Interpolation with Lagrange polynomial

Given points in the plane, we look for the polynomial of interpolation.

i	0	1	2	3
x_i	0.001	0.01	0.1	1
y_i	176.7578	120.3572	63.9566	8.7780

We have 4 points, the degree of the polynomial is not greater than 3.

We write the form of the Lagrange polynomial from the y coordinates.

$$L(x) = 176.7578 l_0(x) + 120.3572 l_1(x) + 63.9566 l_2(x) + 8.7780 l_3(x)$$

Using x coordiantes, we write and simplify the small Lagrange polynomials.

$$l_0(x) = \frac{(x-0.01)\,(x-0.1)\,(x-1)}{(0.001-0.01)\,(0.001-0.1)\,(0.001-1)} = \frac{x^3 - 1.11x^2 + 0.111x - 0.001}{-8.9010900000000015 \times 10^{-4}} =$$
$$= -1123.45791358137 x^3 + 1247.038284075321 x^2 - 124.7038284075321 x + 1.12345791358137$$

$$l_1(x) = \frac{(x-0.001)\,(x-0.1)\,(x-1)}{(0.01-0.001)\,(0.01-0.1)\,(0.01-1)} = \frac{x^3 - 1.101x^2 + 0.1011x - 1.0 \times 10^{-4}}{8.0190000000000014 \times 10^{-4}} =$$
$$= 1247.038284075321 x^3 - 1372.989150766928 x^2 + 126.075570520015 x - 0.12470382840753$$

$$l_2(x) = \frac{(x-0.001)\,(x-0.01)\,(x-1)}{(0.1-0.001)\,(0.1-0.01)\,(0.1-1)} =$$
$$= \frac{x^3 - 1.011x^2 + 0.01101x - 1.0000000000000001 \times 10^{-5}}{-0.008019} =$$
$$= -124.7038284075321 x^3 + 126.0755705200149 x^2 - 1.372989150766928 x + 0.0012470382840753$$

$$l_3(x) = \frac{(x-0.001)\,(x-0.01)\,(x-0.1)}{(1-0.001)\,(1-0.01)\,(1-0.1)} =$$
$$= \frac{x^3 - 0.111x^2 + 0.00111x - 9.9999999999999995 \times 10^{-7}}{0.890109} =$$
$$= 1.123457913581371 x^3 - 0.12470382840753 x^2 + 0.0012470382840753 x - 1.1234579135813704 \times 10^{-6}$$

We use small Lagrange polynomials in the interpolation formula and sum up like powers of x (the final result is in the frame).

$$L(x) =$$
$$= 176.7578\big(-1123.45791358137 x^3 + 1247.038284075321 x^2 - 124.7038284075321 x + 1.1234579135813\big)$$
$$+ 120.3572\big(1247.038284075321 x^3 - 1372.989150766928 x^2 + 126.075570520015 x - 0.12470382840753\big)$$
$$+ 63.9566\big(-124.7038284075321 x^3 + 126.0755705200149 x^2 - 1.372989150766928 x + 0.00124703828407\big)$$
$$+ 8.7780\big(1.123457913581371 x^3 - 0.12470382840753 x^2 + 0.0012470382840753 x - 1.1234579135813704\big)$$

$$= \boxed{-\frac{1255024927298014000}{22230267846633} x^3 + \frac{12664619899898014}{200272683303} x^2 - \frac{69655562309569007}{10013634165150} x + \frac{20413020138676930}{1111513392331650}}$$