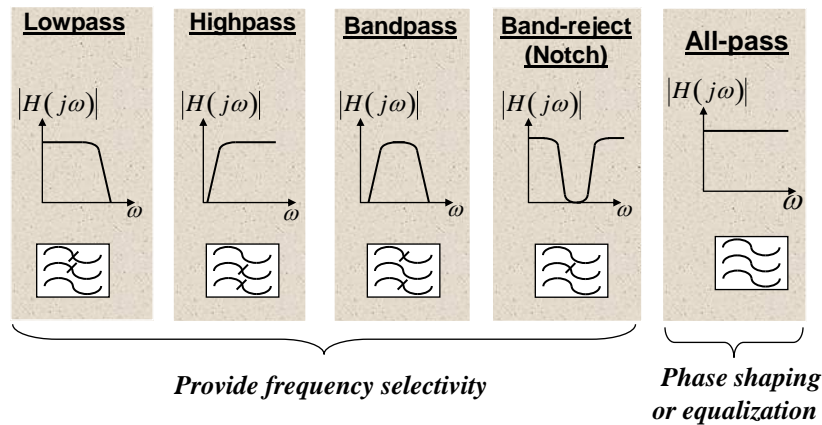


EE247 - Lecture 2 Filters

- Filters:
 - Nomenclature
 - Specifications
 - Quality factor
 - Magnitude/phase response versus frequency characteristics
 - Group delay
 - Filter types
 - Butterworth
 - Chebyshev I & II
 - Elliptic
 - Bessel
 - Group delay comparison example
 - Biquads

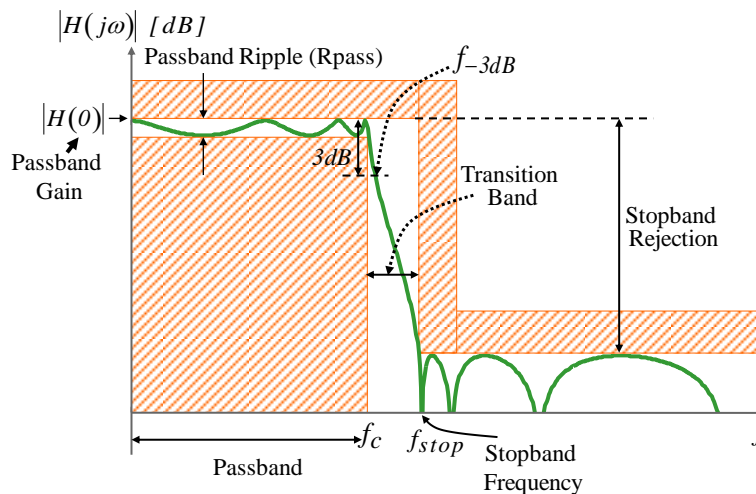
Nomenclature Filter Types wrt Frequency Range Selectivity



Filter Specifications

- Magnitude response versus frequency characteristics:
 - Passband ripple (R_{pass})
 - Cutoff frequency or $-3dB$ frequency
 - Stopband rejection
 - Passband gain
- Phase characteristics:
 - Group delay
- SNR (Dynamic range)
- SNDR (Signal to Noise+Distortion ratio)
- Linearity measures: IM3 (intermodulation distortion), HD3 (harmonic distortion), IIP3 or OIP3 (Input-referred or output-referred third order intercept point)
- Area/pole & Power/pole

Filter Magnitude versus Frequency Characteristics Example: Lowpass



Filters

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Quality Factor (Q)

- The term quality factor (Q) has different definitions in different contexts:
 - Component quality factor (inductor & capacitor Q)
 - Pole quality factor
 - Bandpass filter quality factor
- Next 3 slides clarifies each

Component Quality Factor (Q)

- For any component with a transfer function:

$$H(j\omega) = \frac{I}{R(\omega) + jX(\omega)}$$

- Quality factor is defined as:

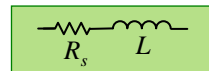
$$Q = \frac{X(\omega)}{R(\omega)} \rightarrow \frac{\text{Energy Stored}}{\text{Average Power Dissipation}} \text{ per unit time}$$

Component Quality Factor (Q) Inductor & Capacitor Quality Factor

- Inductor Q :

❖ $R_s \rightarrow$ series parasitic resistance

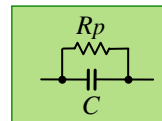
$$Y_L = \frac{I}{R_s + j\omega L} \quad Q_L = \frac{\omega L}{R_s}$$



- Capacitor Q :

