

# Essential Principles of Power Part 1

Voltage, Current and Power – from AC Line to PWM



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# Agenda

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- Defining “Power”
- “Power” Overview
- The Basics – “AC Line” Voltage, Current and Power
- Distorted Waveform Power Calculations
- Three-Phase Power Calculations
- Measurement Example
- Questions & Answers

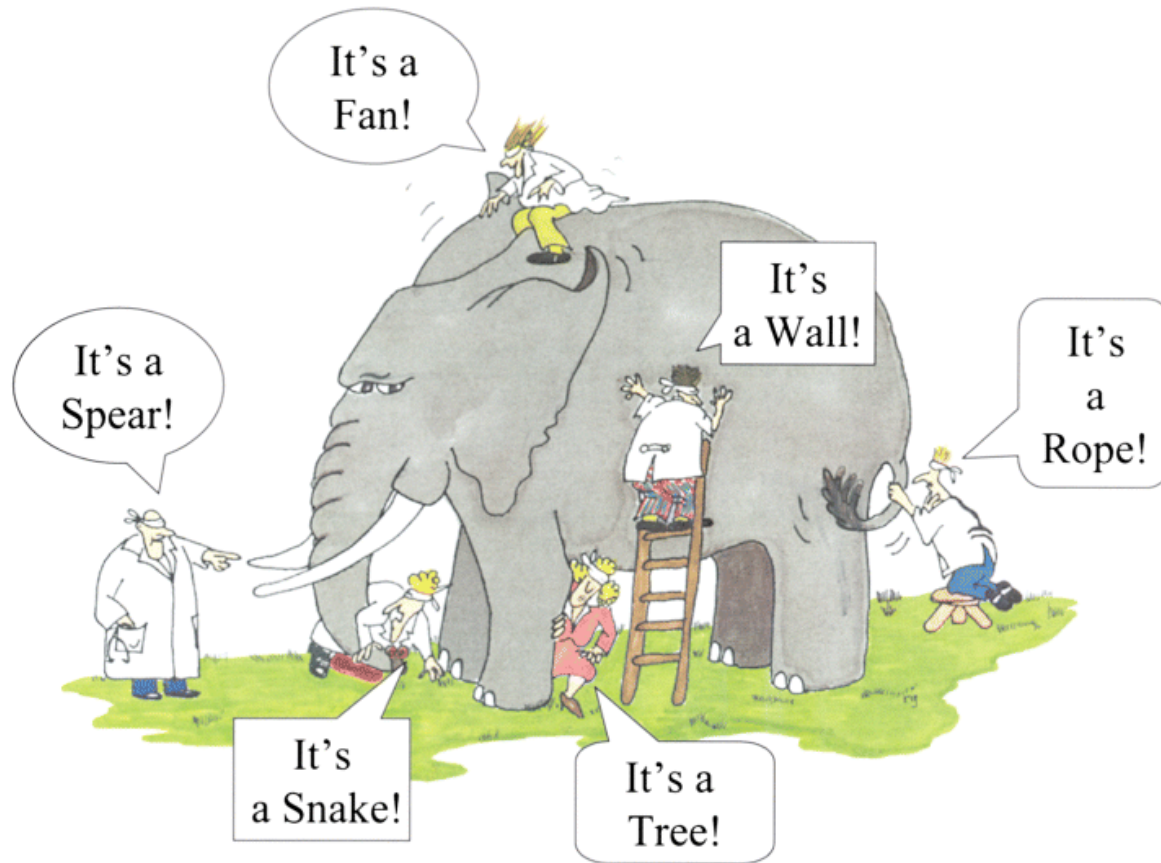
# Defining “Power”

*See the elephant*



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# Defining “Power” Can Be Like Blind Men Describing an Elephant...



- Engineers can mean many different things when they say “power”
- In the next three slides, we’ll define our “power” focus for this presentation...

