

RF Hybrid Devices

M568

V2.00

M/A-COM offers a comprehensive list of standard RF Hybrids through our "HH-XXX" and "JH-XXX" series of products. The following explains the purpose and use of these products.

The 90° and 180° hybrids are two forms of passive, reciprocal four-port devices which have wide application in RF and microwave system design. The devices described in this article are equal power split versions of the general four-port hybrid configuration. Directional couplers, described in a separate article, are actually 180° hybrids having unequal power splits. Power dividers/summers generally represent another variation of the basic 180° hybrid with an internal termination on one port. These devices are also described in detail in a separate article.

The purpose of this article is to provide the designer with basic information describing the function of these devices, the basic specification parameters with possible tradeoffs and the relationships that apply for various combinations of signal inputs and port terminations. Signal flow descriptions in this article assume the ideal, lossless version of the 180° and 90° hybrid. This is a reasonable approximation for this purpose because of the generally low loss, well matched characteristics of the actual circuits.

Functional Description

180° Hybrid

A 180° hybrid is a reciprocal four-port device which provides two equal amplitude in-phase signals when fed from its sum port and two equal amplitude 180° out-of-phase signals when fed from its difference port.

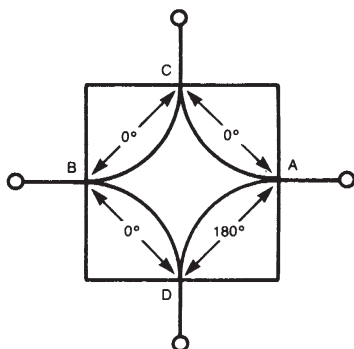


Figure 1. 180° Hybrid

Opposite ports of the hybrid are isolated. Figure 1 is a functional diagram which will be used in this article to represent the 180° hybrid. Port B can be considered the sum port with port A the difference port. Ports A and B and ports C and D are isolated pairs of ports.

Utilizing the functional diagram of Figure 1, we can consider the application of signal at one or more of the ports of the hybrid. The convention used for explaining signal flow is based on Figure 2. The cases that are important to consider are the following:

1. Operation as a power divider - One source operating at ports A, B, C or D.
2. Operation as a power summer - Two sources operating at ports A and B, or C and D.

For these cases, the impedances Z_A , Z_B , Z_C , and Z_D are assumed to be Z_0 , the characteristic impedance of the 180° hybrid. Under this matched condition, the source voltage of $2E \cos \omega t$ will supply a voltage of $E \cos \omega t$ to the input of the hybrid.

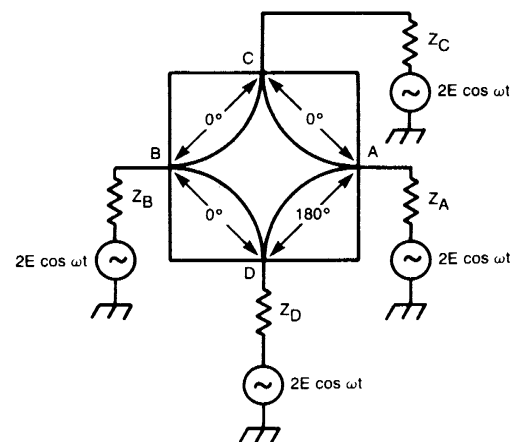


Figure 2. Signal Source Configurations for 180° Hybrid
(Reference Tables I, II, III, IV)

As a power divider, the hybrid will equally split the input signal and deliver one half the power to each load. Since all ports are considered to be at Z_0 impedance, the voltages at the outputs will be proportional to the square root of the output power and will be phase shifted by the amount indicated for that path of the hybrid, since

$$P_{OUT} = \frac{1}{2} P_{IN} = V_{OUT}^2 = \frac{1}{2} V_{IN}^2$$

For example, if an input signal at P_{OUT} A of $E \cos \omega t$ is injected, the resultant

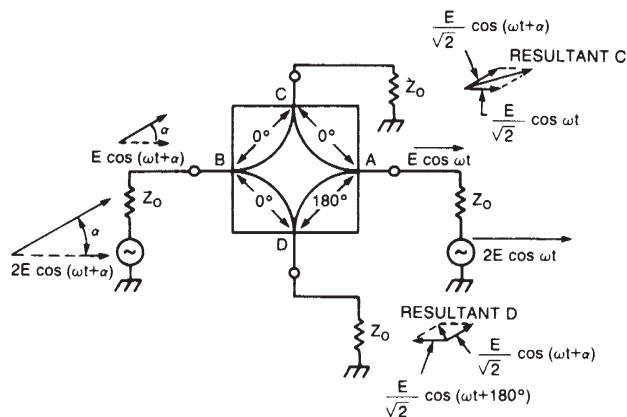
output at Port C is $\frac{1}{\sqrt{2}} E \cos \omega t$ and the output at Port D

is $\frac{1}{\sqrt{2}} E \cos (\omega t - 180^\circ)$. No signal will appear at port B.

