

E 1 (*Aliasing*) Let $x[n]$ be a sampled version of a continuous time signal $x(t) = \sin(2\pi t)$, a sinusoid of 1Hz, consider the sampling rate to be $f_s = 20$ samples per second.

- Find all the sinusoids which give the same sample set when sampled with f_s . In other words find all the aliased frequencies!

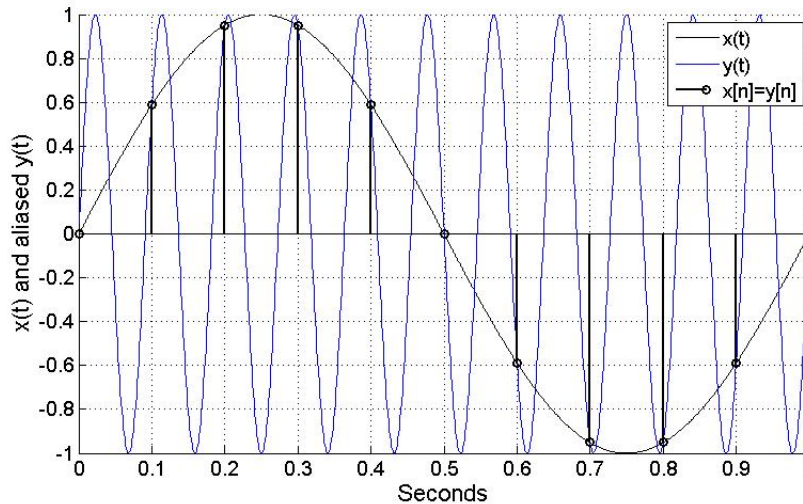


Figure 1: An example of Aliasing

E 2 (*Fourier!*)

1. Given a periodic function $x(t)$ as shown in the Figure.2(dark black continuous pulse train) compute the Fourier series expansion.
2. In Figure.3 a pulse $p(t)$ of width 0.2sec is given. Compute the Fourier transform of the same.
3. Let the sequence $x[n]$ be the sampled version of $x(t)$. For this periodic sequence calculate the Discrete Time Fourier Series.
4. Similarly let $p[n]$ be the sampled $p(t)$ as shown in Figure.3. For this compute the Discrete Time Fourier Transform.
5. Take 10, 20, 100 samples of $p[n]$ starting from $n = -0.1$ and compute the Discrete Fourier Transform in each case.
6. State your observations.

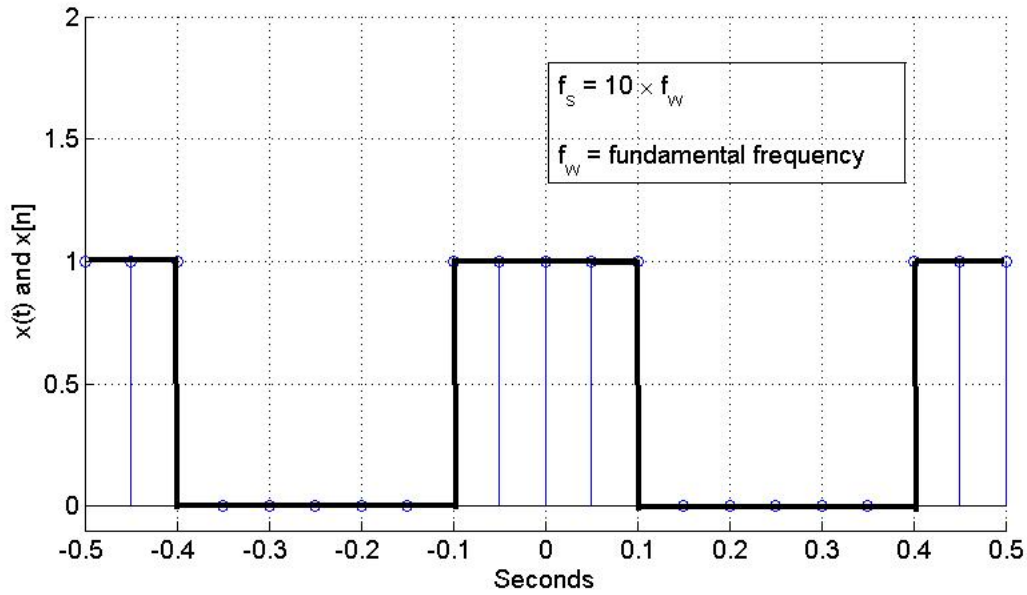


Figure 2: A rectangular pulse train

E 3 (*Filters from Analog to Digital*)

1. Consider the RC network shown in Figure.4. It is a two stage network. For each stage compute the impulse response. Let the impulse response for each stage be $h_1(t) = h_2(t)$.
2. Show that the impulse response of the network in Figure.4 will be

$$h(t) = h_1(t) * h_2(t).$$

3. Also show that $H(j\omega) = H_1(j\omega)H_2(j\omega)$. Plot $|H(j\omega)|^2$.
4. Let the input to the network be $x(t) = U(t)$, a unit step function, as shown in Figure.5. Calculate the output $y(t)$.
5. Let the input be discretized with sampling interval as shown in Figure.5. Now using the approximations for derivatives as discussed in class compute the output $y[n]$ for input $x[n]$. Compare it with the continuous case.

