Colpitts Oscillator

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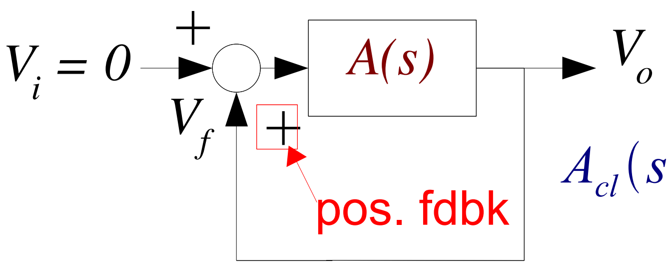
- The Colpitts LC Oscillator circuit
- Open-loop analysis
- Closed-loop analysis
- Root locus
- Oscillation Conditions
- Colpitts design

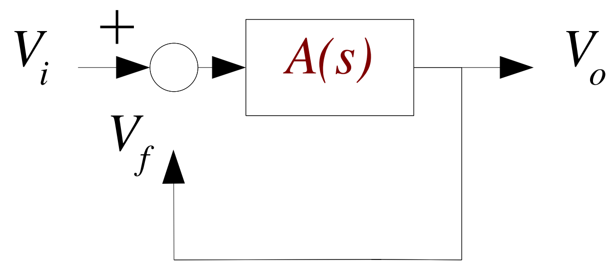
Basic Positive Feedback Oscillator

closed-loop oscillator



$$A_{cl}(s) = \frac{A(s)}{1-A(s)}$$

open-loop: determine loop-gain



$$V_o = A(s)(V_i + V_f) = A(s)(0 + V_o) \Rightarrow$$

$$V_o(1 - A(s)) = 0$$

$$\frac{V_f}{V_i} = \frac{V_o}{V_i} = A(s) = \frac{K\,N(s)}{D(s)}$$

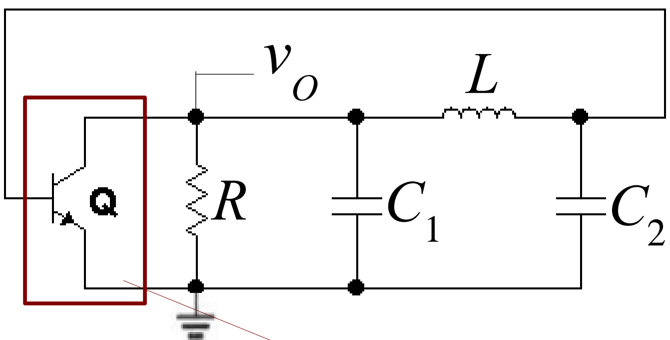
$$A_{cl}(s) = \frac{A(s)}{1-A(s)} = \frac{K\,N(s)}{D(s) - K\,N(s)}$$

Since: $V_o \neq 0 \Rightarrow 1 - A(s) = 0 \Rightarrow A(s) = 1 \Rightarrow$ $\boxed{D(s) - K\,N(s) = 0}$

Condition for oscillation at $s = j\omega_0$: $\boxed{A(s) = 1\, e^{j\pm 2k\pi}}$ Barkhausen criterion