# “Power” Definitions

*These are just a few of them...*

- **Utility, Grid, Household, Line, Power Line, Mains “Power”**
  - This is the 50/60 Hz sinusoidal voltage/current power flowing to your home or business, measured by a kWh meter
- **Power Semiconductor Device “Power”**
  - This is the power consumed by the power semiconductor MOSFET or IGBT device during switching, conduction, or OFF states
- **Digital Power Management “Power”**
  - This is the ON/OFF voltage management of the DC power supply rails on a motherboard or embedded computing system
- **Power Supply Startup Sequencing “Power”**
  - This is the management of the ramp times and sequences of different DC power supply rails on a motherboard or embedded computing system
- **Power Electronics Inverter/Converter “Power” Testing**
  - This is the measurement of a complex mix of line (50/60 Hz) frequency input, variable frequency output, DC and control/sensor signals for debug, troubleshooting and validation purposes
- **Power Analyzer “Power Analysis”**
  - This is the measurement of the Watts or Volt-Amperes that a product (“system”) consumes and/or the efficiency of power consumption for the product

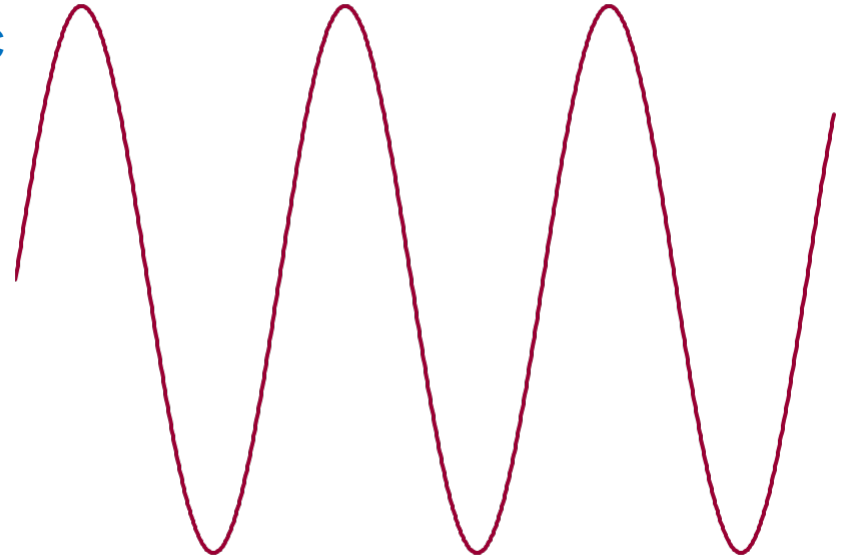
# “Line” V, I, and Power Measurements

*These are 50/60 Hz signals that are input to power conversion systems*

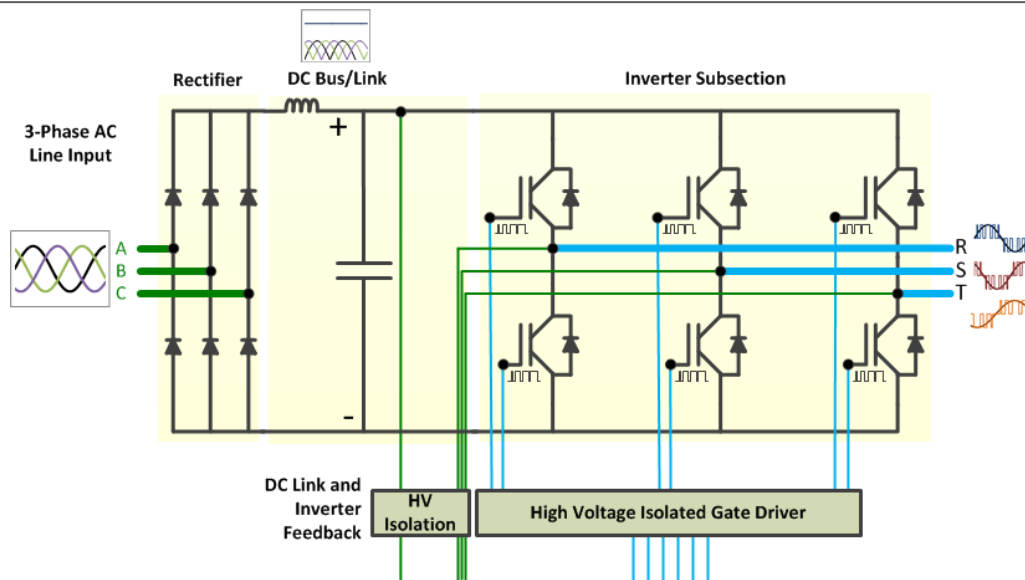
- **Utility, Grid, Household, Line, Power Line, Mains “Power”**
  - This is the 50/60 Hz sinusoidal voltage/current power flowing to your home or business, measured by a kWh meter

The “Line” input of a power conversion (AC-AC or AC-DC) system is typically 50/60 Hz signals.

