An Efficient PML Absorbing Medium in FDTD Simulations of Acoustic Scattering in Lossy Media

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Abstract – The concept of the PML is used to construct a novel, anisotropic, absorber for acoustic scattering FDTD simulations in lossy media. Overcoming the original PML lossless profile, it is proved that by a proper selection of additional attenuation parameters in a stretched coordinate system a theoretically zero reflection can be achieved, independently of frequency and incident angle. Numerical results – addressing several open-region problems of acoustic propagation and scattering in ordinary media – verify the efficiency of the proposed algorithm, and the accuracy of the computations.

I. Introduction

The last years the Finite-Difference Time-Domain (FDTD) method has been widely used for the proper simulation of numerous problems not only in electromagnetics, but also in acoustics. This method was found invaluable along with the enforcement of the absorbing boundary conditions (ABCs), which enabled the method's usage in all open-boundary problems, as it is necessary for the last to be computationally restricted. That is, the propagating waves can be absorbed without reflections and thus the FDTD simulation can be efficient.

Many techniques have been proposed in order to accomplish as better absorption as possible. In this way, J.-P. Berenger suggested the perfectly matched layer (PML), an artificial anisotropic lossy material, which encloses the space of interest, and has been proved to be a very effective ABC [1]. It was also found useful in acoustics and employed in many problems with impressive results [2]-[4].

Despite all the advantages, the PML was firstly designed to cover only lossless propagation media, and hence it was insufficient in lossy cases. This is of great importance in acoustics as losses must be seriously considered in most media. There have been efforts to improve PML and adjust it to absorb travelling waves in lossy materials. Among them, a generalized form, which relies on the assumption of additional attenuation coefficients in a stretched coordinate system, has been presented [5], [6].

In this paper, a FDTD-PML technique is implemented in acoustic scattering to accomplish the desirable dissipation of propagating waves at the truncation boundaries. The acoustic equations are considered and the reflection factor of an incident plane wave striking a lossy medium-PML interface is imposed to be zero for all frequencies and angles of incidence. The proper general PML equations are derived in such a way that can be easily enforced in both lossless and lossy media after slight modification of their parameters. This is verified by several numerical experiments, which show the capabilities of the proposed PMLs as well as their high efficiency in the absorption of travelling waves.

II. The PML in Stretched Coordinates and its Discretized Form

In the general case of a homogeneous, lossy fluid medium, the pressure-velocity acoustic equations are

$$\nabla_s p = -\rho \cdot \partial_t \mathbf{u} - a^* \cdot \mathbf{u} \qquad (1)$$

$$\nabla_s \cdot \mathbf{u} = -\kappa \cdot \partial_t p - a \cdot p \qquad (2)$$

where p is the pressure and **u** the vector of the sound velocity in the fluid, respectively. The coefficients ρ, κ and a symbolize the mass density, the compressibility and the compressibility attenuation of the medium, respectively. The coefficient a^* is a nonphysical attenuation parameter, which is generally zero for the acoustic media. The velocity of the sound in the medium is given by the expression $c = (\kappa\rho)^{-1/2}$. The ∇_s tensor indicates the implementation of a stretched system in Cartesian coordinates that is subsequently described by

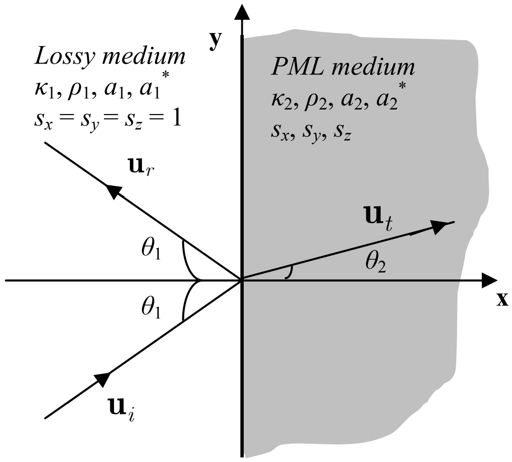


Figure 1: Oblique incidence of the acoustic wave.