❖ $R_p \rightarrow$ parallel parasitic resistance

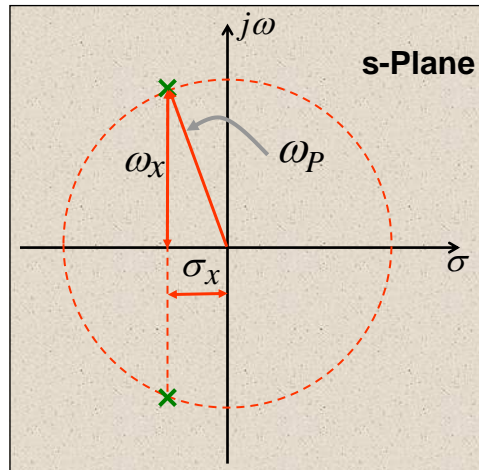
$$Z_C = \frac{I}{\frac{1}{R_p} + j\omega C} \quad Q_C = \omega C R_p$$



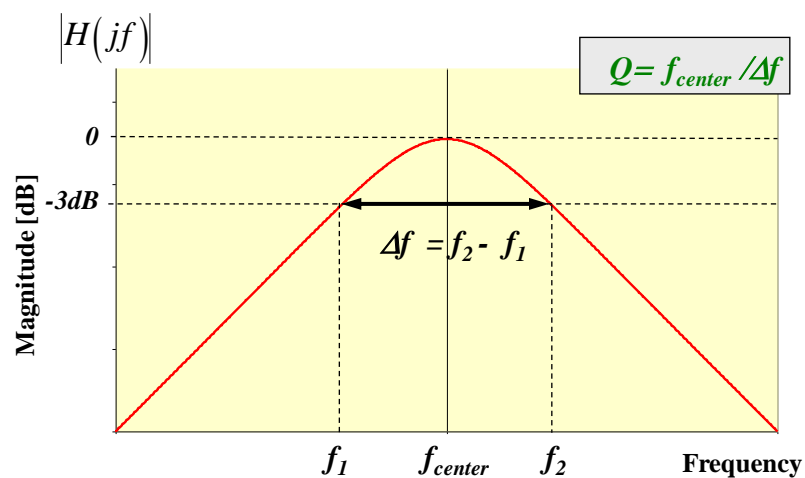
Pole Quality Factor

- Typically filter singularities include pairs of complex conjugate poles.
- Quality factor of complex conjugate poles are defined as:

$$Q_{Pole} = \frac{\omega_p}{2\sigma_x}$$



Bandpass Filter Quality Factor (Q)



Filters

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What is Group Delay?

- Consider a continuous-time filter with s-domain transfer function $G(s)$:

$$G(j\omega) \equiv |G(j\omega)| e^{j\theta(\omega)}$$

- Let us apply a signal to the filter input composed of sum of two sine waves at slightly different frequencies ($\Delta\omega \ll \omega$):

$$v_{IN}(t) = A_1 \sin(\omega t) + A_2 \sin[(\omega + \Delta\omega) t]$$

- The filter output is:

$$v_{OUT}(t) = A_1 |G(j\omega)| \sin[\omega t + \theta(\omega)] +$$

$$A_2 |G[j(\omega + \Delta\omega)]| \sin[(\omega + \Delta\omega)t + \theta(\omega + \Delta\omega)]$$

What is Group Delay?

$$v_{\text{OUT}}(t) = A_1 |G(j\omega)| \sin \left\{ \omega \left[t + \frac{\theta(\omega)}{\omega} \right] \right\} +$$

$$+ A_2 |G[j(\omega+\Delta\omega)]| \sin \left\{ (\omega+\Delta\omega) \left[t + \frac{\theta(\omega+\Delta\omega)}{\omega+\Delta\omega} \right] \right\}$$

Since $\frac{\Delta\omega}{\omega} \ll 1$ then $\left[\frac{\Delta\omega}{\omega}\right]^2 \rightarrow 0$

$$\frac{\theta(\omega+\Delta\omega)}{\omega+\Delta\omega} \cong \left[\theta(\omega) + \frac{d\theta(\omega)}{d\omega} \Delta\omega \right] \left[\frac{1}{\omega} \left(1 - \frac{\Delta\omega}{\omega} \right) \right]$$

$$\cong \frac{\theta(\omega)}{\omega} + \left(\frac{d\theta(\omega)}{d\omega} - \frac{\theta(\omega)}{\omega} \right) \frac{\Delta\omega}{\omega}$$

What is Group Delay?