for $k = 0, 1, 2 ..$

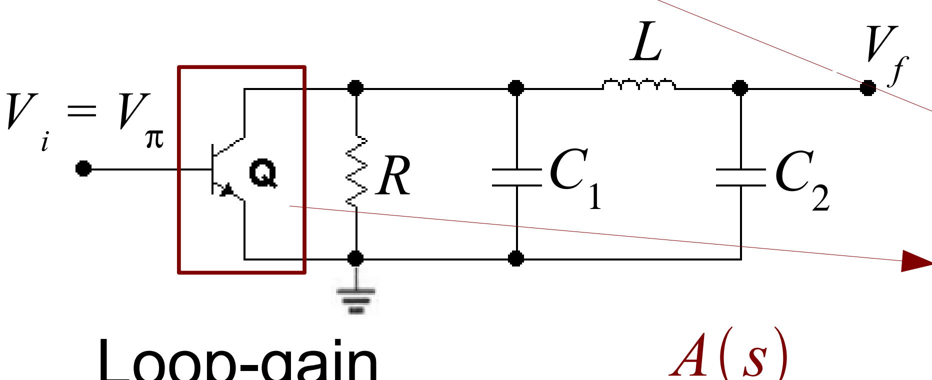
Colpitts Oscillator Basic Schematic

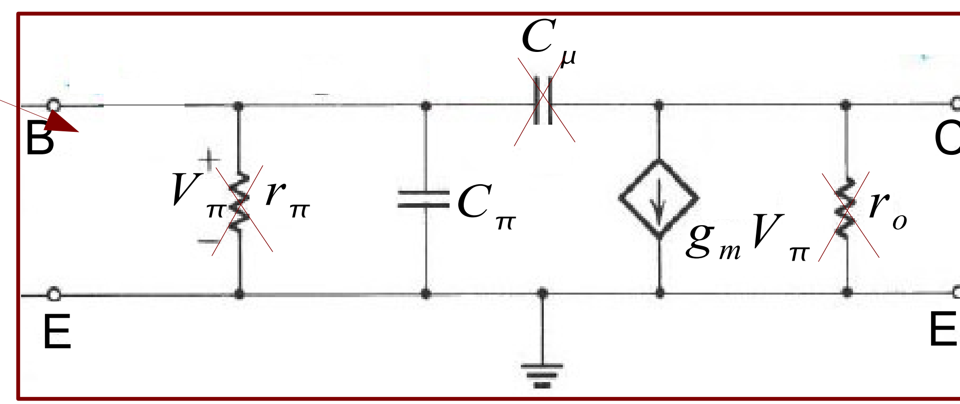


Closed-loop

Assumptions:

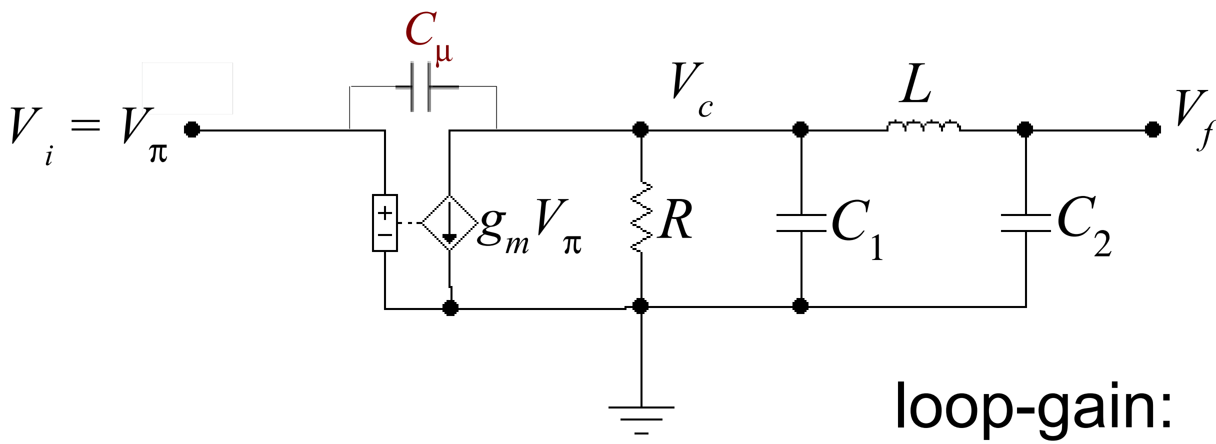
1. r_π large (compared to $1/\omega C_2$).
2. C_μ negligible (compared to C_1, C_2)
3. C_π part of C_2 *(in closed loop)*
4. R represents total resistance in collector circuit, i.e. $R \| r_o \approx R$





Loop-gain

Loop-Gain Analysis



loop-gain: $A(s) = \dfrac{V_f}{V_i}$

Node equation at V_c:

$$|j\omega C_\mu| \ll g_m$$

$$\left(sC_1 + \frac{1}{sL} + \frac{1}{R}\right)V_c - \frac{1}{sL}V_f + (g_m - \cancel{sC_\mu})V_i = 0$$

$$\left(sC_1 + \frac{1}{sL} + \frac{1}{R}\right)V_c - \frac{1}{sL}V_f + g_m V_i = 0 \qquad (1)$$

Node equation at V_f: $\dfrac{-1}{sL}V_c + \left(\dfrac{1}{sL} + sC_2\right)V_f = 0 \qquad (2)$

Note that
$V_f = V_f(s)$
$V_c = V_c(s)$
$V_\pi = V_\pi(s)$
$V_i = V_i(s)$

Open Loop Analysis - cont.

