6.302 Feedback Systems

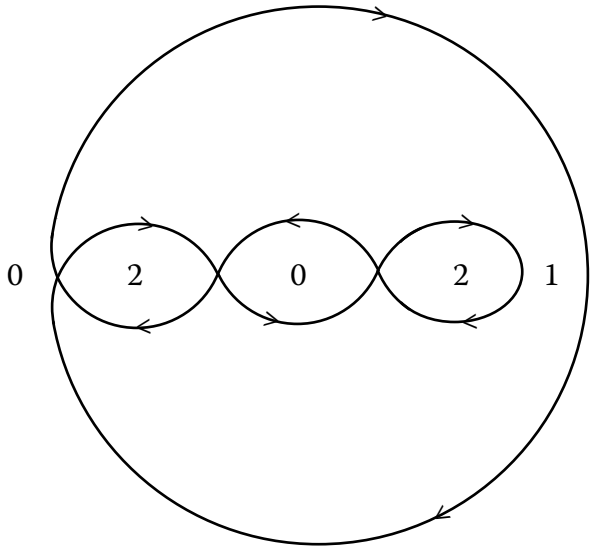
Recitation 10: Nyquist Stability

Prof. Joel L. Dawson

Today's theme is going to be examples of using the Nyquist Stability Criterion. We're going to be counting encirclements of the "-1/k" point.

But first, we must be precise about how we count encirclements.

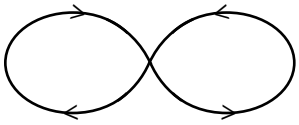
EXAMPLE 1:

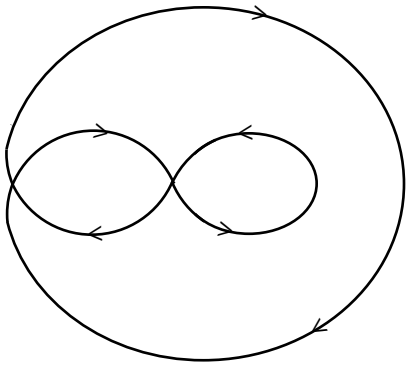


Problem: In each region, how many positive (clockwise) encirclements are there? The numbers in the regions are the answers. Notice that when counting encirclements, we are free to split the contour into any convenient number of continuous circles or loops.

CLASS EXERCISE

Label the number of clockwise encirclements in the Nyquist plots below:

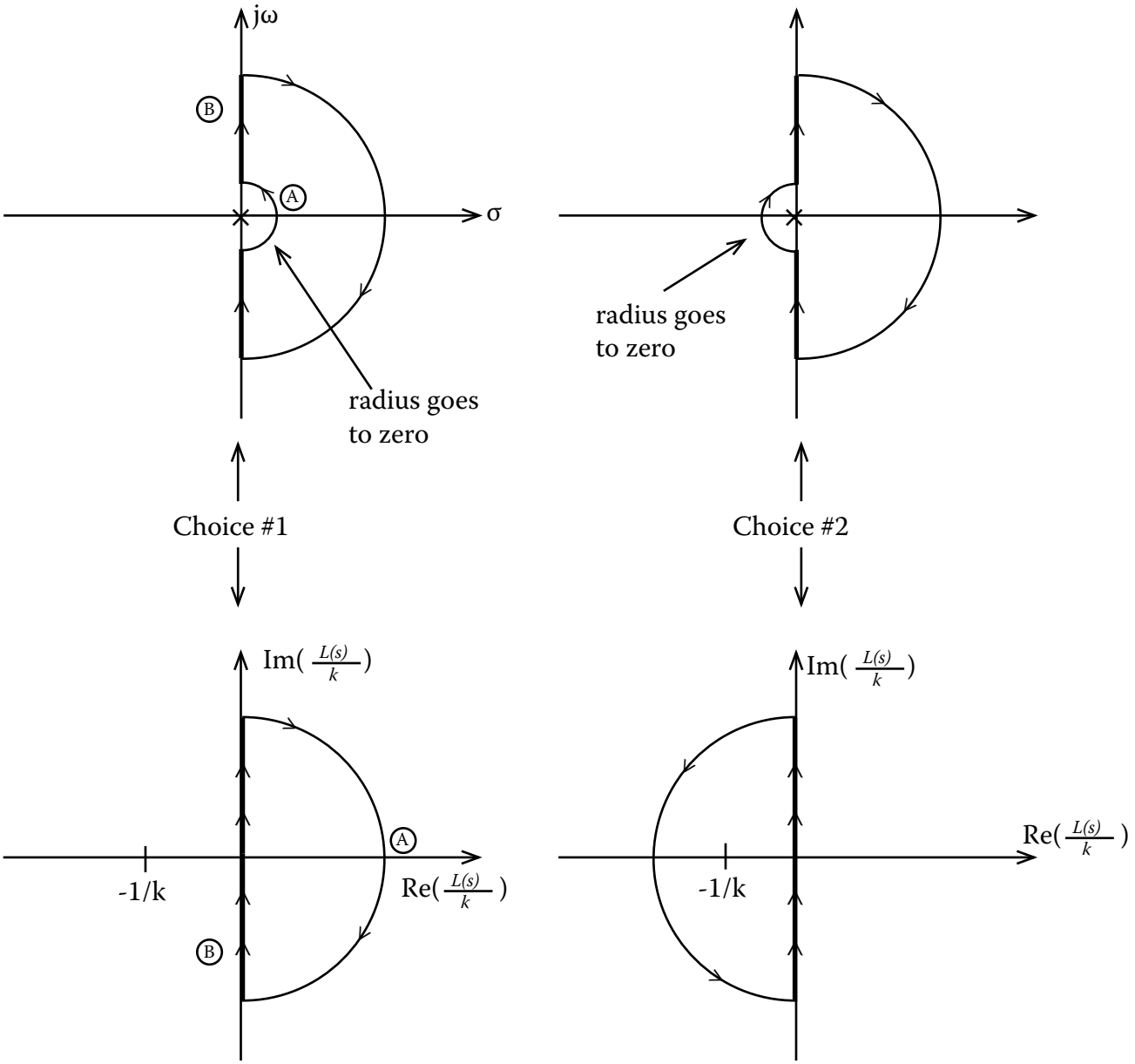




Now there is only one more difficulty to resolve: what to do with singularities on the jω axis?

$$\frac{L(s)}{k} = \frac{1}{s}$$

We actually have two choices:



Let's do the accounting for the two cases. Remember that we're calculating $Z = N + P$, and we want $Z = 0$ if the system is to be stable.

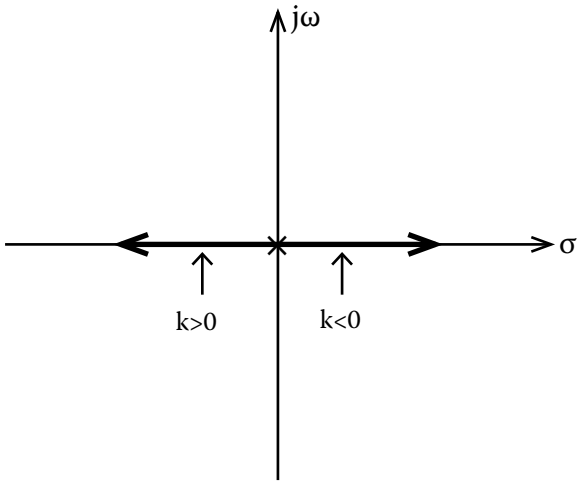
Choice #1

Range of k	P	N	Z	Stable?
$-\infty < k < 0$	0	1	1	NO
$0 < k < \infty$	0	0	0	YES

Choice #2

$-\infty < k < 0$	1	0	1	NO
$0 < k < \infty$	1	-1	0	YES

Either choice gives you the right answer! And both choices give agreement with root locus:



We have all of the basic tools now...we can proceed now to even more complex examples. There's one more trick that can help you, which is that bode plots can be a tremendous help in drawing complicated Nyquist contours.

