15V and $V_{++}$ = +15V.

Applying these two conditions leads to 3 different cases:

**2a: If** $A\ \Delta V_{in} > V_{++}$ **then** $V_{out} = V_{++}$
**2b: If** $A\ \Delta V_{in} < V_{--}$ **then** $V_{out} = V_{--}$
**2c: If** $V_{--} < A\ \Delta V_{in} < V_{++}$ **then** $V_{out} = A\ \Delta V_{in}$

Conditions a) and b) merely restate rule 2 above, namely that if the predicted gain of the op-amp exceeds the supply voltages, it will "max out" (or "saturate") at the supply voltages.

Additionally, op-amps approach nearly ideal input and output impedance conditions. For completeness, the following rules are added:

**3a:** $Z_{out} = 0$
**3b:** $Z_{in-} = \infty$; $Z_{in+} = \infty$.

From 3b, it follows that:

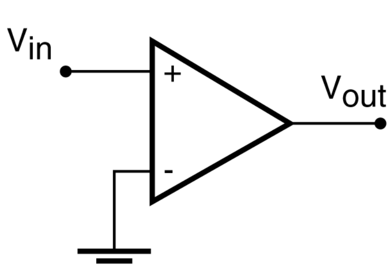
**3c:** $I_{in-} = 0$; $I_{in+} = 0$.

For most applications these assumptions holds since typical input currents are $10^{-9}$ to $10^{-12}$ Amps and the input impedances are in the $10^6$ to $10^9$ $\Omega$ range.

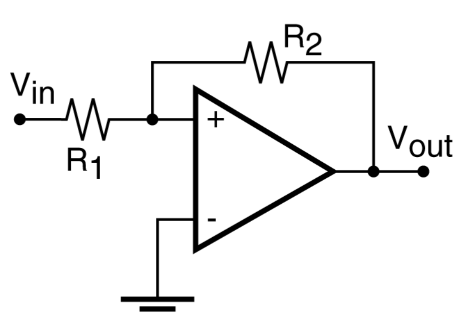
## 2. Feedback

***The most important step in analyzing an op-amp circuit is to determine first the type of feedback that is being employed!***

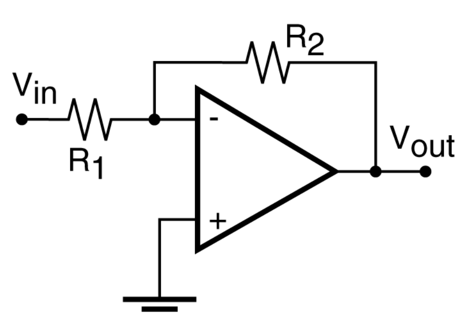
Feedback refers to connecting the output of the op-amp to its input, usually through resistors, and there are three basic ways to do that, shown below:



No Feedback



Positive Feedback



Negative Feedback

Figure A

Determining which feedback method is used will tell us in what function the op-amp is being used, see below:

# 3. No Feedback: Comparator

If no feedback is employed and applying the general rules for an ideal op-amp with A = ∞ shows that only cases 2a) and 2b) need to be considered, i.e.,

If $\Delta V_{in} > 0$, $V_{out} = V_{++}$
If $\Delta V_{in} < 0$, $V_{out} = V_{--}$.

In other words, an ideal op-amp without any feedback will always saturate either at its positive or negative supply voltage! Its output will change its state when $\Delta V_{in}$ changes its sign and it will do so at $\Delta V_{in} = 0$.

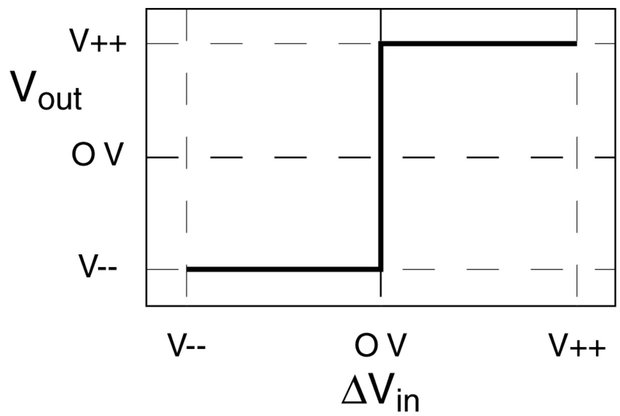



Figure B: $V_{out}$ vs. $\Delta Vin_{in}$. Note: the output switches at $\Delta Vin_{in} = 0$ and otherwise remains at either $V_{++}$ or $V_{--}$.

This type of feedback, or rather the lack thereof, is used in **comparators** circuits where one is interested only in discreet voltages, i.e., outputs that take on one of two states, i.e., $V_{++}$ or $V_{--}$.