from previous slide

$$(sC_1+\frac{1}{sL}+\frac{1}{R})V_c-\frac{1}{sL}V_f+g_mV_i=0 \quad (1)$$

$$\frac{-1}{sL}V_c+(\frac{1}{sL}+sC_2)V_f=0 \quad (2)$$

Rearranging (1) and (2):

$$\left(sC_1+\frac{1}{sL}+\frac{1}{R}\right)V_c-\frac{1}{sL}V_f=-g_mV_i \quad (3)$$

$$-\frac{1}{sL}V_c+\left(\frac{1}{sL}+sC_2\right)V_f=0 \quad (4)$$

Eq. (1) $-g_m V_\pi$

Eq. (2)

Further rearrangement of (3) and (4):

$$\left(\frac{s^2LC_1+1}{sL}+\frac{1}{R}\right)V_c-\frac{1}{sL}V_f=-g_mV_i \quad (5)$$

$$-\frac{1}{sL}V_c+\left(\frac{s^2LC_2+1}{sL}\right)V_f=0 \quad (6)$$

Open Loop Analysis - cont.

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$$\left(\frac{s^2LC_1+1}{sL}+\frac{1}{R}\right)V_c-\frac{1}{sL}V_f=-g_mV_i \qquad (5)$$

$$-\frac{1}{sL}V_c+\left(\frac{s^2LC_2+1}{sL}\right)V_f=0 \qquad (6)$$

Prepare to add the two equations:

$$\frac{1}{sL}\left(\frac{s^2LC_1+1}{sL}+\frac{1}{R}\right)V_c-\left(\frac{1}{sL}\right)^2V_f=\frac{-g_m}{sL}V_i$$

$$Eq(5)*\frac{1}{sL}$$

$$-\frac{1}{sL}\left(\frac{s^2LC_1+1}{sL}+\frac{1}{R}\right)V_c+\left(\frac{s^2LC_1+1}{sL}+\frac{1}{R}\right)\left(\frac{s^2LC_2+1}{sL}\right)V_f=0$$

$$Eq(6)*\left(\frac{s^2LC_1+1}{sL}+\frac{1}{R}\right)$$

Adding (to eliminate V_c terms):

$$\left(-\left(\frac{1}{sL}\right)^2+\left(\frac{s^2LC_1+1}{sL}+\frac{1}{R}\right)\left(\frac{s^2LC_2+1}{sL}\right)\right)V_f=\frac{-g_m}{sL}V_i \qquad (7)$$

Open Loop Analysis – cont.

From previous slide

$$\left(-\left(\frac{1}{sL}\right)^2+\left(\frac{s^2LC_1+1}{sL}+\frac{1}{R}\right)\left(\frac{s^2LC_2+1}{sL}\right)\right)V_f=\frac{-g_m}{sL}V_i \qquad (7)$$

Multiply (7) by $(sL)^2$:

$$\left(-1+\left(s^2LC_1+1+\frac{sL}{R}\right)\left(s^2LC_2+1\right)\right)V_f=-g_m sL V_i$$

Expand and collect terms according to s^n:

$$\left(-1+s^4C_1C_2L^2+s^2\left(LC_1+LC_2\right)+s^3\frac{L^2C_2}{R}+s\frac{L}{R}+1\right)V_f=-g_m sLV_i \qquad (8)$$

Open Loop Analysis - cont.

From previous slide

$$\left(-\cancel{1}+s^4C_1C_2L^2+s^2\left(LC_1+LC_2\right)+\frac{s^3L^2C_2}{R}+\frac{sL}{R}+\cancel{1}\right)V_f=-g_m sLV_i \qquad (8)$$

Canceling (-1 by 1) in (8) and dividing by sL:

$$\left(s^3C_1C_2L+s\left(C_1+C_2\right)+\frac{s^2LC_2}{R}+\frac{1}{R}\right)V_f=-g_mV_i$$

Multiply by R:

$$\left(s^3RC_1C_2L+sR\left(C_1+C_2\right)+s^2LC_2+1\right)V_f=-g_mRV_i \qquad (9)$$

Open Loop Analysis - cont.