PWM voltage signals at the output of a power conversion system have a sinusoid fundamental



# Power Conversion Systems Measurements



- **Power Electronics Inverter/Converter “Power” Testing**
  - This is the measurement of a complex mix of line (50/60 Hz) frequency input, variable frequency output, DC and control/sensor signals for debug, troubleshooting and validation purposes
- **Power Analyzer “Power Analysis”**
  - This is the measurement of the Watts or Volt-Amperes that a product (“system”) consumes and/or the efficiency of power consumption for the product

# “Power” Overview: 100 years in 7 slides



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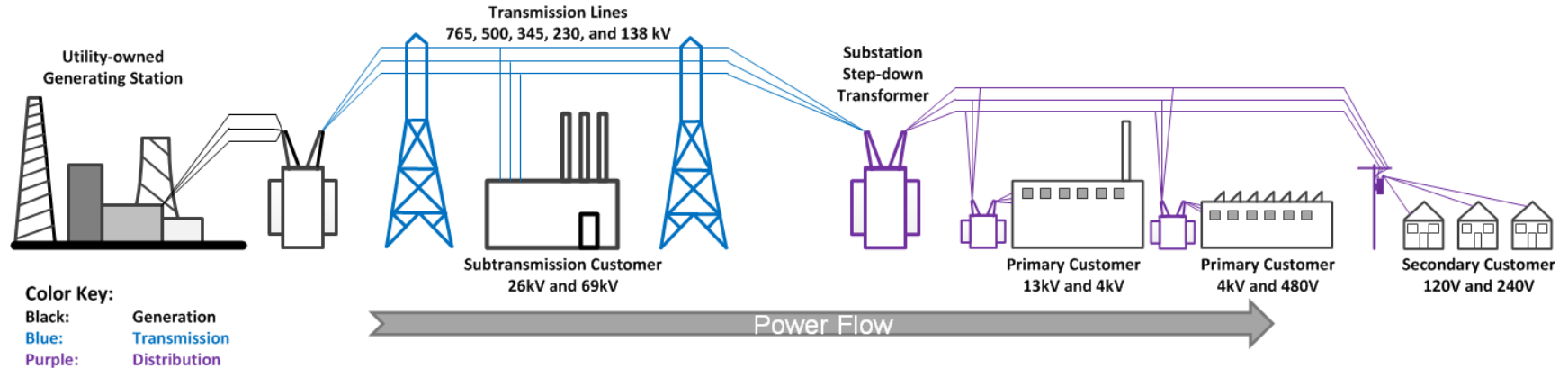
# Generation, Transmission & Distribution (GT&D) and Consumption of Power

- First - Electricity is **Generated**
  - Stationary Generators
    - Utility centralized “generating plants”
    - Distributed Generation (DG) (DC inverted to AC)
- Then – Electricity is **Transmitted and Distributed** to
  - Homes
  - Commercial Locations
  - Industrial Users
- Finally – Power is **Consumed**
  - Directly from the utility AC (50/60 Hz) line (no power conversion)
  - Via AC-AC conversion (variable frequency drives)
  - Via AC-DC conversion (“switch-mode” power supplies)
  - Via DC-AC conversion (inverters)
  - Via DC-DC conversion (converters)



# Historical Generation, Transmission & Distribution System (GT&D)

*Large generation inefficiencies, high T&D losses*



- Centralized power generation, utility delivery to customer
- Overall power delivery efficiency = 32%
  - Generation input/output efficiency = 35% (1 BTU in = 0.35 BTU out)
  - T&D efficiency = 93% (0.35 BTU in = 0.32 BTU out)
    - 7% losses in T&D system components, e.g.
      - Step-up, Power, Substation, and Distribution transformers
      - Power Cables