Signal Magnitude and Phase Impairment

$$v_{\text{OUT}}(t) = A_1 |G(j\omega)| \sin \left\{ \omega \left[t + \frac{\theta(\omega)}{\omega} \right] \right\} +$$

$$+ A_2 |G[j(\omega+\Delta\omega)]| \sin \left\{ (\omega+\Delta\omega) \left[t + \frac{\theta(\omega)}{\omega} + \underbrace{\left(\frac{d\theta(\omega)}{d\omega} - \frac{\theta(\omega)}{\omega} \right) \frac{\Delta\omega}{\omega}}_{\delta} \right] \right\}$$

- $\tau_{PD} \equiv -\theta(\omega)/\omega$ is called the “phase delay” and has units of time
 - If the delay term δ is zero \rightarrow the filter’s output at frequency $\omega+\Delta\omega$ and the output at frequency ω are each delayed in time by $-\theta(\omega)/\omega$
 - If the term δ is non-zero \rightarrow the filter’s output at frequency $\omega+\Delta\omega$ is time-shifted differently than the filter’s output at frequency ω
- \rightarrow “Phase distortion”

What is Group Delay? Signal Magnitude and Phase Impairment

- Phase distortion is avoided only if:

$$\frac{d\theta(\omega)}{d\omega} - \frac{\theta(\omega)}{\omega} = 0$$

- Clearly, if $\theta(\omega) = k\omega$, k a constant, \rightarrow no phase distortion
- This type of filter phase response is called “linear phase”
 \rightarrow Phase shift varies linearly with frequency
- $\tau_{GR} \equiv -d\theta(\omega)/d\omega$ is called the “group delay” and also has units of time. For a linear phase filter $\tau_{GR} \equiv \tau_{PD} = -k$
 $\rightarrow \tau_{GR} = \tau_{PD}$ implies linear phase
- Note: Filters with $\theta(\omega) = k\omega + c$ are also called linear phase filters, but they're not free of phase distortion

What is Group Delay? Signal Magnitude and Phase Impairment

- If $\tau_{GR} = \tau_{PD} \rightarrow$ No phase distortion

$$\begin{aligned} v_{OUT}(t) = & A_1 |G(j\omega)| \sin \left[\omega (t - \tau_{GR}) \right] + \\ & + A_2 |G[j(\omega + \Delta\omega)]| \sin \left[(\omega + \Delta\omega) (t - \tau_{GR}) \right] \end{aligned}$$

- If also $|G(j\omega)| = |G[j(\omega + \Delta\omega)]|$ for all input frequencies within the signal-band, v_{OUT} is a scaled, time-shifted replica of the input, with no “signal magnitude distortion”
- In most cases neither of these conditions are exactly realizable

Summary Group Delay

- Phase delay is defined as:

$$\tau_{PD} \equiv -\theta(\omega)/\omega \quad [\text{time}]$$

- Group delay is defined as :

$$\tau_{GR} \equiv -d\theta(\omega)/d\omega \quad [\text{time}]$$

- If $\theta(\omega)=k\omega$, k a constant, \rightarrow no phase distortion

- For a linear phase filter $\tau_{GR} \equiv \tau_{PD} = -k$

Filters

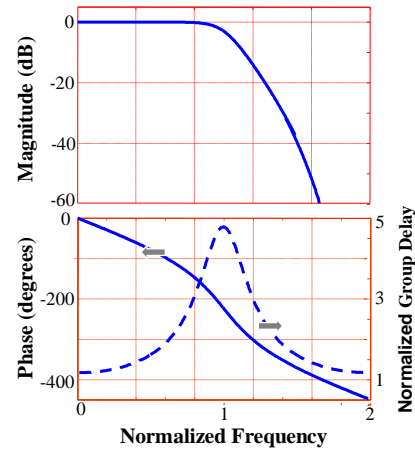
- Filters:
 - Nomenclature
 - Specifications
 - Magnitude/phase response versus frequency characteristics
 - Quality factor
 - Group delay
 - ➡ – Filter types (examples considered all lowpass, the highpass and bandpass versions similar characteristics)
 - Butterworth
 - Chebyshev I & II
 - Elliptic
 - Bessel
 - Group delay comparison example
 - Biquads

Filter Types wrt Frequency Response Lowpass Butterworth Filter

- Maximally flat amplitude within the filter passband

$$\left. \frac{d^N |H(j\omega)|}{d\omega} \right|_{\omega=0} = 0$$

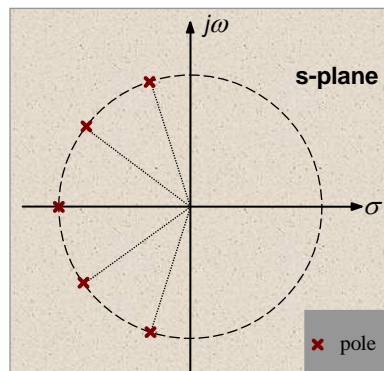
- Moderate phase distortion



Example: 5th Order Butterworth filter

Lowpass Butterworth Filter

- All poles
- Number of poles equal to filter order
- Poles located on the unit circle with equal angles