The various power divider relationships are summarized in Table 1.

When used as a power summer, the function of the 180° hybrid is somewhat less obvious due to the vector addition of the two signals. Figure 3 shows the signal flow and resultant outputs for the general case of two equal amplitude, equal frequency signals of arbitrary phase. The vector representation of the input signals as well as the resultant output signals is shown graphically together with the algebraic expression for the signal. It should be noted that, in the general case, the phases of the resultant outputs are in quadrature. Table II lists the relationships for various combinations of signals applied in pairs to Ports A and B or Ports C and D.

- Equal Amplitude, Equal Frequency Input
- Resultant Output Phases are in Quadrature
- Resultant Output Magnitudes vary based on Input Phase



Resultant C

$$\begin{aligned}
 &= \frac{E}{\sqrt{2}} [\cos \omega t + \cos (\omega t + \alpha)] \\
 &= \frac{E}{\sqrt{2}} [2 \cos \frac{1}{2} (2\omega t + \alpha) \cos \frac{1}{2} (-\alpha)] \\
 &= \sqrt{2} E \cos (-\frac{\alpha}{2}) \cos (\omega t + \frac{\alpha}{2})
 \end{aligned}$$

Resultant D

$$\begin{aligned}
 &= \frac{E}{\sqrt{2}} [\cos (\omega t + 180^\circ) + \cos (\omega t + \alpha)] \\
 &= \frac{E}{\sqrt{2}} [2 \cos \frac{1}{2} (2\omega t + \alpha + 180^\circ) \cos \frac{1}{2} (180^\circ - \alpha)] \\
 &= \sqrt{2} E \cos (90^\circ - \frac{\alpha}{2}) \cos (\omega t + \frac{\alpha}{2})
 \end{aligned}$$

∴ Phase of Resultant D = Phase of Resultant C + 90°

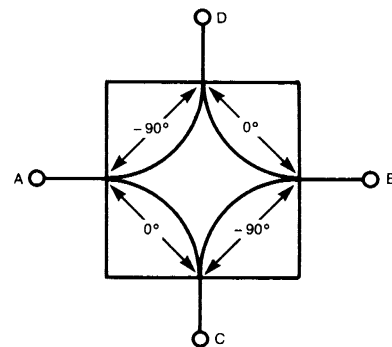
Figure 3. 180° Hybrid

Tables III and IV provide useful relations for determining isolation and VSWR under varying loading conditions. One point to note from the expressions in these tables is that equal mismatches on opposite ports of the hybrid do not effect isolation since the reflected signal will cancel at the isolated port while they will add at the port where the signal is injected. Thus, for example, if we inject a signal at Port A with equal mismatches at Ports C and D ($P_C = P_D$), then no signal will appear at Port B because the reflected components are 180° out-of-phase. The VSWR at Port A will be degraded because the reflections add in phase at this port. In general, equal mismatches may not be present. The relations in Tables III and IV may be used to calculate VSWR and isolation for any known combination of load impedance.

90° Hybrid

A 90° hybrid functions in much the same manner as 180° hybrid since it is also a reciprocal four-port device. Equal amplitude outputs result when a signal is fed to one of the inputs. Opposite ports of the 90° hybrid are also isolated as in the 180° hybrid. The different phase relationship of the 90° hybrid does however, cause important functional differences.

Figure 4 shows the circuit diagram and truth table that will be used in explaining the operation of the 90° hybrid. As can be seen from this diagram, a signal applied to any input will result in two quadrature or 90° outputs. Ports A and B and Ports C and D are isolated. Following an analysis similar to that applied to the 180°



	A	B	C	D
A		ISO.	0	-90
B	ISO.		-90	0
C	0	-90		ISO.
D	-90	0	ISO.	

ISO. = ISOLATION

Truth Table

Figure 4. 90° Hybrid

Power Divider

The relationships given in Table I apply when the 180° hybrid is used as a power divider. The following conditions apply.