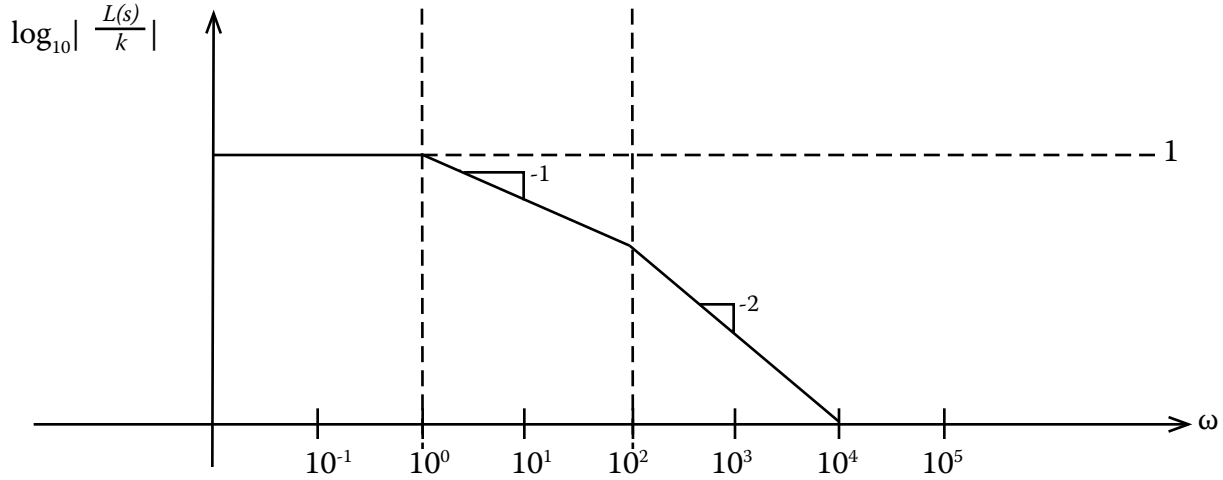
EXAMPLE: $\frac{L(s)}{k} = \frac{1}{(s+1)(10^{-2}s+1)}$