From previous slide

$$\left(s^3 R C_1 C_2 L + s R\left(C_1 + C_2\right) + s^2 L C_2 + 1\right) V_f = -g_m R V_i \qquad (9)$$

Divide (9) by RC_1C_2L:

$$\left(s^3 + s\frac{\left(C_1 + C_2\right)}{C_1 C_2 L} + s^2 \frac{1}{R C_1} + \frac{1}{R C_1 C_2 L}\right) V_f = \frac{-g_m \cancel{R}}{\cancel{R} C_1 C_2 L} V_i$$

The loop-gain transfer function:

$$\boxed{\frac{V_f}{V_i} = A(s) = \frac{\dfrac{-g_m}{C_1 C_2 L}}{s^3 + s^2 \dfrac{1}{R C_1} + s\dfrac{\left(C_1 + C_2\right)}{C_1 C_2 L} + \dfrac{1}{R C_1 C_2 L}} = \frac{K N(s)}{D(s)}} \qquad (10)$$

Closed Loop Analysis

The closed loop equation:

$$\frac{V_f}{V_i} = A(s) = \frac{\dfrac{-g_m}{C_1 C_2 L}}{s^3 + s^2 \dfrac{1}{RC_1} + s\dfrac{(C_1+C_2)}{C_1 C_2 L} + \dfrac{1}{RC_1 C_2 L}} = \frac{K\,N(s)}{D(s)} \quad (10)$$

$$A_{cl}(s) = \frac{A(s)}{\boxed{1 - A(s)}} = \frac{K\,N(s)}{\boxed{D(s) - K\,N(s)}} = \frac{K\,N(s)}{\boxed{D'(s)}}$$

where:

$$A(s) = \frac{K\,N(s)}{D(s)} \qquad K = \frac{-g_m}{C_1 C_2 L} \qquad N(s) = 1$$

$$D(s) = s^3 + s^2 \frac{1}{RC_1} + s\frac{(C_1+C_2)}{C_1 C_2 L} + \frac{1}{RC_1 C_2 L} \quad (11)$$

then:

$$D'(s) = D(s) - \underbrace{\boxed{\frac{-g_m}{C_1 C_2 L}}}_{KN(s)} = s^3 + s^2 \frac{1}{RC_1} + s\frac{(C_1+C_2)}{C_1 C_2 L} + \frac{1 + g_m R}{RC_1 C_2 L} \quad (12)$$

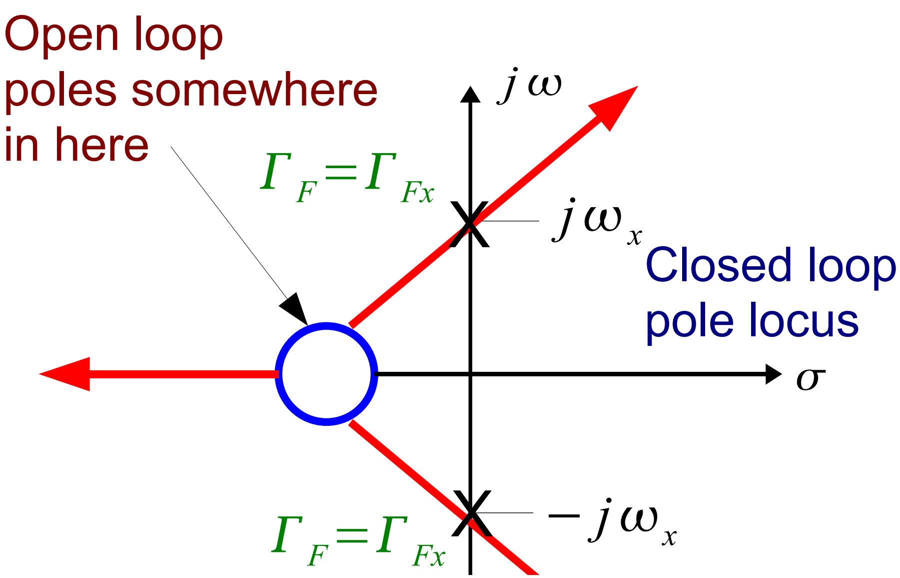
Closed Loop Analysis - cont.

We know that the open loop system $A(s)$ is stable. It has poles in the left half plane, since it is a passive RLC circuit. We also know that it has 3 poles. One is negative-real, the other 2 can be negative-real or LHP complex conjugates.