# Transmission & Distribution System Loss Measurements

*T&D equipment suppliers would validate equipment losses prior to shipment to utility*

- Transformer power frequency loss measurements
  - 50 or 60 Hz
  - Load (Copper, or  $I^2R$ ) Losses
  - Excitation (Core) Losses
  - Efficiencies
- Validation Test
  - Loss validation
  - Efficiency measurements
- Report provided to end utility customer as part of sale



# Power Consumption – Motors

*Motors have represented the largest single opportunity to reduce energy consumption*

- 45% of worldwide delivered electricity is consumed by electric motors
  - 9% of this by small motors
    - Up to 750W (**90% of motors**)
    - AC Induction, BLDC, PMSM
  - 68% of this by medium motors
    - Up to 375 kW (**9.9% of motors**)
    - Mostly AC Induction
  - 23% of this by large motors
    - Up to 1000 kW (**0.03% of motors**)
    - AC Induction
- Motors were essentially only line-powered prior to the 1990s
  - Power semiconductor-based “drives” revolutionized motor speed and torque control
  - Various government mandates were enacted to increase motor efficiency



AC induction motor



Permanent Magnet Synchronous Motor



Brushless DC Motor



# Power Analysis of Electric Motors (1990s and earlier)

*Focus was on the larger motors (10% of unit volume) that consumed 91% of the electricity*

- Dynamometer Test Stand
  - “**Static**” load testing
  - Analog or digital (pulse) tachometer
  - Analog torque transducer
- Rudimentary Test Validation and Reporting
  - **Efficiency** measurements – one speed/load
  - “**Numbers only**”
- Not an Integrated Design Tool
  - No (or very limited) waveform capture
  - No “Dynamic” load measurement
  - No “Complete System” test with controls debug
  - Not well-suited for small motor test and debug





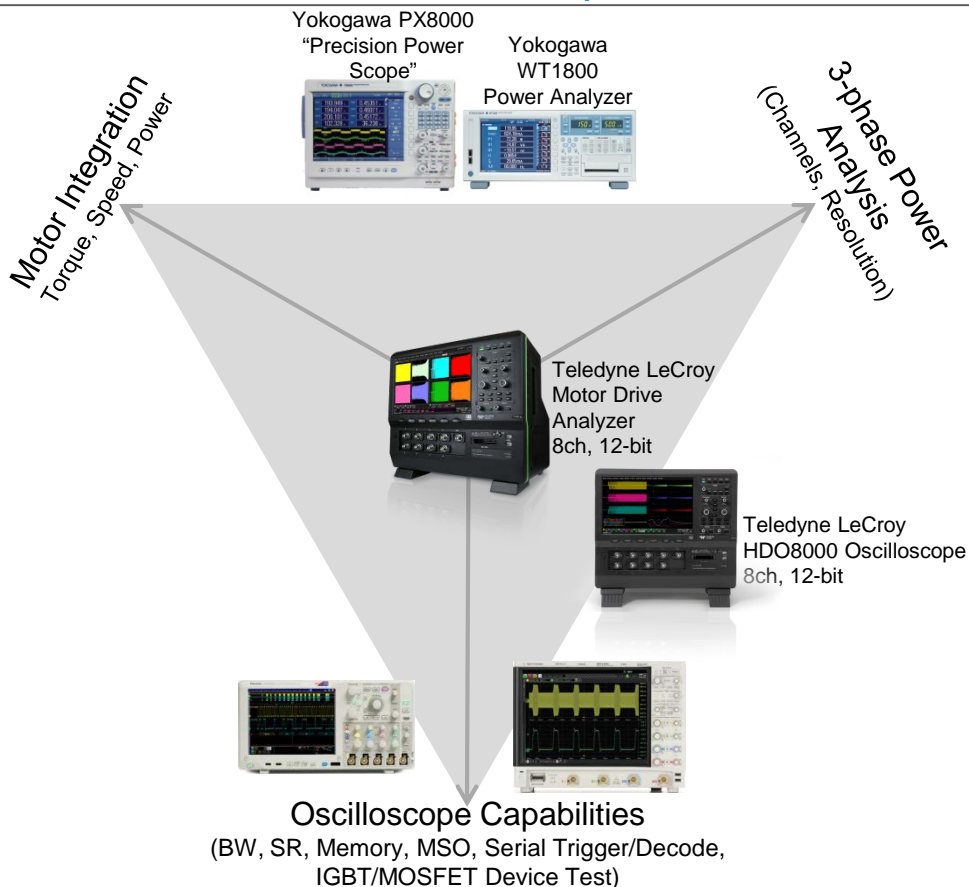
# The Power Electronics / Power Conversion Revolution