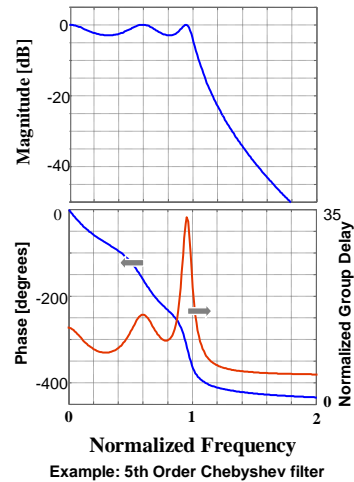


Example: 5th Order Butterworth Filter

Filter Types

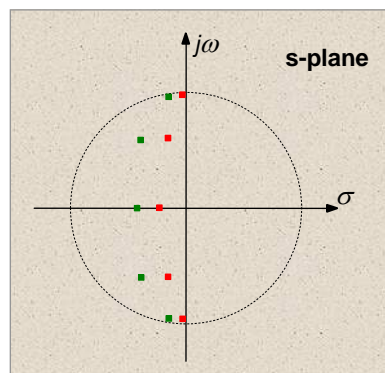
Chebyshev I Lowpass Filter

- Chebyshev I filter
 - Ripple in the passband
 - Sharper transition band compared to Butterworth (for the same number of poles)
 - Poorer group delay compared to Butterworth
 - More ripple in passband → poorer phase response



Chebyshev I Lowpass Filter Characteristics

- All poles
- Poles located on an ellipse inside the unit circle
- Allowing more ripple in the passband:
 - ⇒ Narrower transition band
 - ⇒ Sharper cut-off
 - ⇒ Higher pole Q
 - ⇒ Poorer phase response



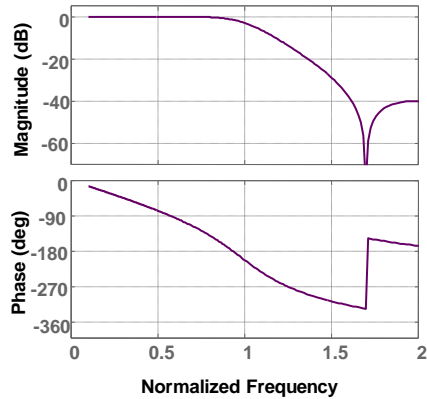
- Chebyshev I LPF 3dB passband ripple
- Chebyshev I LPF 0.1dB passband ripple

Example: 5th Order Chebyshev I Filter

Filter Types

Chebyshev II Lowpass

- Chebyshev II filter
 - No ripple in passband
 - Nulls or notches in stopband
 - Sharper transition band compared to Butterworth
 - Passband phase more linear compared to Chebyshev I

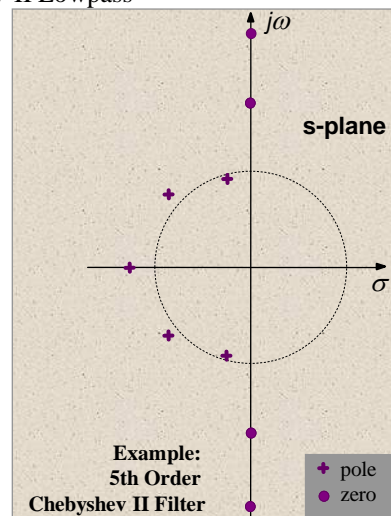


Example: 5th Order Chebyshev II filter

Filter Types

Chebyshev II Lowpass

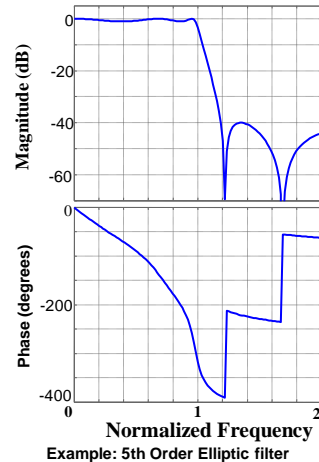
- Poles & finite zeros
 - No. of poles n ($n \rightarrow$ filter order)
 - No. of finite zeros: $n-1$
- Poles located both inside & outside of the unit circle
- Complex conjugate zeros located on $j\omega$ axis
- Zeros create nulls in stopband



Filter Types

Elliptic Lowpass Filter

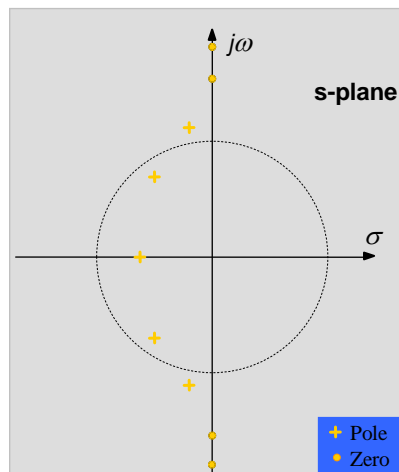
- Elliptic filter
 - Ripple in passband
 - Nulls in the stopband
 - Sharper transition band compared to Butterworth & both Chebyshevs
 - Poorest phase response