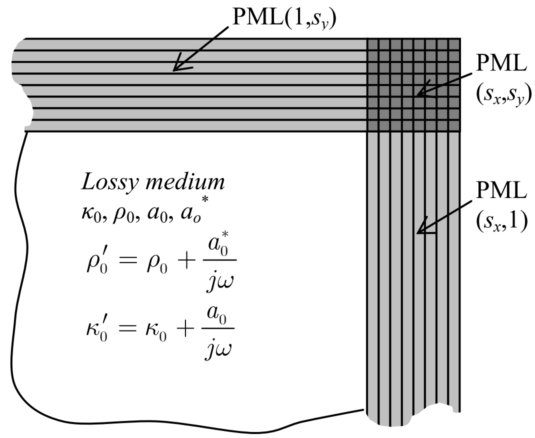


Figure 2: The PML at the corner of the domain.

$$\nabla_s = \hat{\mathbf{x}}\frac{1}{s_x}\cdot\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{1}{s_y}\cdot\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{1}{s_z}\cdot\frac{\partial}{\partial z}$$

If all stretching coefficients are set equal to unity i.e. $s_x = s_y = s_z = 1$, analysis turns to that of the ordinary lossless acoustic material. In the frequency domain, for acoustic waves of angular frequency ω, (1) and (2) can be written as

$$\nabla_s p = (-j\omega\rho - a^*)\mathbf{u} = -j\omega\rho'\mathbf{u} \tag{3}$$

$$\nabla_s \cdot \mathbf{u} = (-j\omega\kappa - a)p = -j\omega\kappa' p \tag{4}$$

where

$$\rho' = \rho + a^* / j\omega \ , \qquad \kappa' = \kappa + a / j\omega \tag{5}$$

The main idea of the proposed PML is to split the acoustic pressure into two additive components p_x, p_y and assume the compressibility attenuation a simply anisotropic: $a = \mathrm{diag}\{a_x, a_y\}$. That is, the acoustic equations in the PML medium can take the form

$$s_x^{-1}\partial_x(p_x + p_y) = -j\omega\rho_2' u_x \tag{6}$$

$$s_y^{-1}\partial_y(p_x + p_y) = -j\omega\rho_2' u_y \tag{7}$$

$$s_x^{-1}\partial_x u_x = -j\omega\kappa_2' p_x \tag{8}$$

$$s_y^{-1}\partial_y u_y = -j\omega\kappa_2' p_y \tag{9}$$

In order to achieve a perfect absorption of the incident waves in the PML, the parameters κ_2, ρ_2, a_2, a_2^* and s_x, s_y, should be determined in such a way that the interface will be reflectionless. For an oblique incident angle θ_1, as shown in figure 1, plane waves propagating in the acoustic medium are considered.

Imposing the continuity conditions for pressure and the normal component of velocity at the interface of the two media – and as no coordinate stretching has been chosen towards the y direction ($s_y = 1$) – we get $\kappa_1 = \kappa_2$, $\rho_1 = \rho_2$, $a_1 = \alpha_2$, $a_1^* = a_2^*$. Thus, the whole problem reduces to the choice of s_x such that the wave that is transmitted within the PML, having passed through the normal to the x-axis interface, is further attenuated. The coefficient s_x, which is generally a complex number, can either be one of the forms

$$\frac{s_x(x)}{s_{0x}(x)} = 1 + \frac{a_x(x)}{j\omega\kappa_1}, \quad \frac{s_x(x)}{s_{0x}(x)} = 1 + \frac{a_x^*(x)}{j\omega\rho_1} \tag{10),(11}$$

where s_{0x}, a_x, a_x^* are functions of x and they will be selected properly considering numerical experiments as well as bibliography. The s_x in (6) is the same as in (11), whereas the s_x in (8) is that of (10). However, it is the same s_x and the new matching condition is

$$a_x(x) / \kappa_1 = a_x^*(x) / \rho_1 \tag{12}$$

So, the parameters κ_1, ρ_1, a_1, a_1^* are arbitrary, and there is no need to satisfy any relation of the form $a_1^*/\rho_1 = a_1/\kappa_1$ – the matching condition of the original PML – responsible for the limitation of the method performing in lossless media only.