*As costs were reduced and reliability increased, motor drives became pervasive*



# Teledyne LeCroy Motor Drive Analyzers

*It's an 8ch/12-bit Oscilloscope, and it's also a Power Analyzer with Motor Integration*



- **Traditional “AC Power Analyzers”**
  - Only calculate “**static**” (steady-state) “mean” power values
  - Some don’t integrate motor torque and speed data (mechanical power)
- **General-purpose 4ch, 8-bit scopes** don’t have enough channels or resolution for three-phase systems
- **Motor Drive Analyzers perform every function**
  - **Static** (steady-state) “mean value” power tables, like a power analyzer
  - **Dynamic** (transient) power analysis
  - **Complete** embedded control debug (i.e. it is a fully-functional oscilloscope)
    - Viewing 3-phase waveform systems
    - High SR, BW, Memory
    - MSO
    - Serial Trigger & Decode

# The Basics – “AC Line” Voltage, Current, and Power

This is the sinusoidal 50 or 60 Hz voltage and current is supplied to residential, commercial and industrial customers.



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# “AC” Sinusoidal Line Voltage



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# “AC” Sinusoidal Voltage – The Basics

- Other “Names”
  - “Grid”, “Household”, “Line”, “Power Line”, “Utility”, “Mains”...
  - Regardless, we mean what comes off the utility wires to your home or business
- Unit Value
  - $V_{\text{RMS}}$  **ALWAYS**, but typically stated as  $V_{\text{AC}}$ 
    - These units/terms are used interchangeably in this context
- # Phases
  - Single-phase (two-wire)
  - Single-phase (three-wire)
  - Three-phase (three-wire or four-wire)
  - >Three-phase
    - Not common...but
    - 4, 5 or 6 phases can be used for redundancy (e.g. aircraft, military applications)
- Measurement Reference Point
  - To “Neutral” (Single-phase or Two-phase)
  - To another “Line” (or Phase) (Two-phase or Three-Phase)
  - “Ground” is a safety connection from a chassis to earth as a protection against faults – it is not “Neutral”
- Shape
  - Nominally a sinewave, but not a pure sinewave – always contains some distortion in real-world systems

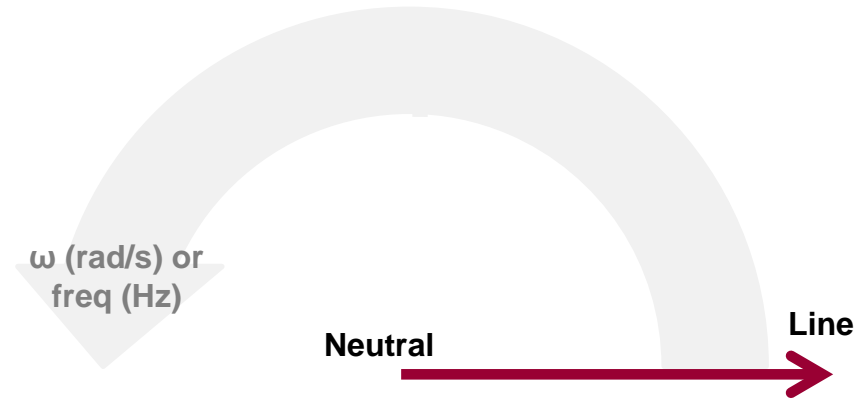
# Single-Phase “AC” Sinusoidal Voltage – The Basics