Filter Types

Elliptic Lowpass Filter

- Poles & finite zeros
 - No. of poles: n
 - No. of finite zeros: $n-1$
- Zeros located on $j\omega$ axis
- Sharp cut-off
 - ⇒ Narrower transition band
 - ⇒ Pole Q higher compared to the previous filter types

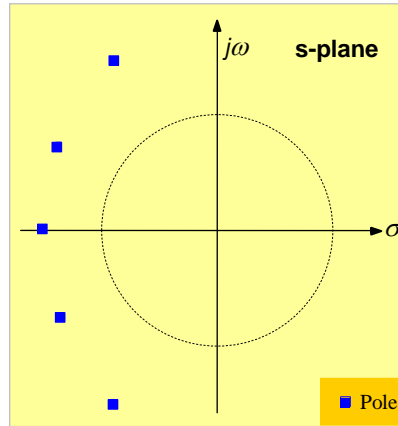


Example: 5th Order Elliptic Filter

Filter Types

Bessel Lowpass Filter

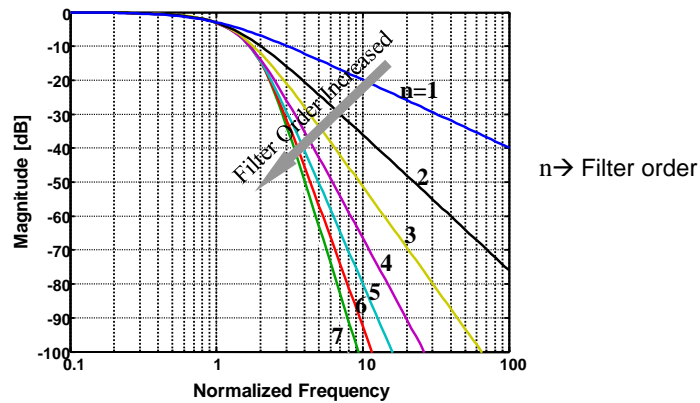
- Bessel
 - All poles
 - Poles outside unit circle
 - Relatively low Q poles
 - **Maximally flat group delay**
 - Poor out-of-band attenuation



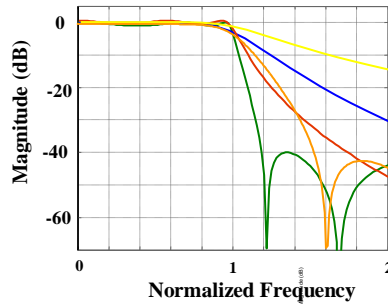
Example: 5th Order Bessel filter

Magnitude Response Behavior as a Function of Filter Order

Example: Bessel Filter



Filter Types Comparison of Various Type LPF Magnitude Response

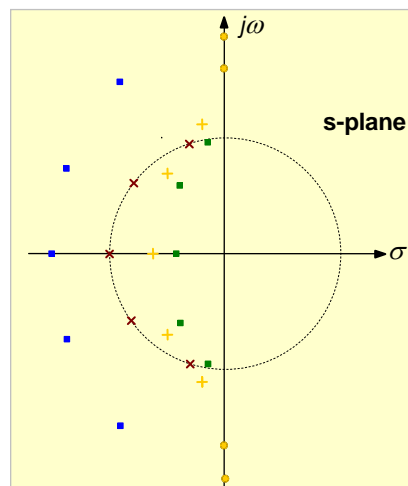


All 5th order filters with same corner freq.

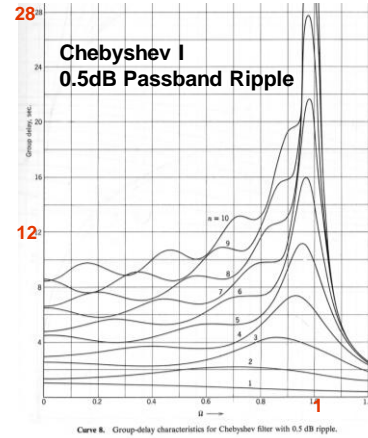
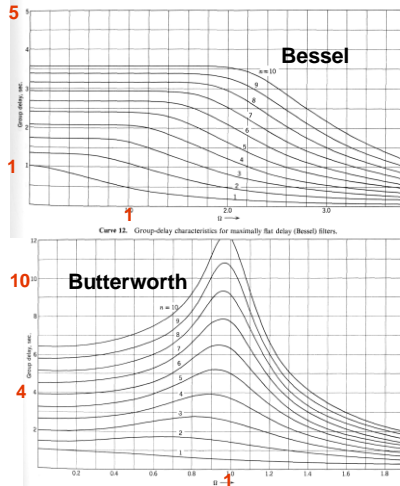
Bessel
 Butterworth
 Chebyshev I
 Chebyshev II
 Elliptic

Filter Types Comparison of Various LPF Singularities

■ Poles Bessel
 × Poles Butterworth
 + Poles Elliptic
 ● Zeros Elliptic
 ■ Poles Chebyshev I 0.1dB



Comparison of Various LPF Groupdelay



Ref: A. Zverev, *Handbook of filter synthesis*, Wiley, 1967.