While the first circuit in Figure A switches its output each time $V_{in}$ crosses 0 volt, a small modification, shown below, allows the circuit to switch its output when $V_{in}$ crosses a certain preset voltage level, often called the **threshold voltage**, $V_{th}$.

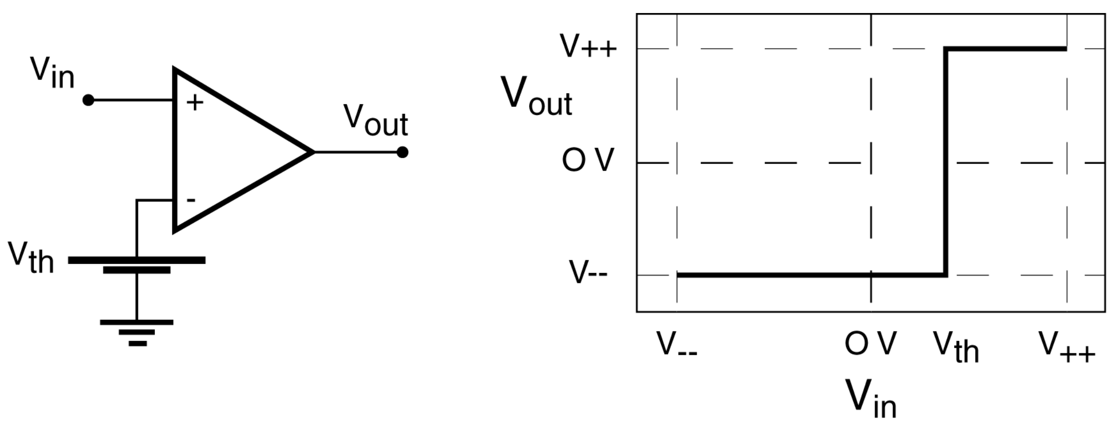
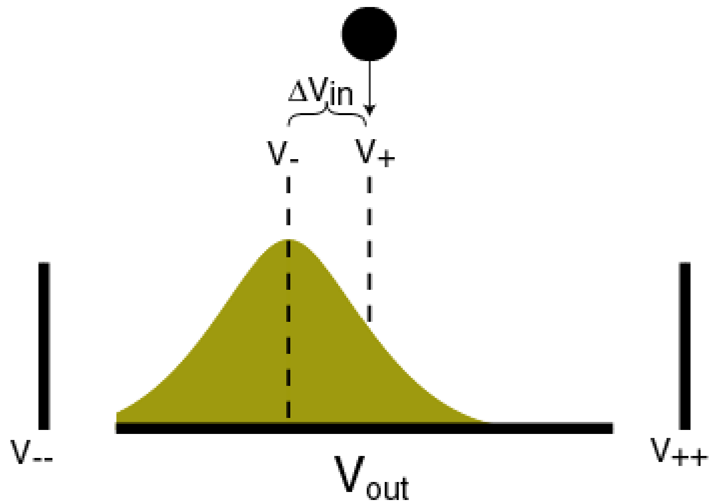



Figure C: Op-Amp comparator circuit with threshold voltage. Note the new switching point at $V_{th}$.

Typical applications of this circuit are crossover detectors, analog to digital converters or counting applications where one wants to count pulses that exceed a certain voltage level.

## 3.1. Comparator Analogy

A different way of looking at this type of circuit is to think of it in a gravitational analog which involves balls dropped on a hill shaped potential (see below.) Assume that the x-axis position corresponds to a voltage level: the position from which ball is dropped represents $V_+$ and the center of the hill is $V_-$; $V_{out}$ corresponds to final state of the ball along the x-axis. Depending on the two input variables, $V_+$ and $V_-$, the ball will always end up either all the way to the left or right of the center of the hill, i.e., the system is said to be **bistable**.




So far only comparator circuits with ideal op-amps have been considered. Luckily, for all practical purposes non-ideal (but real) op-amps behave almost identical. Given the typical open loop gain of $10^6$ to $10^8$ shows that only when $-15$ uV < $\Delta Vin_{in}$ < +15uV case 2c) would have to be considered, i.e., the op-amp is no longer saturated and it behaves like a "real" amplifier. Since this range is so small, for all practical purposes it can, or should, be ignored.

### 3.1.1 Exercises:

Figure out $V_{out}$ vs. $V_{in}$ for $V_+$ and $V_-$ reversed from the previous example.
Figure out an analog to TTL converter, i.e., for TTL: $V_{out}$ is either 0 or 5V.