- Single-phase AC voltage consists of one voltage vector with
  - Magnitude (voltage)
  - Angle
- Typically, the single-phase is referred to as “Line” voltage



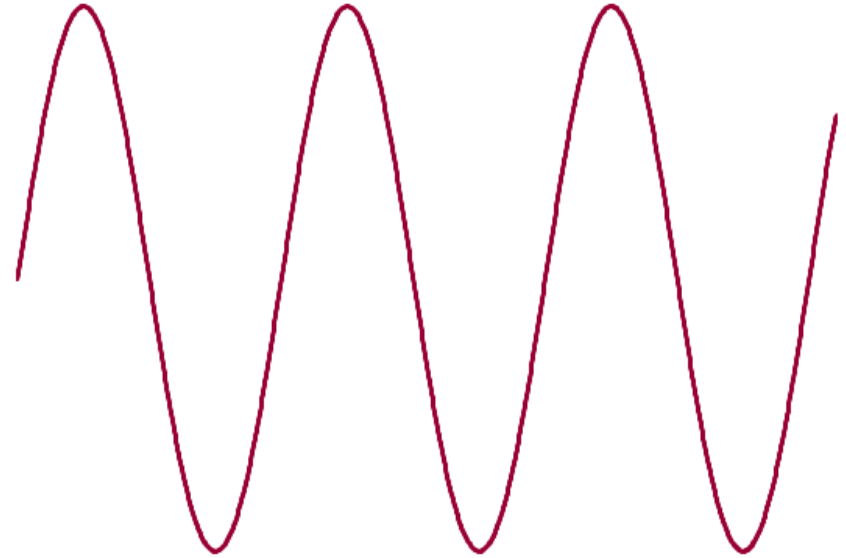
# Single-Phase “AC” Sinusoidal Voltage – The Basics

- The single-phase voltage vector rotates at a given frequency
  - Typically, 50 or 60 Hz for utility-supplied voltage
- At any given moment in time, the voltage magnitude is  $V \sin(\alpha)$ 
  - $V$  = magnitude of voltage vector
  - $\alpha$  = angle of rotation, in radians



# Single-Phase “AC” Sinusoidal Voltage – The Basics

- The resulting time-varying “rotating” voltage vector appears as a sinusoidal waveform with a fixed frequency
  - 50 Hz in Europe
  - 60 Hz in US
  - Either 50 or 60 Hz in Asia
  - Other frequencies sometimes used in non-utility supplied power, e.g.
    - 400 Hz (Naval)
    - 25 Hz (Mining)