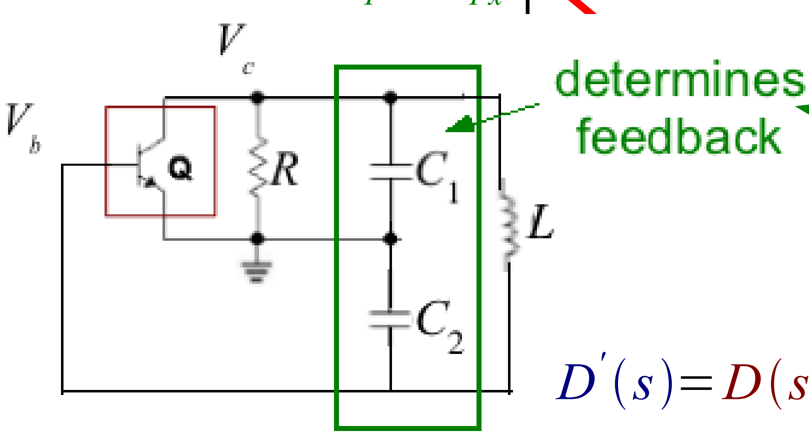
$$D(s)=s^3+s^2\frac{1}{RC_1}+s\frac{(C_1+C_2)}{C_1C_2L}+\frac{1}{RC_1C_2L} \qquad (11)$$

So, let's do a rough sketch of the root locus for a feedback system with a 3 pole $A(s)$.

Root Locus Characteristic



- The loop will become unstable for any value of $\Gamma_F > \Gamma_{Fx}$.
- Rather than sketch the root locus in more exacting detail – it has served its purpose by verifying that oscillation is possible.
- Let's solve for the required Γ_{Fx} .



$$D(s) = s^3 + s^2 \frac{1}{RC_1} + s\frac{(C_1 + C_2)}{C_1 C_2 L} + \frac{1}{RC_1 C_2 L} \quad (11)$$

$$D'(s) = D(s) - KN(s) = s^3 + s^2 \frac{1}{RC_1} + s\frac{(C_1 + C_2)}{C_1 C_2 L} + \frac{1 + \boxed{g_m R}}{RC_1 C_2 L} \quad (12)$$

Closed Loop Oscillation Conditions

If the closed loop system is at the stability limit, the complex conjugate are poles on jω axis:

$$D'(s)=D(s)+K\,N(s)=(s+a)(s^2+\omega_x^2)=0$$

to oscillate at $\omega=\omega_x$

Multiplying terms:

$$D'(s)=s^3+a\,s^2+\omega_x^2 s+a\,\omega_x^2=0\Rightarrow D'(j\omega)=\underbrace{(a\,\omega_x^2-a\,\omega^2)}_{=0}+j\underbrace{(\omega_x^2\omega-\omega^3)}_{=0}=0$$

Match term by term with symbolic circuit equation:

$$D'(s)=s^3+s^2\overset{a}{\boxed{\frac{1}{RC_1}}}+s\overset{\omega_x^2}{\boxed{\frac{(C_1+C_2)}{C_1C_2L}}}+\overset{a\,\omega_x^2}{\boxed{\frac{1+g_mR}{RC_1C_2L}}}$$

$$a=\frac{1}{RC_1}\qquad \omega_x^2=\frac{(C_1+C_2)}{C_1C_2L}\qquad a\,\omega_x^2=\frac{1+g_mR}{RC_1C_2L}$$ s.t. $a*\omega_x^2=a\,\omega_x^2$

Closed Loop Oscillation Conditions cont.

$$a = \frac{1}{RC_1} \qquad \omega_x^2 = \frac{C_1 + C_2}{C_1 C_2 L} \qquad a\omega_x^2 = \frac{1 + g_m R}{RC_1 C_2 L} \qquad a * \omega_x^2 = a\omega_x^2$$

To find the "gain" requirement for oscillation, equate $a * \omega_x^2 = a\omega_x^2$

$$\underbrace{\boxed{\frac{1}{RC_1}}}_{a} \underbrace{\boxed{\frac{C_1 + C_2}{C_1 C_2 L}}}_{\omega_x^2} = \frac{C_1 + C_2}{RC_1^2 C_2 L} = \underbrace{\boxed{\frac{1 + g_m R}{RC_1 C_2 L}}}_{a\omega_x^2}$$

$$\frac{C_1 + C_2}{RC_1^2 C_2 L} = \frac{1 + g_m R}{RC_1 C_2 L} \Rightarrow \cancel{1} + g_m R = \frac{C_1 + C_2}{C_1} = \cancel{1} + \frac{C_2}{C_1}$$

$$\boxed{g_m R = \frac{C_2}{C_1}} \qquad \boxed{\omega_x = \sqrt{\frac{C_1 + C_2}{C_1 C_2 L}}}$$