1. $Z_A = Z_B = Z_C = Z_D = Z_O$
2. Only one generator at a time is operating
3. Infinite isolation is assumed in the junction

Table I – Power Divider Relationships for 180° Hybrids

Input Signal	Input Port	Output Signals			
		Port A	Port B	Port C	Port D
$E \cos \omega t$	A	–	0	$\frac{1}{\sqrt{2}} E \cos \omega t$	$\frac{1}{\sqrt{2}} E \cos(\omega t + 180^\circ)$
	B	0	–	$\frac{1}{\sqrt{2}} E \cos \omega t$	$\frac{1}{\sqrt{2}} E \cos \omega t$
	C	$\frac{1}{\sqrt{2}} E \cos \omega t$	$\frac{1}{\sqrt{2}} E \cos \omega t$	–	0
	D	$\frac{1}{\sqrt{2}} E \cos(\omega t + 180^\circ)$	$\frac{1}{\sqrt{2}} E \cos \omega t$	0	–

Power Summer

The relationships given in Table II apply when the 180° hybrid is used as a power summer. The following conditions apply.

1. $Z_A = Z_B = Z_C = Z_D = Z_O$
2. Two generators are operating simultaneously.
3. Infinite isolation is assumed in the junction.

* Reference Figure 3.

Table II – Power Summer Relationships for 180° Hybrids

Input Signal	Input Port	Output Signals			
		Port A	Port B	Port C	Port D
$*E \cos \omega t$	A	–	–	$\sqrt{2} E \cos(-\alpha/2)$	$\sqrt{2} E \cos(90^\circ - \alpha/2)$
$*E \cos(\omega t + \alpha)$	B			$[\cos(\omega t + \alpha/2)]$	$[\cos(\omega t + \alpha/2 + 90^\circ)]$
$E \cos \omega t$	A	–	–	$\sqrt{2} E \cos \omega t$	0
$E \cos \omega t$	B				
$E \cos(\omega t + 180^\circ)$	A	–	–	0	$\sqrt{2} E \cos \omega t$
$E \cos \omega t$	B				
$E \cos \omega t$	C	0	$\sqrt{2} E \cos \omega t$	–	–
$E \cos \omega t$	D				
$E \cos \omega t$	C	$\sqrt{2} E \cos \omega t$	0	–	–
$E \cos(\omega t + 180^\circ)$	D				
$E \cos \omega_1 t$	A	–	–	$\frac{1}{\sqrt{2}} E(\cos \omega_1 t + \cos \omega_2 t)$	$\frac{1}{\sqrt{2}} E(\cos \omega_1 t + \cos \omega_2 t)$
$E \cos \omega_2 t$	B				

Table III – Isolation Between Ports of 180° Hybrids

Terminations				Isolation (dB)	
A	B	C	D	A to B	C to D
Z_O	Z_O	Z_C	Z_D	$6 + 20 \log \frac{1}{ \rho_c - \rho_d }$	∞
Z_O	Z_O	Z_O	Z_D	$6 + 20 \log \frac{1}{ \rho_d } = 6 + \text{return loss of } Z_D$	∞
Z_A	Z_B	Z_O	Z_O	∞	$6 + 20 \log \frac{1}{ \rho_a - \rho_d }$
Z_O	Z_B	Z_O	Z_O	∞	$6 + 20 \log \frac{1}{ \rho_a } = 6 + \text{return loss of } Z_a$

Isolation

Isolation between ports is expressed by those relationships given in Table III. The following conditions and definitions apply.

1. Only one generator is operating at a time.
2. $\rho = \frac{Z - Z_O}{Z + Z_O}$

VSWR-With Various Terminations

Voltage Standing Wave Ratio (VSWR) with various terminations is expressed by using those relationships given in Table IV.

Table IV – VSWR with Various Terminations for 180° Hybrids

Port Terminations				VSWR	
A	B	C	D	A to B	C to D
Z_O	Z_O	Z_O	Z_O	1:1	1:1
Z_A	Z_B	Z_O	Z_O	1:1	$1 + \frac{ \rho_A + \rho_B }{2}$ $1 - \frac{ \rho_A + \rho_B }{2}$
Z_O	Z_O	Z_C	Z_D	$1 + \frac{ \rho_C + \rho_D }{2}$ $1 - \frac{ \rho_C + \rho_D }{2}$	1:1

hybrid, we can apply signal sources in various combinations to ports of the 90° hybrid and determine the resultant outputs. Figure 5 shows the arrangement used for this analysis, and once again we will consider the operation of the hybrid as a power divider and power summer, with all terminating impedances assumed equal to Z_0 .