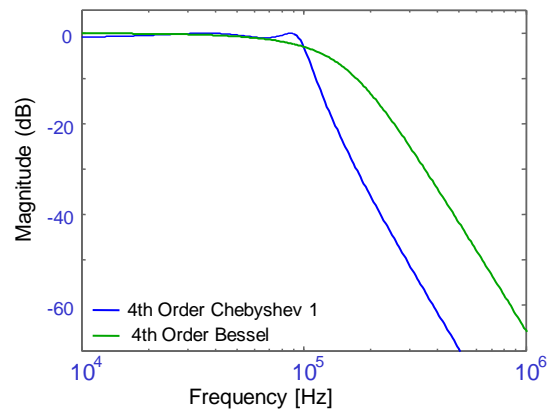
Filters

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 - Bessel
- – Group delay comparison example
- Biquads

Group Delay Comparison Example

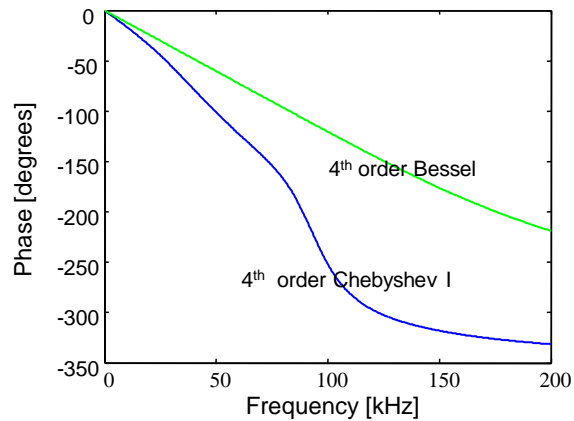
- Lowpass filter with 100kHz corner frequency
- Chebyshev I versus Bessel
 - Both filters 4th order- same **-3dB** point
 - Passband ripple of **1dB** allowed for Chebyshev I