## 4. Positive Feedback: Comparator with Hysteresis, Oscillators

The middle circuit in Figure A has positive feedback yet it still acts like a comparator but, in addition, it now has memory, i.e., it exhibits hysteresis. Though positive feedback is generally associated with oscillation, this circuit is not an oscillator because the input to the circuit ($V_-$) is independent of the output. To create a real oscillator, the input to the circuit must also be in some way connected to the output as, for example, in the RC relaxation oscillator circuit shown in the lab manual.
To understand what causes the hysteresis let's analyze the circuit in Figure E using the same rules as in the previous section for the comparator. (Though at this point that is only an assumption but our results will ultimately show that indeed it acts similar to a comparator.) The key in understanding this circuit will again be in calculating the voltages that cause its output to switch.

The output will still be $V_{++}$ or $V_{--}$, depending on whether $\Delta V > 0$ or $\Delta V < 0$. Switching of the output will occur when the value $\Delta V$ changes sign, i.e., at $\Delta V = 0$.

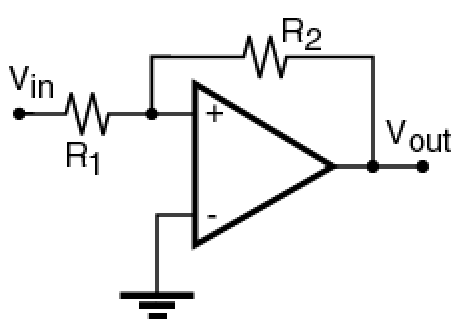


Figure E: Positive Feedback

For the particular circuit above, $\Delta V = V_+$ since $V_- = 0$. So,

- if $V_+ > 0$, $V_{out} = V_{++}$,
- if $V_+ < 0$, $V_{out} = V_{--}$.

Since $V_{out}$ change its state whenever $V_+$ crosses 0V, we need to find what value of $V_{in}$ results in $V_+ = 0$. Since $V_+$ is a voltage divider formed by $R_1$ and $R_2$between $V_{in}$ and $V_{out}$ it follows that:

- $V_+ = (V_{in} R_2 + V_{out} R_1)/(R_1 + R_2)$

Combining these equations and solving for $V_+ = 0$ yields $V_{in} = -V_{out} R_1/R_2$. Since $V_{out}$ can only be either $V_{++}$ or $V_{--}$, $V_{out}$ switches from $V_{++}$ to $V_{--}$ when $V_{in} = -V_{++} R_1/R_2$ and it switches from $V_{--}$ to $V_{++}$ when $V_{in} = -V_{--}R_1 / R_2$. In other words, it always takes "more effort" to reverse the output than was initially required.

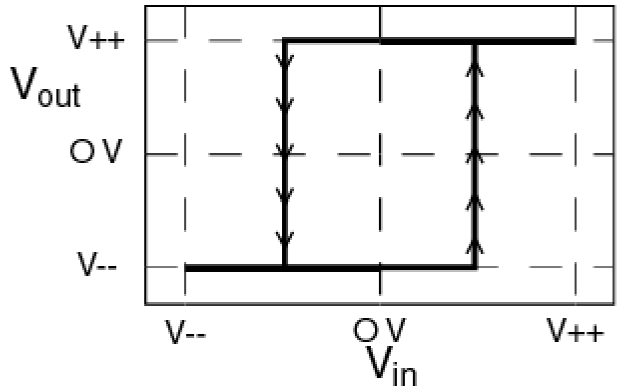



Figure E1*: Vout vs Vin for the Positive Feedback Circuit*

### *4.1. Gravitational Analogy*

If this approach was too mathematical, consider a gravitational analog again. Again assume the $V_{in}$ corresponds to the x-position along the x-axis of balls being dropped on a hill potential from which they either roll down on the left or the right side. While in the previous example the mountain (and its peak) where rigid assume that it itself can move along the x-axis and remain in one of two positions: if a ball previously rolled down its right side, it will be pushed all the way to the left, see figure 3 and remain there till a ball rolls down its left side at which point it will move all the way to the right as shown in Figure 6. From this analogy you can see that it takes some extra effort to reverse the output state.

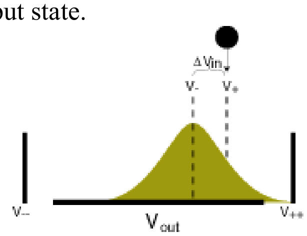



*Figure 1*

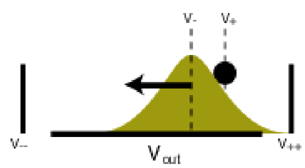



*Figure 2*

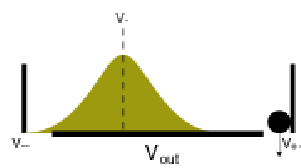



*Figure 3*

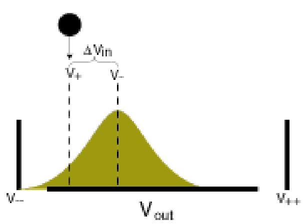



*Figure 4*

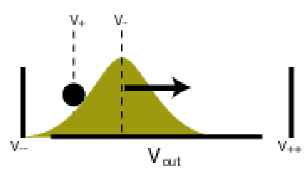



*Figure 5*

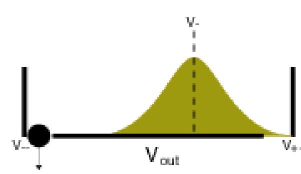



*Figure 6*

With some small modifications to the above circuit, positive feedback circuits can also be used for oscillator circuits. For the sake of brevity, this topic will be covered at a later time.