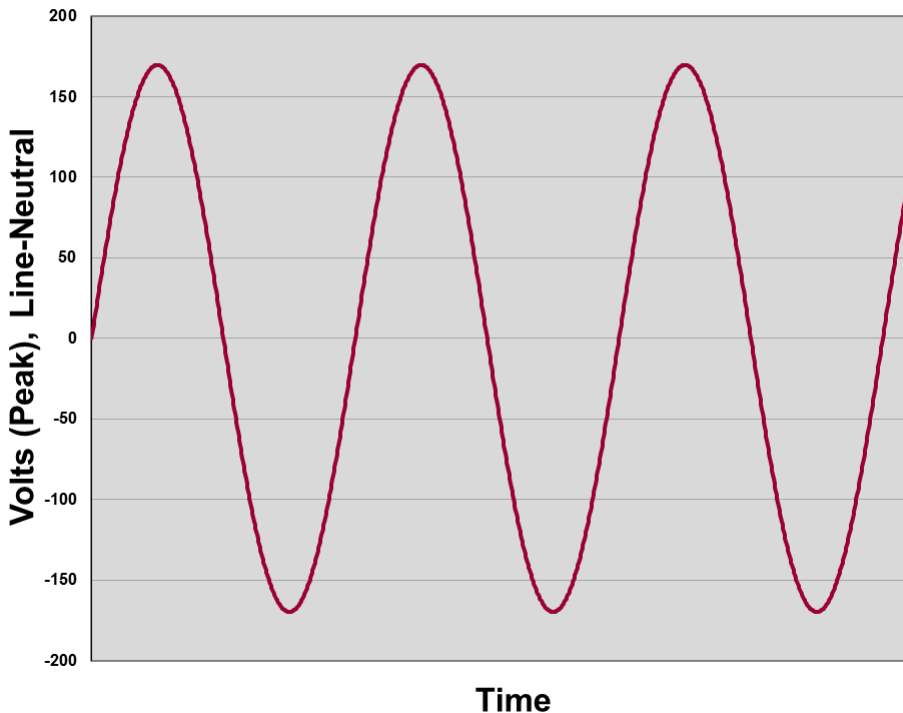


# Single-Phase “AC” Sinusoidal Voltage – The Basics

120V<sub>AC</sub> example

- Important to Know
  - Voltage is stated as “V<sub>AC</sub>”, but this is really V<sub>RMS</sub>
  - Rated Voltage is Line-Neutral
- $V_{\text{PEAK}} = \sqrt{2} * V_{\text{AC}}$  (or  $\sqrt{2} * V_{\text{RMS}}$ )
  - 169.7 V in this case
- $V_{\text{PK-PK}} = 2 * V_{\text{PEAK}}$
- If rectified and filtered
  - $V_{\text{DC}} = \sqrt{2} * V_{\text{AC}} = V_{\text{PEAK}}$

AC Single-Phase "Utility" Voltage  
120V<sub>AC</sub>

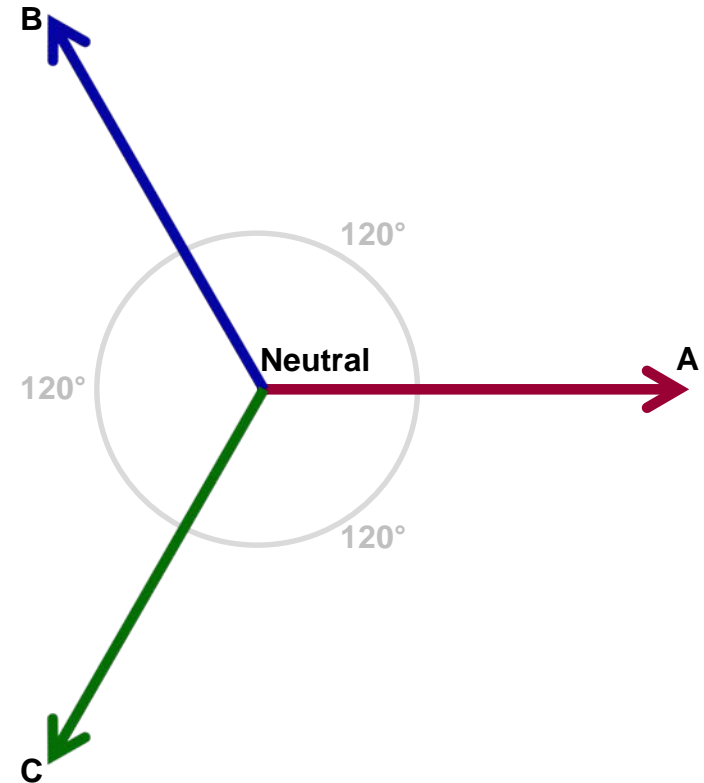


# What is “True RMS”?

- “True RMS” is not an engineering definition
  - It is a “marketing” definition to describe a mathematically correct RMS calculation as compared to a “measurement shortcut” taken in inexpensive instruments
    - “What is described as RMS is often just a  $V_{PK-PK}/\sqrt{2}$  calculation in an inexpensive multi-meter
    - This should really be referred to as “false RMS”
    - It is only a good calculation for a pure sinewave, which can rarely be assumed to be present
- Any “sampling technology” (oscilloscope, power analyzer, etc.) will calculate  $V_{RMS}$  or  $I_{RMS}$  correctly
  - But they won’t “market” their  $V_{RMS}$  calculation as “True RMS”