Magnitude Response 4th Order Chebyshev I versus Bessel



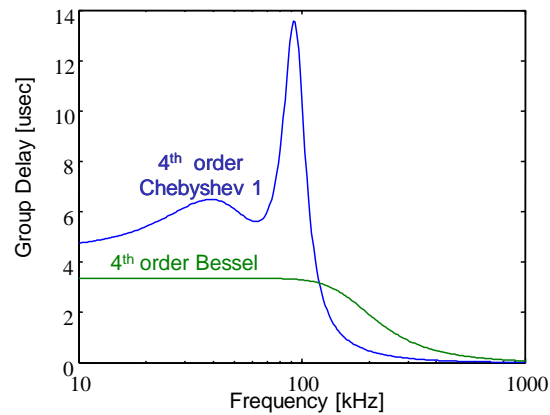
Phase Response

4th Order Chebyshev I versus Bessel



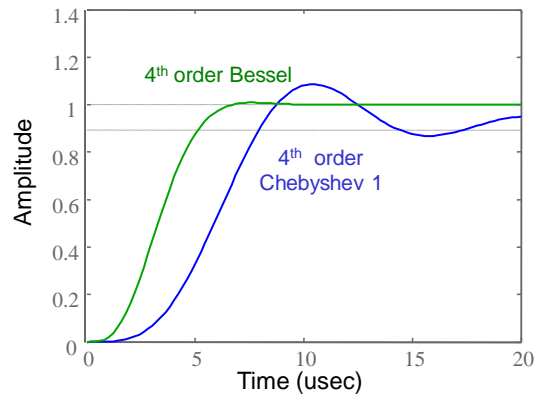
Group Delay

4th Order Chebyshev I versus Bessel



Step Response

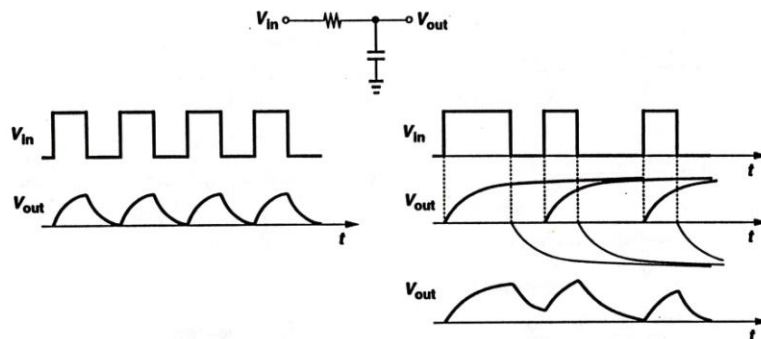
4th Order Chebyshev I versus Bessel



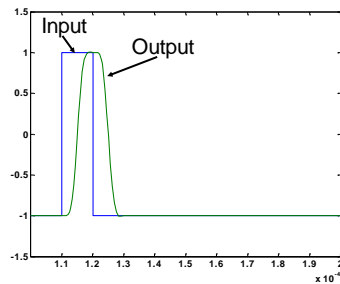
Intersymbol Interference (ISI)

ISI → Broadening of pulses resulting in interference between successive transmitted pulses

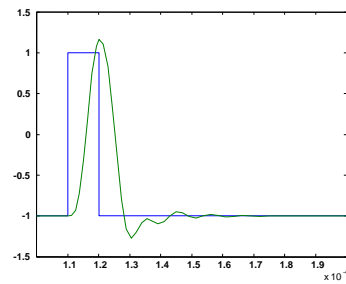
Example: Simple RC filter



Pulse Impairment Bessel versus Chebyshev



4th order Bessel



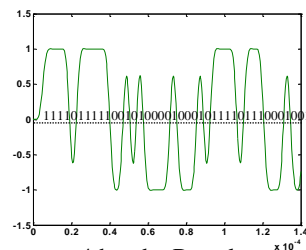
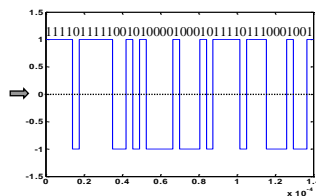
4th order Chebyshev I

Note that in the case of the Chebyshev filter not only the pulse has broadened but it also has a long tail

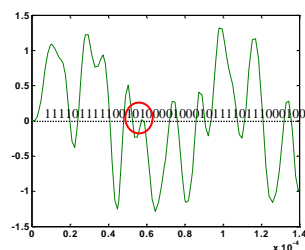
→ More ISI for Chebyshev compared to Bessel

Response to Pseudo-Random Data Chebyshev versus Bessel

Input Signal:
Symbol rate 1/130kHz



4th order Bessel




4th order Chebyshev I

Summary Filter Types

- Filter types with high signal attenuation per pole \Rightarrow poor phase response
- For a given signal attenuation, requirement of preserving constant group delay \rightarrow Higher order filter
 - In the case of passive filters \Rightarrow higher component count
 - For integrated active filters \Rightarrow higher chip area & power dissipation
- In cases where filter is followed by ADC and DSP
 - In some cases possible to digitally correct for phase impairments incurred by the analog circuitry by using digital phase equalizers & thus possible to reduce the required analog filter order

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RLC Filters

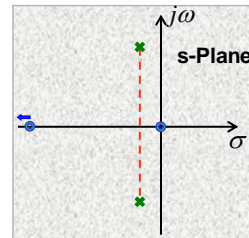
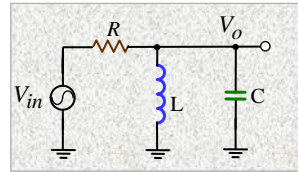
- Bandpass filter (2nd order):

$$\frac{V_o}{V_{in}} = \frac{\frac{s}{RC}}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$\omega_0 = 1 / \sqrt{LC}$$

$$Q = \omega_0 RC = \frac{R}{L\omega_0}$$

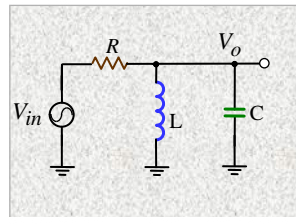
Singularities: Pair of complex conjugate poles
Zeros @ $f=0$ & $f=\infty$.



RLC Filters Example

- Design a bandpass filter with:

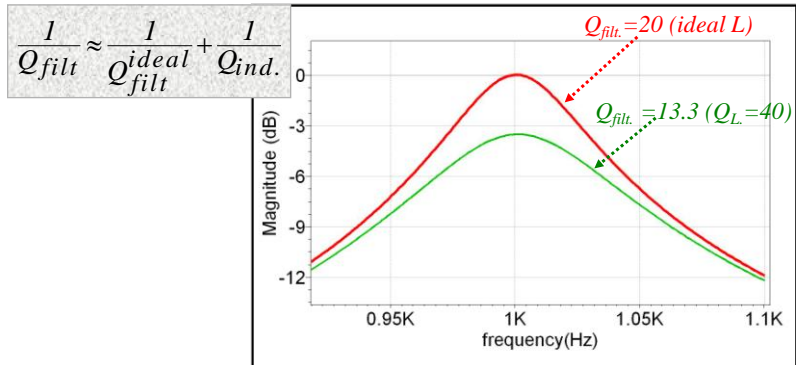
- Center frequency of 1kHz
- Filter quality factor of 20



- First assume the inductor is ideal
- Next consider the case where the inductor has series R resulting in a finite inductor Q of 40
- What is the effect of finite inductor Q on the overall filter Q?