The analysis of the 90° hybrid as a power divider is straightforward and, as previously mentioned, two equal amplitude outputs result when any one of the ports is fed by a signal source. These outputs are in quadrature as indicated in Table V.

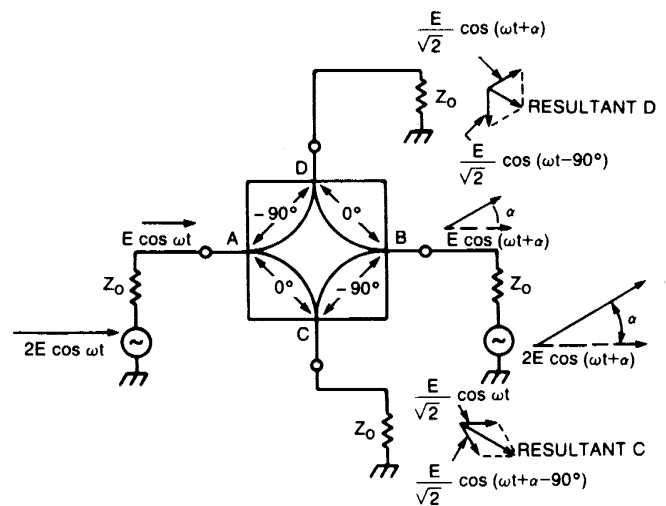
To analyze the 90° hybrid as a power summer, we will once again make use of a diagram showing the vector and algebraic relationships of the signals at all ports when two equal amplitude, equal frequency, arbitrary phase signals are applied.

In Figure 6, these two signals are shown applied to Ports A and B of the 90° hybrid. The amplitudes of the resultant outputs at Ports C and D vary based on the phase of the inputs, while the phases of the outputs are always equal. This can be a useful property in certain applications since the relative phase of the input signals can be determined by measuring the relative amplitudes of the outputs. The relationships for a 90° hybrid with signals applied to Ports A and B or C and D are shown in Table VI.

Tables VII and VIII provide the relations for analysis of VSWR and Isolation in 90° hybrids. If we consider the same condition described for the 180° hybrid, two equal mismatches on opposite ports of the hybrid, we get an interesting result. The reflected signal will appear at the normally isolated port, but will not be present at the

input. Thus, if we inject a signal at Port A, and apply equal mismatches at Ports C and D with a Z_0 termination at Port B, the reflected components from the mismatches will be in phase at Port B and will be 180° out of phase at Port A. The isolation between Ports A and B is reduced, but the VSWR at Port A is unchanged by the presence of the mismatches. This property allows 90° hybrids to be used in applications such as balanced

- Equal Amplitude, Equal Frequency Input
- Resultant Output Phases are Equal
- Resultant Output Magnitudes very based on Input Phase



Resultant C

$$\begin{aligned}
 &= \frac{E}{\sqrt{2}} [\cos \omega t + \cos (\omega t + \alpha - 90^\circ)] \\
 &= \frac{E}{\sqrt{2}} [2 \cos \frac{1}{2} (2\omega t + \alpha - 90^\circ) \cos \frac{1}{2} (-\alpha + 90^\circ)] \\
 &= \sqrt{2} E \cos (45^\circ - \frac{\alpha}{2}) \cos (\omega t + \frac{\alpha}{2} - 45^\circ)
 \end{aligned}$$

Resultant D

$$\begin{aligned}
 &= \frac{E}{\sqrt{2}} [\cos (\omega t - 90^\circ) + \cos (\omega t + \alpha)] \\
 &= \frac{E}{\sqrt{2}} [2 \cos \frac{1}{2} (2\omega t + \alpha - 90^\circ) \cos \frac{1}{2} (-\alpha - 90^\circ)] \\
 &= \sqrt{2} E \cos (-45^\circ - \frac{\alpha}{2}) \cos (\omega t + \frac{\alpha}{2} - 45^\circ)
 \end{aligned}$$

$$\therefore \text{Phase of Resultant C} = \text{Phase of Resultant D} = (\omega t + \frac{\alpha}{2} - 45^\circ)$$