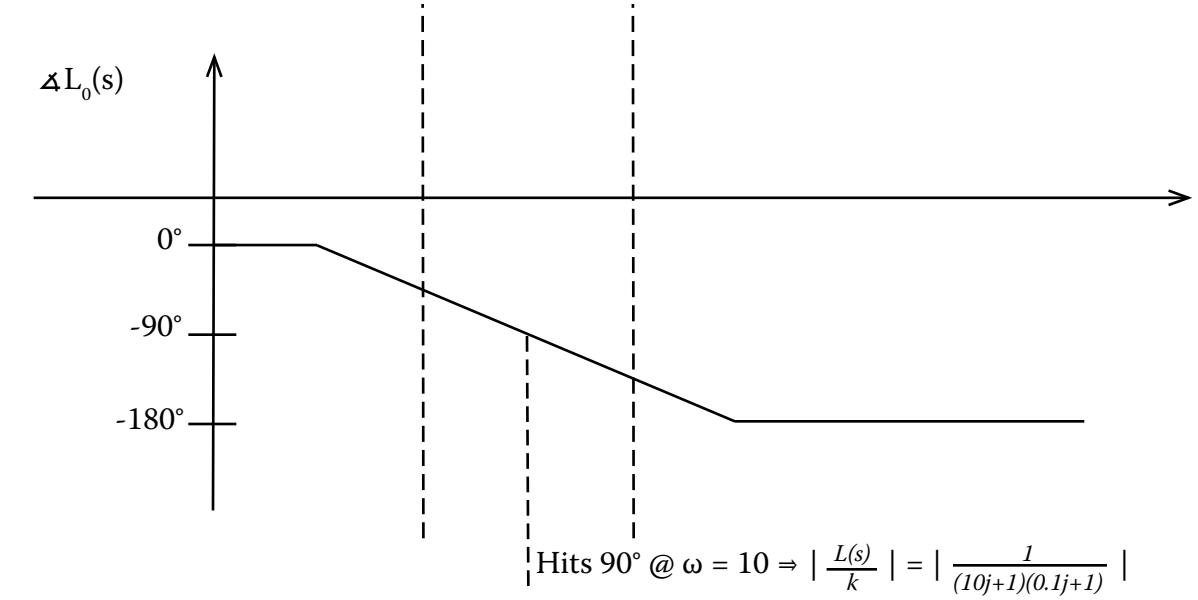
6.302 Feedback Systems

Recitation 10: Nyquist Stability

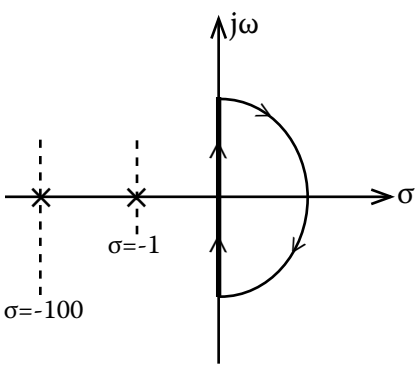
Prof. Joel L. Dawson

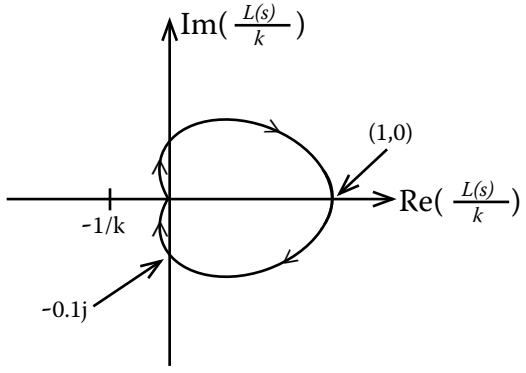
Consider the Bode Plot: $\frac{1}{(s+1)(10^{-2}s+1)}$





Hits 90° @ $\omega = 10 \Rightarrow \left| \frac{L(s)}{k} \right| = \left| \frac{1}{(10j+1)(0.1j+1)} \right| \approx 0.1$