In the case of an interface, which is normal to the y-axis, s_x is set equal to unity, whereas s_y takes a form analogous to that mentioned above. In the corner regions, both s_x and s_y are different from unity, and considered to be of such a form. A corner section of the FDTD domain is depicted in figure 2.

Replacing (10),(11) in (6)-(9), the PML equations can take a new form, with the first of them written as

$$\partial_x(p_x + p_y) = -s_{0x}(x)\left[j\omega\rho_1 + a_1^* + a_x^*(x) + \frac{a_1^* a_x^*(x)}{j\omega\rho_1}\right]u_x \quad (13)$$

Such generalized equations are more difficult to be discretized, since they involve an additional term of the form $1/j\omega$, which in the time domain implies temporal integration. The most usual way to tackle with this is to introduce proper auxiliary variables.

$$u_x^I = (j\omega)^{-1}u_x \Rightarrow \partial_t u_x^I = u_x \quad (14)$$

The PML equations can now be discretized using a proper FDTD scheme and computationally solved at every time step. That is, (13) is transformed

$$\begin{aligned} u_x^{n+1/2}(i+1/2,j) &= C_1^* u_x^{n-1/2}(i+1/2,j) \\ &- C_2^* u_{x,I}^n(i+1/2,j) - C_3^*[p_x^n(i+1,j) \\ &+ p_y^n(i+1,j) - p_x^n(i,j) - p_y^n(i,j)] \end{aligned} \quad (15)$$

when (14) becomes

$$\begin{aligned} u_{x,I}^n(i+1/2,j) &= u_{x,I}^{n-1}(i+1/2,j) \\ &+ \Delta t u_x^{n-1/2}(i+1/2,j) \end{aligned} \quad (16)$$

where $C_1^* = \dfrac{\rho_0/\Delta t - (a_0^* + a_x^*)/2}{\rho_0/\Delta t + (a_0^* + a_x^*)/2}$,

$$C_2^* = \frac{a_0^* a_x^*/\rho_0}{\rho_0/\Delta t + (a_0^* + a_x^*)/2},$$

$$C_3^* = \left\{s_{0x}\Delta x\left[\rho_0/\Delta t + (a_0^* + a_x^*)/2\right]\right\}^{-1}$$

III. SIMULATION RESULTS

Three kinds of problems have been examined with the presented technique to show its ability in the absorption of acoustic waves and the minimization of the reflections caused by the truncation boundaries.

The first one concerns the propagation of the acoustic waves in a lossless medium. PML absorbers of the original form are constructed ($s_x = s_y = 1$) to enclose the 143×143 domain, at the centre of which a Gaussian pulse excitation is considered. The PML medium consists of m layers and the parameter of losses is chosen to be of parabolic variation along them in order to accomplish the best absorption. Therefore, from the inner to the outer layer the value of a increases from zero to $a_{\max}$ according to

$$a(i) = a_{\max}\left(\frac{m+1-i}{m}\right)^2, \quad (i = 1,2,\ldots,m) \quad (17)$$

The maximum value of the attenuation coefficient $a_{\max}$ has been selected after some computational effort estimating the global error. What is more, the great ability of the PML is the possibility for further reduction of the reflections, by increasing the number of the PML layers in the FDTD grid. Figure 3 shows the reduction of the global error of the acoustic pressure as the number of the PML layers increases.

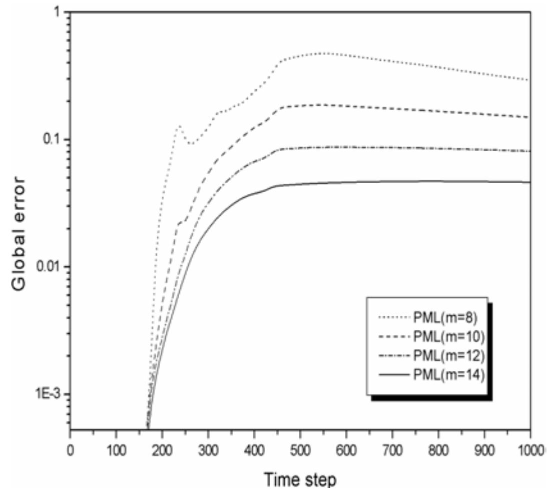


Figure 3: Global error for various PML layers.