Figure 6. 90° Hybrid

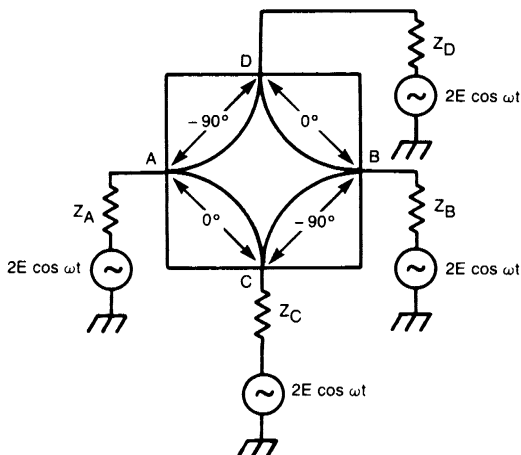


Figure 5. Signal Source Configuration for 90° Hybrid
(Reference Tables V, VI, VII, VIII)

Power Divider

Table V shows applicable relationships when the 90° hybrid is used as a power divider. The following conditions apply:

1. $Z_A = Z_B = Z_C = Z_D = Z_O$
2. Only one generator at a time is operating.

Table V – 90° Hybrid as a Power Divider

Input Signal	Input Port	Output Signals			
		Port A	Port D	Port C	Port B
$E \cos \omega t$	A	/	$\frac{E}{\sqrt{2}} \cos(\omega t - 90^\circ)$	$\frac{E}{\sqrt{2}} \cos \omega t$	0
	B	$\frac{E}{\sqrt{2}} \cos(\omega t - 90^\circ)$	/	0	$\frac{E}{\sqrt{2}} \cos \omega t$
	C	$\frac{E}{\sqrt{2}} \cos \omega t$	0	/	$\frac{E}{\sqrt{2}} \cos(\omega t - 90^\circ)$
	D	0	$\frac{E}{\sqrt{2}} \cos \omega t$	$\frac{E}{\sqrt{2}} \cos(\omega t - 90^\circ)$	/

Power Summer

The relationships given in Table II apply when the 180° hybrid is used as a power summer. The following conditions apply:

1. $Z_A = Z_B = Z_C = Z_D = Z_O$
2. Two generators are operating simultaneously.

Table VI – 90° Hybrid as a Power Summer

Input Signal	Input Port	Output Ports			
		Port A	Port D	Port C	Port B
$E \cos \omega t$	A	/	$\sqrt{2} E \cos(-45^\circ - \alpha/2)$	$\sqrt{2} E \cos(45^\circ - \alpha/2)$	/
$E \cos(\omega t + \alpha)$	B	/	$[\cos(\omega t + \alpha/2 - 45^\circ)]$	$[\cos(\omega t + \alpha/2 + 45^\circ)]$	/
$E \cos \omega t$	A	/	0	$\sqrt{2} E \cos \omega t$	/
$E \cos \omega t$	B	/	$\sqrt{2} E \cos \omega t$	0	/
$E \cos(\omega t + 90^\circ)$	A	/	0	0	/
$E \cos \omega t$	B	/	$\sqrt{2} E \cos \omega t$	0	/
$E \cos \omega t$	D	0	/	/	$\sqrt{2} E \cos \omega t$
$E \cos(\omega t + 90^\circ)$	C	0	/	/	$\sqrt{2} E \cos \omega t$
$E \cos \omega t$	D	$\sqrt{2} E \cos \omega t$	/	/	0
$E \cos(\omega t + 90^\circ)$	C	$\sqrt{2} E \cos \omega t$	/	/	0
$E \cos \omega_1 t$	B	/	$\frac{E}{\sqrt{2}} [\cos \omega_1 t + \cos(\omega_1 t - 90^\circ)]$	$\frac{E}{\sqrt{2}} [\cos \omega_1 t + \cos(\omega_1 t + 90^\circ)]$	/
$E \cos \omega_2 t$	A	/	$\frac{E}{\sqrt{2}} [\cos \omega_2 t + \cos(\omega_2 t - 90^\circ)]$	$\frac{E}{\sqrt{2}} [\cos \omega_2 t + \cos(\omega_2 t + 90^\circ)]$	/