E 4 (*DFT*)

- Find a 5 point sequence $x[n]$ for which if $\hat{X} = DX$, where D is the 5 point DFT matrix and $X := [x[0] \dots x[4]]^T$, then $\hat{X} \propto X$. (Proportional upto a complex multiple)

OR

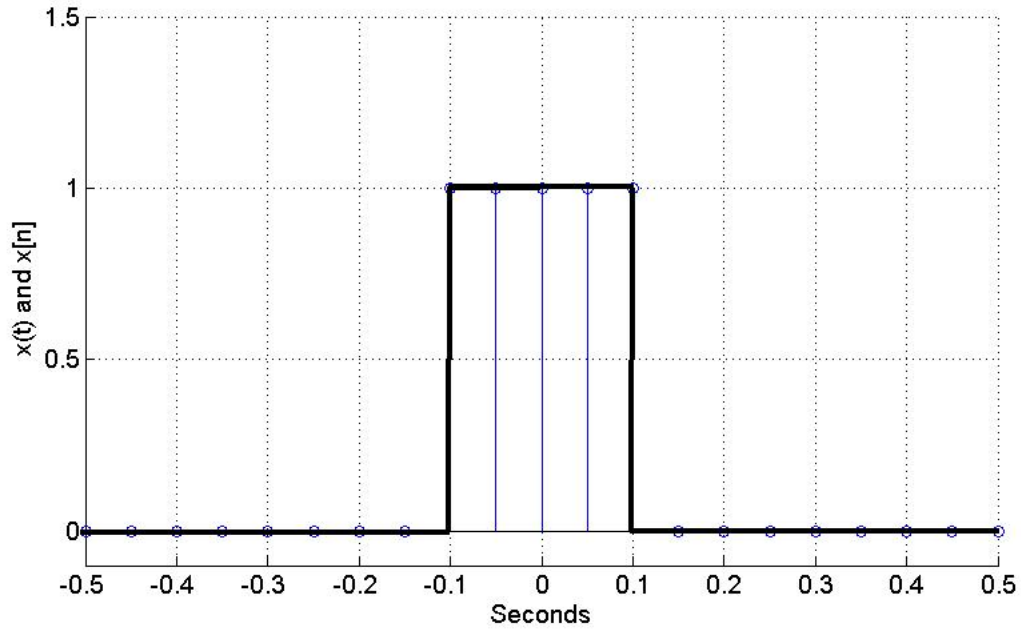


Figure 3: A pulse

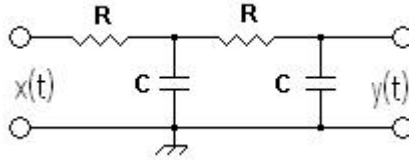


Figure 4: A 2-stage RC network with $RC = 1$

- Let the support of a sequence be defined as the set of indices where the values of sequence is not zero, for example let $x[n] = \{1, 0, 1, 0\}$, $n = 0, \dots, 3$ then $\text{supp } x := \{0, 2\}$. We have discussed in class that, if $\hat{x}[k] = \text{DFT}(x[n])$ then

$$|\text{supp } \hat{x}| |\text{supp } x| \geq N \quad (1)$$

where N is the length of the sequence $x[n]$ and $|\cdot|$ represents the cardinality of the set. For above given $x[n]$ the $\hat{x}[k] = \{2, 0, 2, 0\}$, and $|\text{supp } \hat{x}| |\text{supp } x| = 4$. Give an example of a sequence of length 6 such that the equality in the Equation.1 is attained.

E 5 (DFT as an instance of change of basis.)

1. Write down a matrix form, an operator $H : \mathbb{R}^4 \rightarrow \mathbb{R}^4$, of a 4-tap FIR filter. Consider 4-point input as a vector $X = [x[0] \dots x[3]]^T$. Convolution is to be performed in a circular fashion.
2. Find all the eigen values and eigen vectors of H .

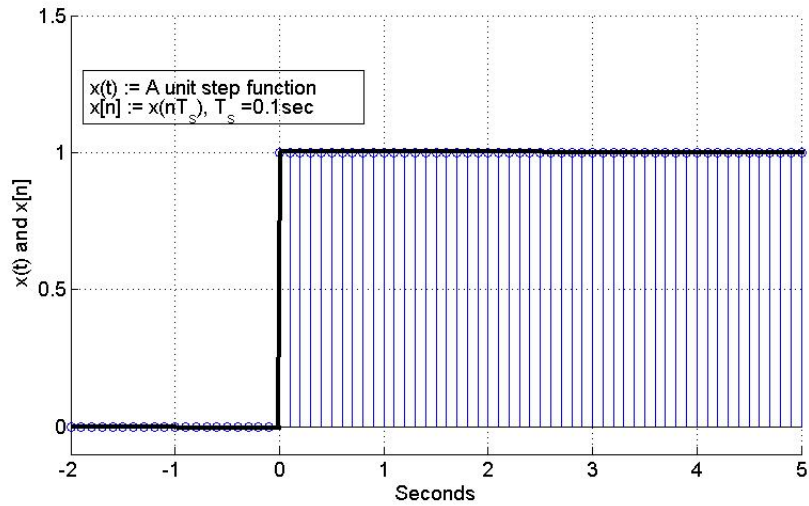


Figure 5: A unit step input

3. Show that under similarity transformation H is diagonalizable.
4. Show that all permutation matrices related to the circular shifts are simultaneously diagonalizable with the same similarity transform.

E 6 (Short Time Fourier Transform and location of Discontinuity)

Let $x[n] = \cos(2\pi \times 5 \times 10^{-2} \times n)$ and $n = 0, 1, \dots, 399$. With $x[101] = 3$,

1. Compute spectrogram of $x[n]$ with Hamming window of length 16 and 32, with no overlap and 50% overlap.
2. State your observations.

E 7 (Fast Fourier Transform)

For 8-point FFT,

1. Show the bit reversal pattern with each step of decimation.
2. Draw the butterfly diagram.
3. Compute the number of complex multiplications involved.
4. For $x[n] = \{1, 0, 1, 0, 1, 0, 1, 0\}$ compute $FFT(x[n])$.