# 5. Negative Feedback: Amplifiers and Golden Rule of Op-Amps

Only when negative feedback is applied to the op-amp it (may) act(s) as a linear amplifier. To be precise, it can only act as a linear amplifier as long as its output is not saturated, i.e., it does not exceed its supply voltages.

Nevertheless, when these two conditions are met, (negative feedback AND not saturated) a new rules, called the **"Golden Rule of Op-Amps"** can be applied. It says:

**4: $V_+ \cong V_-$.**

As already mentioned, be careful when applying this rule that both conditions stated above are met!

The Golden Rule of Op-amps makes the analysis of negative feedback circuits straightforward:

1. Remove the op-amp altogether from your circuit
2. State that $V_+ = V_-$.
3. Solve.

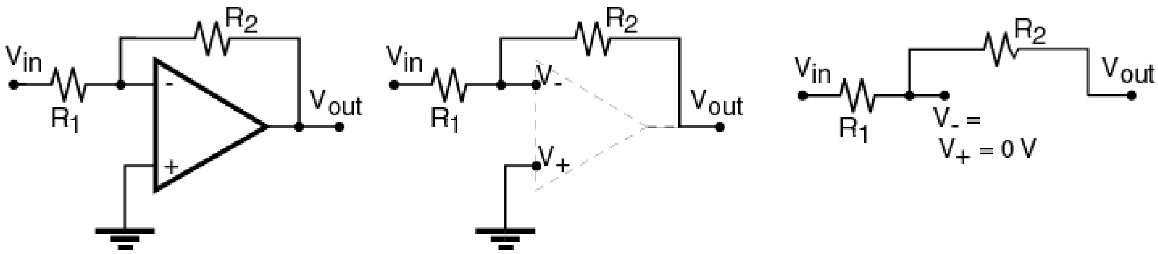



Figure G: *The circuit shown is the negative feedback circuit from Figure A. Step 1 is shown in the middle picture and step 2 is at right.*

After applying steps 1 and 2, all that is usually left for the analysis is some sort of resistive voltage divider, as shown in Figure G at the right. Noting that the current through $R_1$ must be identical to the current through $R_2$ leads as immediately to the application of Ohm's law, namely:

- $(V_{in} - V_-)/R_1 = (V_- - V_{out})/R_2$

Since $V_- = 0$

- $V_{out} = (-R_2/R_1)\, V_{in}$

Before applying the golden rule blindly, even where it applies, one should ask where does it come from and does it make sense?

To start with the second question, consider once more the inverting amplifier solved previously. We found by applying the golden rules that $V_{out} = (-R_2/R_1)\, V_{in}$. On the other hand, when applying these rules to obtain this result, they were used to draw the conclusion that $V_+ = V_- = 0$ V. Since $\Delta V \equiv V_+ - V_-$ it follows then that for the above case $\Delta V = 0$. Applying now our primary rule, namely that $V_{out} = A\Delta V$, results in $V_{out} = 0$ which is in direct contradiction to the result obtained by applying the golden rules of op-amps. So which calculation is right?

Since we stated that rule #1, $V_{out} = A\Delta V$ always holds, (at least if $V_{out}$ is not in saturation) we should really apply it to solve for $V_{out}$. Also for now assume that A is finite. Solving the circuit in Figure G using only rule #1, (and NOT applying the Golden Rules of Op-Amps) one would find: (please check for yourself)

- $$V_{out} = \frac{-R_2/R_1}{1 + \left[\dfrac{1 + R_2/R_1}{A}\right]} V_{in}$$

We see that when $A \gg R_2/R_1$ we arrive at the same answer that was previously obtained applying the Golden Rules. In other words, the Golden Rule assumes A being infinite, or at least much larger than the negative feedback gain of the circuit. The negative feedback is usually called the "**closed loop gain.**"

The infinite value of A also explains the contradiction we arrived at previously. Solving Rule #1 for $\Delta V$, we find that $\Delta V = V_{out}/A$. If we assume that A is infinite, well then the $\Delta V$ also will approach 0 V. In other words, we are dealing now with the singularities of infinity and 0. Using "realistic" values for $A = 10^8$, we find that $\Delta V = 10^{-8}\, V_{out}$, under any circumstances, a very, very small signal but not quite 0!