

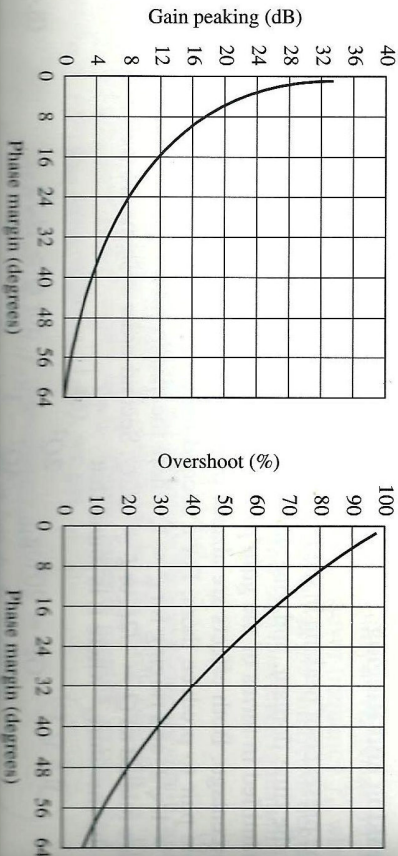
FIGURE 8.2
Illustrating gain peaking GP and overshoot OS.

$$\phi_m = \cos^{-1}(\sqrt{4\zeta^4 + 1 - 2\zeta^2}) = \cos^{-1}(\sqrt{1 + 1/4Q^4 - 1/2Q^2}) \quad (8.6)$$

Combining these equations yields the graphs of Fig. 8.3, which relate peaking and ringing to the phase margin. We observe that peaking occurs for $\phi_m \leq \cos^{-1}(\sqrt{2} - 1) = 65.5^\circ$, and ringing for $\phi_m \leq \cos^{-1}(\sqrt{5} - 1) = 76.3^\circ$. It is also worth keeping in mind the following frequently encountered values of GP (ϕ_m) and OS (ϕ_m):

$$\begin{array}{ll} \text{GP } (60^\circ) \cong 0.3 \text{ dB} & \text{OS } (60^\circ) \cong 8.8\% \\ \text{GP } (45^\circ) \cong 2.4 \text{ dB} & \text{OS } (45^\circ) \cong 23\% \end{array}$$

Depending on the case, a closed-loop response may have a single pole, a pole pair, or a higher number of poles. Mercifully, the response of higher-order circuits is often dominated by a single pole pair, so the graphs of Fig. 8.3 provide a good starting point for a great many circuits of practical interest.



The Rate of Closure (ROC)

We are now ready to develop a quick means for assessing stability from magnitude Bode plots for *minimum-phase systems*, that is, for systems having no roots in the right half of the s plane. To this end, let us first study the plots of Fig. 8.4, which pertain to the single-root function $H(jf) = (1 + jf/f_0)^{-1}$, where -1 holds for a pole frequency, and $+1$ for a zero. Denoting the slope of $|H|$ as $\text{Slope}(|H|)$, we observe that for $f \leq f_0/10$, $\text{Slope}(|H|) \rightarrow 0 \text{ dB/dec}$ and $\angle H \rightarrow 0^\circ$; for $f = f_0$, $\text{Slope}(|H|) \rightarrow \pm 20 \text{ dB/dec}$ and $\angle H \rightarrow \pm 90^\circ$; for $f = 10f_0$, $\text{Slope}(|H|) \rightarrow 0 \text{ dB/dec}$ and $\angle H \rightarrow \pm 90^\circ$. We can empirically derive phase (in degrees) from slope (in decibels per decade) as

$$\angle H \cong 4.5 \times \text{Slope}(|H|) \quad (8.7)$$

This correlation holds also if $H(s)$ has more than one root, provided the root is *real, negative, and well separated*, say, at least a decade apart.

Next, suppose both $|a|$ and $|1/\beta|$ have been graphed. Observe the slopes of the two curves at the crossover frequency f_x , and call the magnitude of their difference the *rate of closure*,

$$\text{ROC} = |\text{Slope}(|a|) - \text{Slope}(|1/\beta|)|_{f=f_x} \quad (8.8)$$

Considering that $\angle T(jf_x) = \angle a(jf_x) - \angle \beta^{-1}(jf_x)$, we can use the ROC to estimate ϕ_m via Eq. (8.7). The following cases arise so frequently that it is worth keeping them in mind.

$$\text{ROC} \cong 20 \text{ dB/dec} \Rightarrow \phi_m \cong 90^\circ \quad (8.9a)$$

$$\text{ROC} \cong 30 \text{ dB/dec} \Rightarrow \phi_m \cong 45^\circ \quad (8.9b)$$

$$\text{ROC} \cong 40 \text{ dB/dec} \Rightarrow \phi_m \cong 0^\circ \quad (8.9c)$$

$$\text{ROC} > 40 \text{ dB/dec} \Rightarrow \phi_m < 0^\circ \quad (8.9d)$$

We shall also make frequent use of the property that for any two frequencies f_1 and f_2 located within a region of constant slope of $\pm n$ dB/dec, we have

$$|H(jf_1)|/|H(jf_2)| = (f_1/f_2)^{\pm n} \quad (8.10)$$