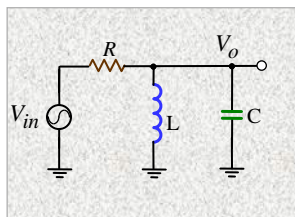
RLC Filters

Effect of Finite Component Q



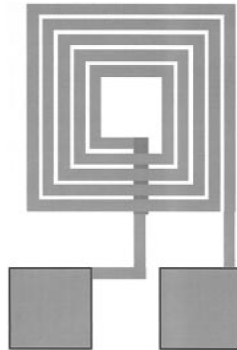
⇒ Need to have component Q much higher compared to desired filter Q

RLC Filters



Question:
Can RLC filters be integrated on-chip?

Monolithic Spiral Inductors



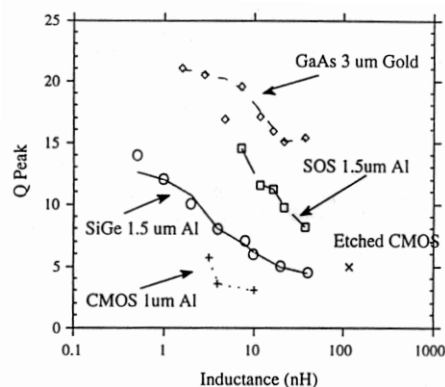
Top View

Monolithic Inductors Feasible Quality Factor & Value

Typically, on-chip inductors built as spiral structures out of metal/s layers

$$Q_L = (\omega L/R)$$

Q_L measured at frequencies of operation ($>1\text{GHz}$)



⇒ Feasible monolithic inductor in CMOS tech. $<10\text{nH}$ with $Q < 7$

❖Ref: "Radio Frequency Filters", Lawrence Larson; Mead workshop presentation 1999

Integrated Filters

- Implementation of RLC filters in CMOS technologies requires on-chip inductors
 - Integrated $L < 10\text{nH}$ with $Q < 10$
 - Combined with max. cap. 20pF
 - *LC filters in the monolithic form feasible: $\text{freq} > 350\text{MHz}$*
 - *(Learn more in EE242 & RF circuit courses)*
- Analog/Digital interface circuitry require fully integrated filters with critical frequencies $\ll 350\text{MHz}$
- Hence:

⇒ Need to build active filters without using inductors

Filters

2nd Order Transfer Functions (Biquads)

- Biquadratic (2nd order) transfer function:

$$H(s) = \frac{1}{1 + \frac{s}{\omega_P Q_P} + \frac{s^2}{\omega_P^2}}$$

$$|H(j\omega)| = \frac{1}{\sqrt{\left(1 - \frac{\omega^2}{\omega_P^2}\right)^2 + \left(\frac{\omega}{\omega_P Q_P}\right)^2}} \longrightarrow \begin{cases} |H(j\omega)|_{\omega=0} = 1 \\ |H(j\omega)|_{\omega \rightarrow \infty} = 0 \\ |H(j\omega)|_{\omega=\omega_P} = Q_P \end{cases}$$

$$\text{Biquad poles @ } s = -\frac{\omega_P}{2Q_P} \left(1 \pm \sqrt{1 - 4Q_P^2}\right)$$

Note: for $Q_P \leq \frac{1}{2}$ poles are real, complex otherwise

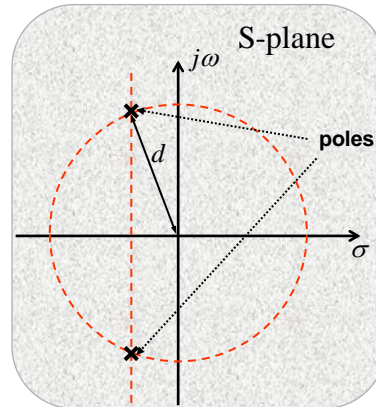
Biquad Complex Poles

$Q_P > \frac{1}{2} \rightarrow$ Complex conjugate poles:

$$s = -\frac{\omega_P}{2Q_P} \left(1 \pm j\sqrt{4Q_P^2 - 1} \right)$$

Distance from origin in s-plane:

$$\begin{aligned} d^2 &= \left(\frac{\omega_P}{2Q_P} \right)^2 (1 + 4Q_P^2 - 1) \\ &= \omega_P^2 \end{aligned}$$



s-Plane