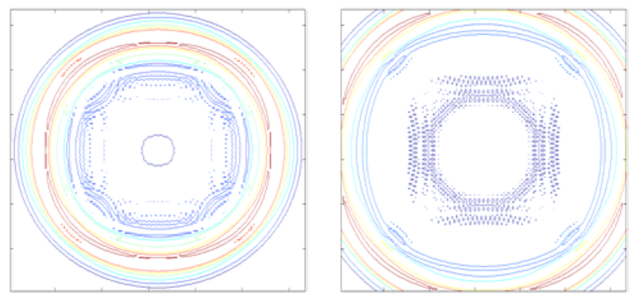
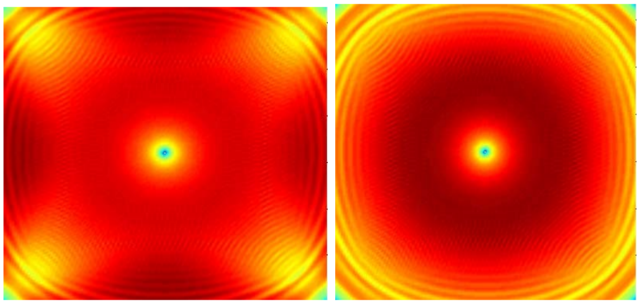


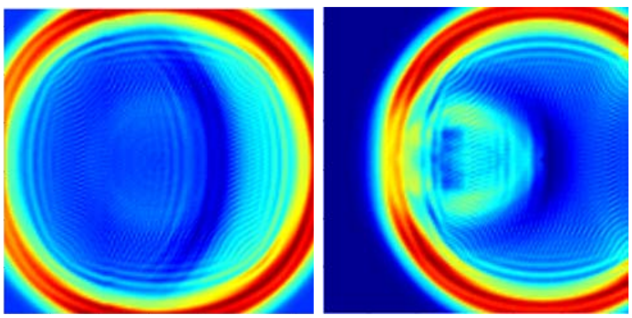
Figure 4: The acoustic pressure before and after wave impingement on the PML absorber.

The evolution of the acoustic waves before and after their impingement on the medium-PML interface is shown in figure 4. It is obvious that the propagation of the acoustic waves remains unobstructed as if the domain has not been truncated.



(a) (b)

Figure 5: Acoustic pressure calculated via (a) existing PML schemes and (b) the proposed technique.



(a) (b)

Figure 6: Snapshots of acoustic scattering in the case of a (a) two-media interface and (b) square scatterer.

However, usual PMLs do not work satisfyingly when the propagation medium is lossy, which usually occurs in acoustics. In figure 5(a), it is apparent that significant reflections can be caused at the truncation boundaries when losses are considered. To overcome this, the stretched coordinate system is assumed (s_x, s_y $\neq 1$) and s_{0x}, s_{0y} are set equal to unity while a_x, a_y are given by (17). The acoustic wave propagation in a lossy medium with $a_0 = 3.212*10^{-4}$ m^{2}Nt^{-1}sec^{-1} and $a_0^* = 0$, is simulated for the same FDTD grid, the same number of PML layers with maximum attenuation coefficient $a_{max} = 3.212*10^{-3}$ m^{2}Nt^{-1}sec^{-1} and the same excitation. Figure 5(b) presents how the modified PML worked in this case as the reflections observed in figure 5(a) are completely suppressed and the acoustic field is no more distorted.

The behavior of the PML is further studied in acoustic scattering from an interface of two different lossy media and an obstacle of square cross-section existing into the medium of propagation. The interface is placed at the 50th column of the FDTD grid, when the 20×20 cells scatterer placed on the left side of an identical FDTD grid. The scattering acoustic field is presented for both cases in figures 6(a) and 6(b) respectively, which indicate high absorption of travelling waves and verify the usefulness of our method as an ABC.

IV. Conclusions

The development of an efficient PML absorption technique in open-problems of acoustic scattering in lossy media is presented in this paper. It has been proved that zero reflections can occur in such a case, while numerical results verify the theoretical prescripts, creating new aspirations for the precise simulations in the field of acoustic propagation.

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