Oscillator Design Summary

The oscillation frequency:

$$\boxed{\omega_x = \sqrt{\frac{C_1 + C_2}{C_1 C_2 L}}}$$

The required feedback gain $\Gamma_F = g_m R$: $\Gamma_{Fx} = (g_m)_x R = \frac{C_2}{C_1}$ poles at $\pm j\omega_x$

To insure the start-up of oscillation:

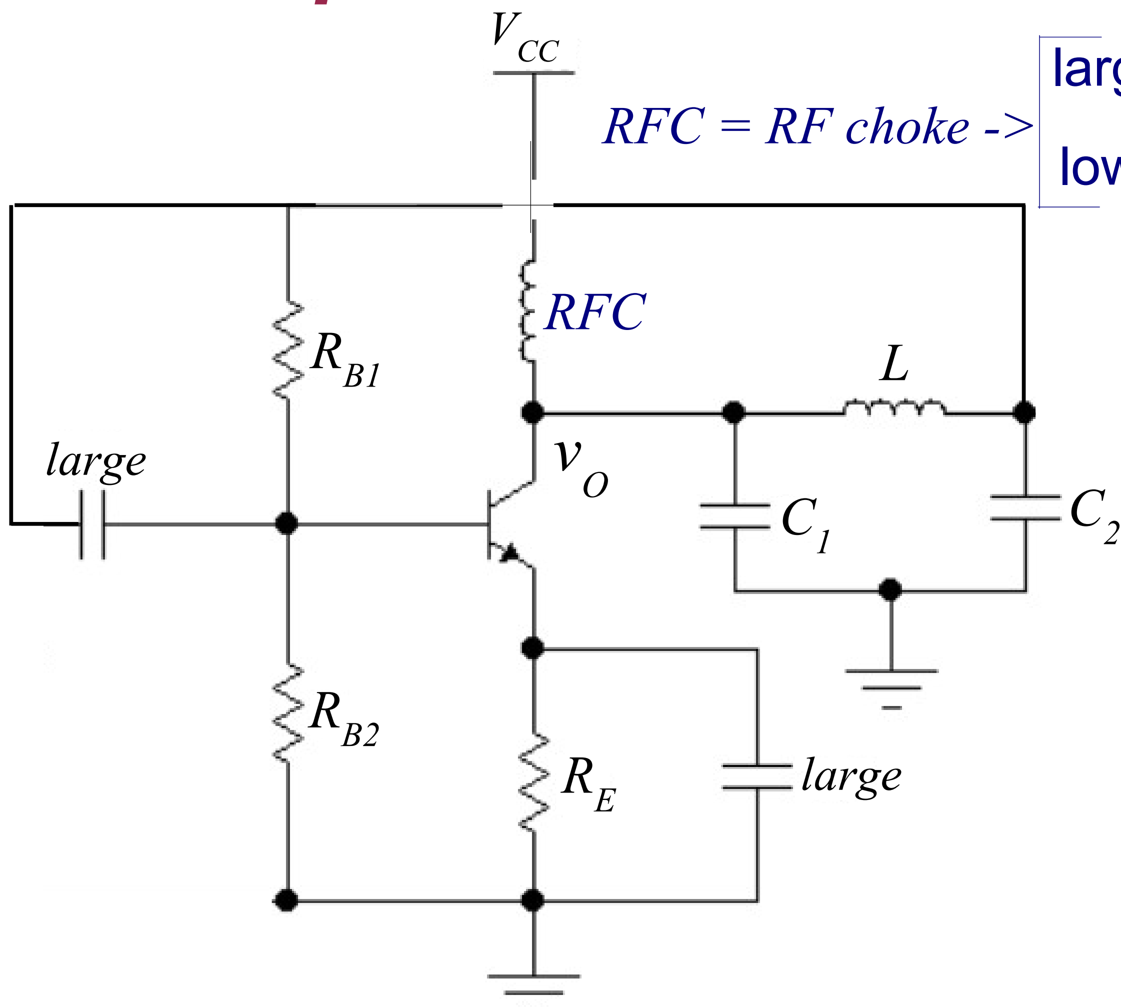
$$\boxed{(g_m)_x R > \frac{C_2}{C_1}}$$

Comments:

1. Unfortunately we don't control "R".

2. Set C_2 & C_1 to convenient values, say $C_2 = C_1 = C$, and adjust g_m using I_C s.t. $g_m R > C_2/C_1$; then adjust "L" to maintain ω_x at the desired oscillation frequency.

Practical Colpitts Oscillator Circuit



$RFC = RF\ choke \rightarrow$ large reactance at ω_x; low resistance at dc