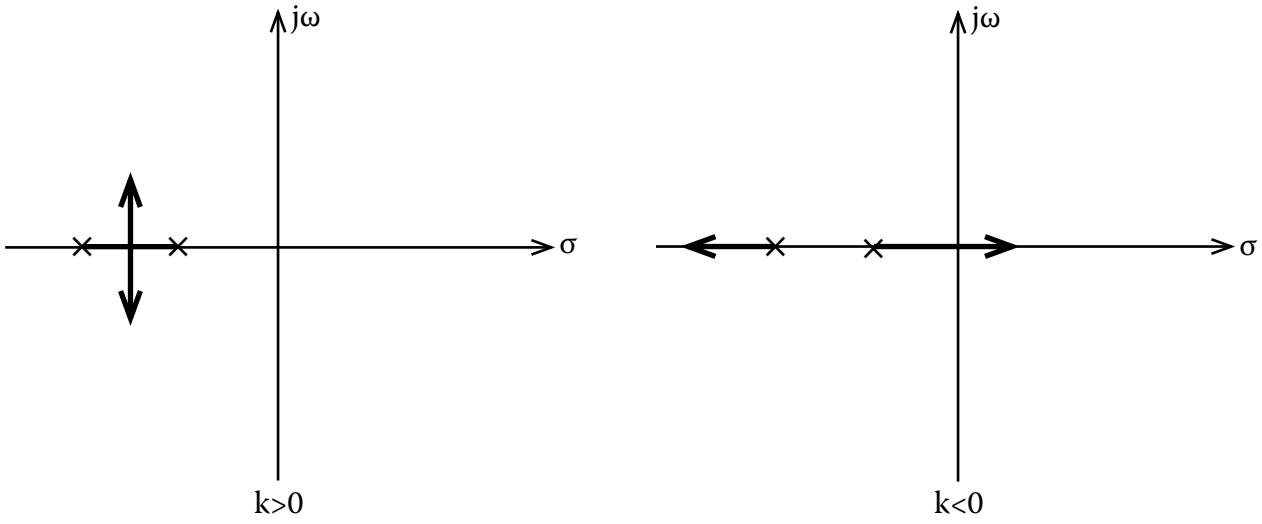




We can now do the normal accounting:

Range of k	P	N	Z	Stable?
$-\infty < k < -1$	0	1	1	NO
$-1 < k < 0$	0	0	0	YES
$0 < k < \infty$	0	0	0	YES

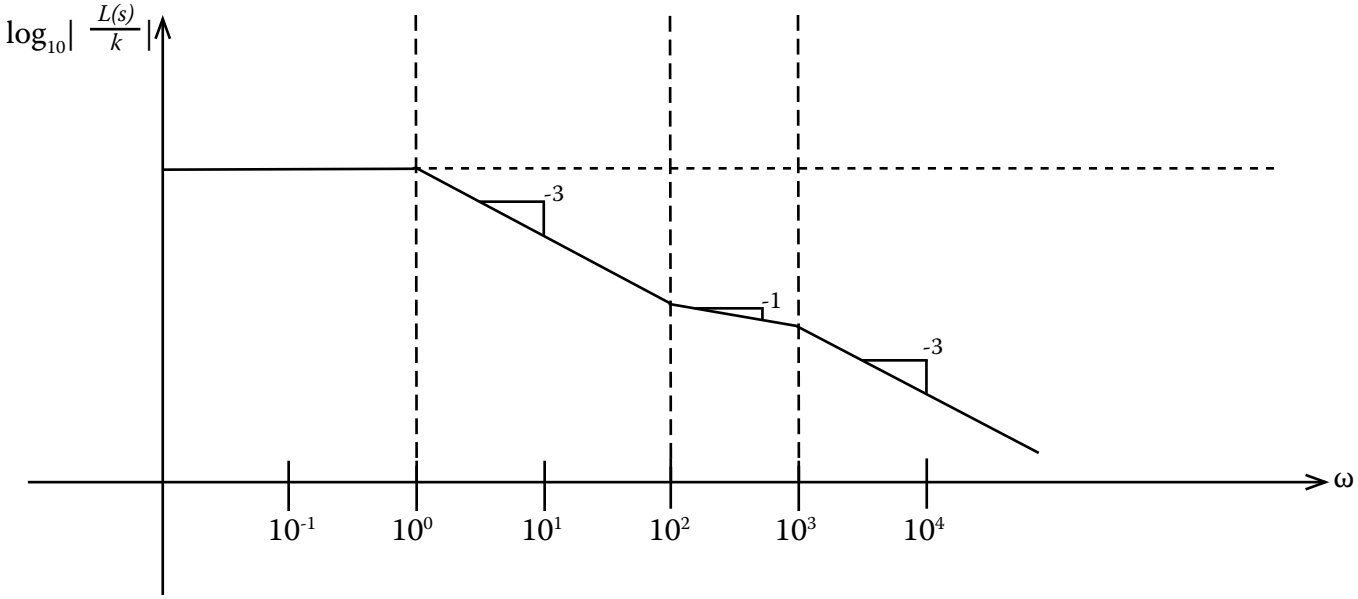
And check our results with root locus:

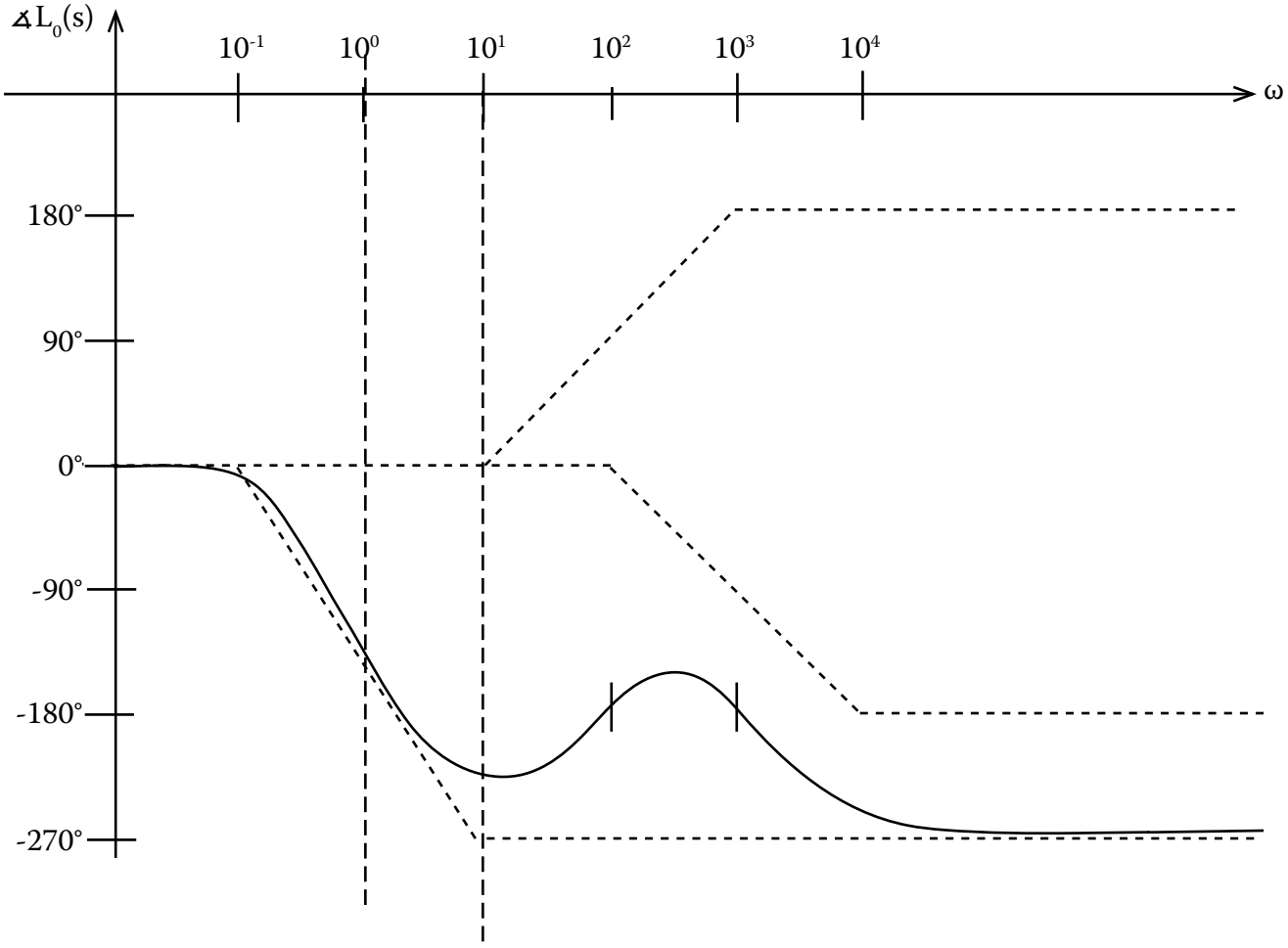


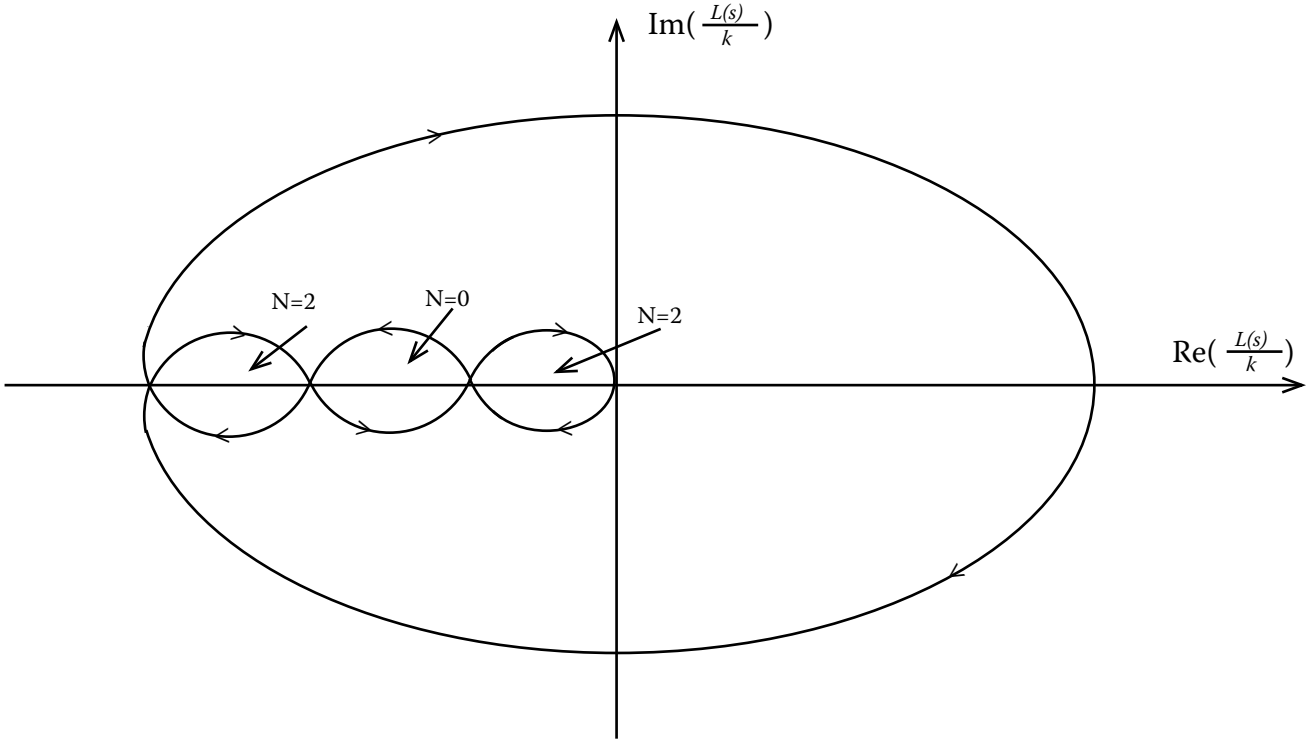
And now an impressive example, just in case you thought EXAMPLE 1 was some kind of joke...!

