

From an application notes by Osram-Sylvania about the tungsten filament lamps, supplying a lamp with two different voltages V1 and V2, the following equations applies (when in steady-state);

$$\frac{I1}{I2} = \left(\frac{V1}{V2}\right)^m$$

$$\frac{R1}{R2} = \left(\frac{V1}{V2}\right)^n$$

$$\frac{\text{Lumen1}}{\text{Lumen2}} = \left(\frac{V1}{V2}\right)^a$$

the actual exponents vary from lamp to lamp, however the average values are m=0.6, n=0.4 and a=3.4

This means we can write:

$$I = K_I * V^m$$

$$R = K_R * V^n$$

actually the last equation should also consider the additive term R0, that is the resistance when no voltage is applied. However this term can be neglected.

First of all we can try to estimate the two coefficients KI and KR knowing the nominal voltage and power of a lamp.

$$P = R * I^2 = K_R * V^n * (K_I * V^m)^2 = K_R * K_I^2 * V^{2m+n}$$

$$P = V * I = V * (K_I * V^m) = K_I * V^{m+1}$$

from the last equation:

$$K_I = \frac{P}{V^{m+1}}$$

and from the previous one:

$$K_R = \frac{P}{K_I^2 * V^{2m+n}}$$

We need to express now the resistance as function of the current. Combining the first two equations of this post:

$$R = K_R * \left(\frac{I}{K_I}\right)^{\frac{n}{m}}$$

We have a series circuit with 3 resistors R1, R2 and R3 supplied with Vin, then

Vin=(R1+R2+R3)*I, then using the equation from the resistance:

$$V_{in} = I * [K_{R1} * (\frac{I}{K_{I1}})^{\frac{n}{m}} + K_{R2} * (\frac{I}{K_{I2}})^{\frac{n}{m}} + K_{R3} * (\frac{I}{K_{I3}})^{\frac{n}{m}}]$$

that is

$$V_{in} = [\frac{K_{R1}}{(K_{I1})^{\frac{n}{m}}} + \frac{K_{R2}}{(K_{I2})^{\frac{n}{m}}} + \frac{K_{R3}}{(K_{I3})^{\frac{n}{m}}}] * I^{\frac{n}{m}+1}$$

solving with respect to I:

$$I = (\frac{V_{in}}{[\frac{K_{R1}}{(K_{I1})^{\frac{n}{m}}} + \frac{K_{R2}}{(K_{I2})^{\frac{n}{m}}} + \frac{K_{R3}}{(K_{I3})^{\frac{n}{m}}]})^{\frac{m}{m+n}}$$

the total power will be, simply $P_{tot} = V_{in} * I$

Numerically. Lamp1 nominal 200W, 250V, then

$$K_{I1} = \frac{200}{250^{1.6}} = 0.029$$

$$K_{R1} = \frac{200}{0.029^2 * 250^{1.6}} = 34.6$$

In the same way Lamp2 and Lamp3

$$K_{I2} = K_{I3} = 0.0145$$

$$K_{R2} = K_{R3} = 69.3$$

then

$$I = (\frac{250}{[\frac{34.6}{(0.029)^{0.67}} + \frac{69.3}{(0.0145)^{0.67}} + \frac{69.3}{(0.0145)^{0.67}}]})^{0.6} = 0.238 \text{ A}$$

From which $P = 60 \text{ W}$

From the current "I" it's also possible to calculate R1, R2 and R3 and the voltage across each lamp. It's also possible to estimate the relative lumen. If at the nominal voltage (in our case 250V) we set lumen=100% then

$$Lumen = 100 (\frac{V}{250})^{3.4}$$