VSWR

Table VII shows relationships used in determining port Voltage Standing Wave Ratios (VSWR's) for 90° hybrids. The following condition applies:

$$\rho = \frac{Z - Z_O}{Z + Z_O}$$

Table VII – VSWR of 90° Hybrids

Port Terminations				Port VSWR's			
A	B	C	D	Port A	Port D	Port C	Port B
Z_O	Z_O	Z_O	Z_O	1:1	1:1	1:1	1:1
Z_O	Z_O	Z_C	Z_O	$1 + \frac{ \rho_d - \rho_c }{2}$ $1 - \frac{ \rho_d - \rho_c }{2}$	1:1	1:1	$1 + \frac{ \rho_d - \rho_c }{2}$ $1 - \frac{ \rho_d - \rho_c }{2}$
Z_A	Z_O	Z_O	Z_B	1:1	$1 + \frac{ \rho_A - \rho_B }{2}$ $1 - \frac{ \rho_A - \rho_B }{2}$	$1 + \frac{ \rho_A - \rho_B }{2}$ $1 - \frac{ \rho_A - \rho_B }{2}$	

Isolation

Table VIII shows relationships used in determining isolation between ports of 90° hybrids. The following conditions apply:

1. Only one generator at a time is operating.

$$2. \rho = \frac{Z - Z_O}{Z + Z_O}$$

$$3. \text{Return Loss} = 20 \log \frac{1}{|\rho|}$$

Table VIII – Isolation for 90° Hybrids

Terminations				Isolation (dB)	
A	B	C	D	Ports A to B	Ports C to D
Z_O	Z_O	Z_O	Z_O	∞	∞
Z_O	Z_O	Z_C	Z_O	$6 + 20 \log \frac{1}{ \rho_d + \rho_c }$	∞
Z_O	Z_O	Z_O	Z_O	$6 + 20 \log \frac{1}{ \rho_d } = 6 + \text{return loss of } Z_O$	∞
Z_A	Z_O	Z_O	Z_B	∞	$6 + 20 \log \frac{1}{ \rho_B + \rho_A }$
Z_O	Z_O	Z_O	Z_B	∞	$6 + 20 \log \frac{1}{ \rho_B } = 6 + \text{return loss of } Z_B$

amplifiers, where two equal impedance but mismatched amplifier stages are combined at inputs and outputs with 90° hybrids to achieve a low VSWR. VSWR and Isolation can be determined based on the relationships in Tables VII and VIII.

Performance Parameters

The 180° or 90° hybrid electrical parameters of principal importance to the designer or components engineer and commonly specified by manufacturers are the following:

Frequency Range

This is the range over which specifications are guaranteed for the particular device.

Insertion Loss

The amount of attenuation, in excess of signal splitting losses, of an input signal from a source of characteristic impedance Z_O measured at an output port terminated in Z_O .

Isolation

Isolation between two ports of a passive device is the amount of attenuation that a signal from a source of characteristic impedance Z_O applied to one port undergoes when measured at the other port terminated in Z_O .

Impedance

This is the nominal characteristic impedance (Z_O) for the device.

VSWR

Voltage Standing Wave Ratio - VSWR is a measure of the impedance of a device relative to Z_O .

It can be expressed as
$$VSWR = \frac{1 + |\rho|}{1 - |\rho|}$$

where $|\rho|$ is the magnitude of the reflection coefficient at the frequency of interest.

Amplitude Balance

The difference in attenuation between two or more output signals fed from a common input generally expressed as a maximum variation.

Phase Balance

The difference in phase between two or more output signals fed from a common input generally expressed as a maximum variation relative to the nominal phase difference between the paths. This nominal phase difference may be 0, 90, or 180°.

Some performance tradeoffs may be made between certain of these parameters. The principal tradeoff is between frequency range, insertion loss and amplitude balance for 90° hybrids. Several different design approaches are used for quadrature hybrids. These can generally be separated into narrow band and broadband designs. For single frequency applications the 10% bandwidth design can achieve very low insertion loss (0.1 to 0.2 dB), but the amplitude balance will degrade rapidly away from the center frequency. Octave bandwidth designs have slightly more loss, but the amplitude balance is maintained over the octave range. For this design, two crossover points occur where the output signals are equal. The broadband design is normally used only where frequency ranges of a decade or more are required. It is a more complex design generally consisting of a pair of 180° hybrids interconnected with a pair of phase tracking 90° all pass lattice filter networks, and will usually have higher insertion loss because of this complexity.

Conclusion

The 90° and 180° hybrids are basic system and component building blocks. Understanding their basic relationships allows the designer, through clearer application, to overcome system problems and/or improve system performance with these simple devices. The reader is urged to study the tables of relationships carefully since some of the hybrid's unique capabilities are not inherently obvious. There are special cases of these devices discussed in more detail in the RF power divider, coupler, and RF multi-function assembly sections of this catalog.