$$\frac{L(s)}{k} = \frac{(10^{-2}s + 1)^2}{(s+1)^3(10^{-3}s + 1)^2}$$

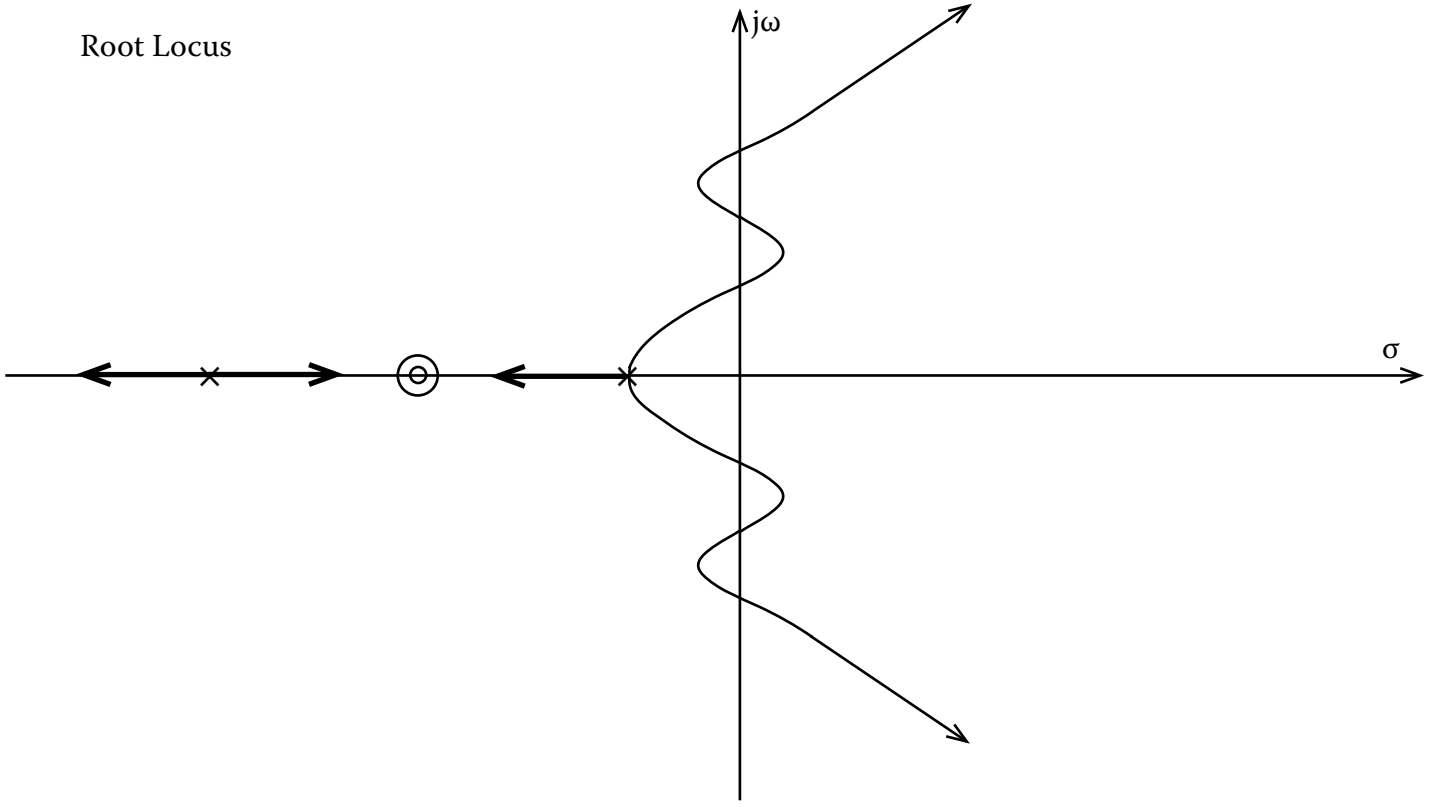
Begin by doing a Bode Plot ⇒


log10| L(s)/k |
-3
-1
-3
ω
10-1
100
101
102
103
104


∠L0(s)
10-1
100
101
102
103
104
ω
180°
90°
0°
-90°
-180°
-270°



Root Locus



This is a PRIME EXAMPLE of a conditionally stable system.

Now look back at the Bode Plot, and observe where the phase crossings of 180˚ are taking place. The "bubble" on the Nyquist plot where N = 0 is located at around $1/k = 10^{-6.5}$.

Let $k=10^{6.5}$then, and return to the Bode Plot. You'll discover that the first time the phase hits 180˚, the magnitude of the loop transmission is greater than unity!! But for this value of k, the Nyquist Criterion says the system is stable.

Nyquist is right...we can't trust our intuition in this case.