Sabancı University
Faculty of Engineering and Natural Sciences

EE 568 - DETECTION AND ESTIMATION THEORY

Problem Set No. 1

Spring 2012-2013

Issued: Monday, March 4, 2013 **Due:** Tuesday, March 12, 2013

Problem 1.1

Consider the random variables X and Y whose joint probability density function is given by

$$p_{X,Y}(x,y) = \begin{cases} 0.5 & |x+y| \leq 1 \ \ \& \ \ |x-y| \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(a) Specify the covariance matrix Λ_{XY}.

(b) Are X and Y uncorrelated?

(c) Are X and Y independent?

Problem 1.2

Let X_1 and X_2 be zero-mean jointly Gaussian random variables with covariance matrix

$$\Lambda_X = \begin{bmatrix} 34 & 12 \\ 12 & 41 \end{bmatrix}$$

where $X = [X_1 \ \ X_2]^T$.

(a) Verify that Λ_X is a valid covariance matrix.

(b) Find the marginal probability density for X_1.

(c) Find the probability density for $Y = 2X_1 + X_2$.

(d) Find a linear transformation defining two new variables

$$\begin{bmatrix} X_1' \\ X_2' \end{bmatrix} = \mathbf{P} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

such that X_1' and X_2' are statistically independent and such that

$$\mathbf{P}\mathbf{P}^T = \mathbf{I}$$

where $\mathbf{I}$ is the 2×2 identity matrix.

Problem 1.3

A group of students is taking a multiple-choice test. For a particular question on the test, the fraction of students who know the answer is p. The fraction that will have to guess the answer is $(1-p)$. If a student knows the answer, then he/she will certainly answer the question correctly. If a student does not know the answer and must guess, then the probability of answering the question correctly is $1/n$, where n is the number of choices for the given question.

(a) Compute the probability P_c that a student who answers the question correctly actually knew the answer.

(b) Suppose that the professor believes that $p = 0.85$, i.e. that 85% of the students actually know the answer. Furthermore, suppose that he/she wants to design the multiple choice question such that $P_c \geq 0.95$, i.e. such that correct answers on the question indicate actual knowledge at least with 95% probability. How many choices n should the problem have?

Problem 1.4

Let X be a continuous *uniform* random variable taking values between 0 and 0.5. Assume that X is discretized by rounding with a step length of 0.1 between quantization levels and a discrete random variable X_d is obtained. Plot the pdf of X and the pmf of X_d. Find the mean and variance values for both X and X_d.