# Three-Phase “AC” Sinusoidal Voltage – The Basics

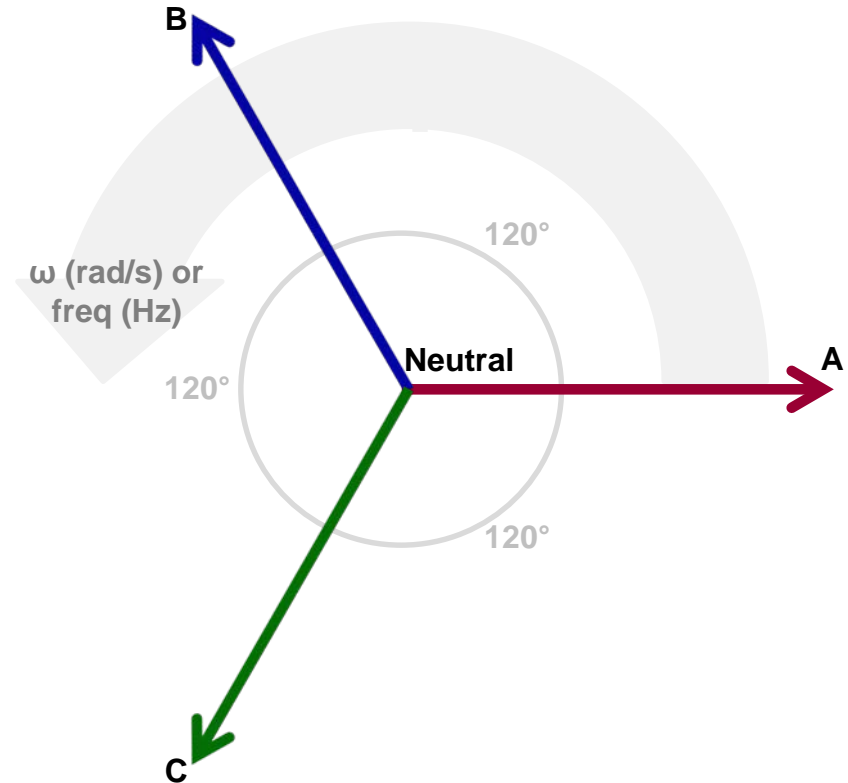
- Three-phase AC voltage consists of three voltage vectors
  - By definition, the system is “balanced”
    - Vectors are separated by  $120^\circ$
    - Vectors are of equal magnitude
    - Sum of all three voltages = 0V at Neutral
- Typically, the three phases are referred to as A, B, and C, but other conventions are also used
  - 1, 2, and 3, L1, L2, L3
  - R, S, and T





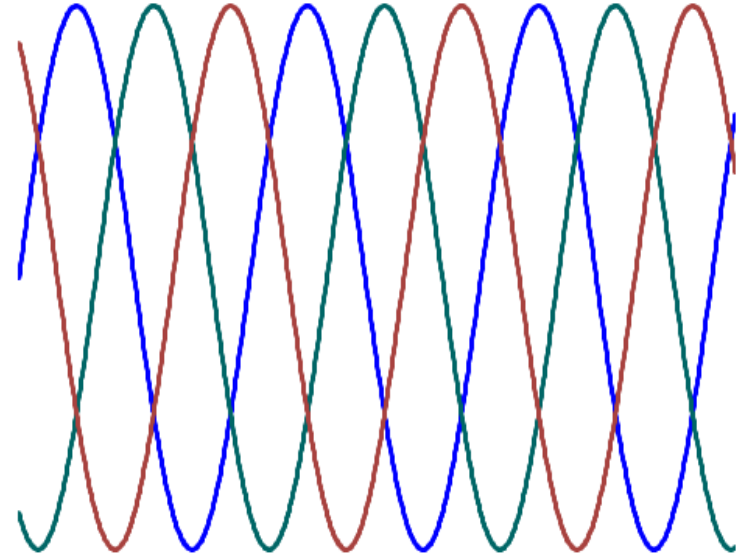
# Three-Phase “AC” Sinusoidal Voltage – The Basics

- A three-phase AC voltage is generated by the utility using a rotating machine (i.e. a “generator”)
  - A generator uses a rotating magnetic field to induce a voltage in the stator
  - The resulting voltage vectors have magnitude and phase
- Therefore, the three different phase voltage vectors rotate at a given frequency
  - Typically, 50 or 60 Hz for utility-supplied voltage



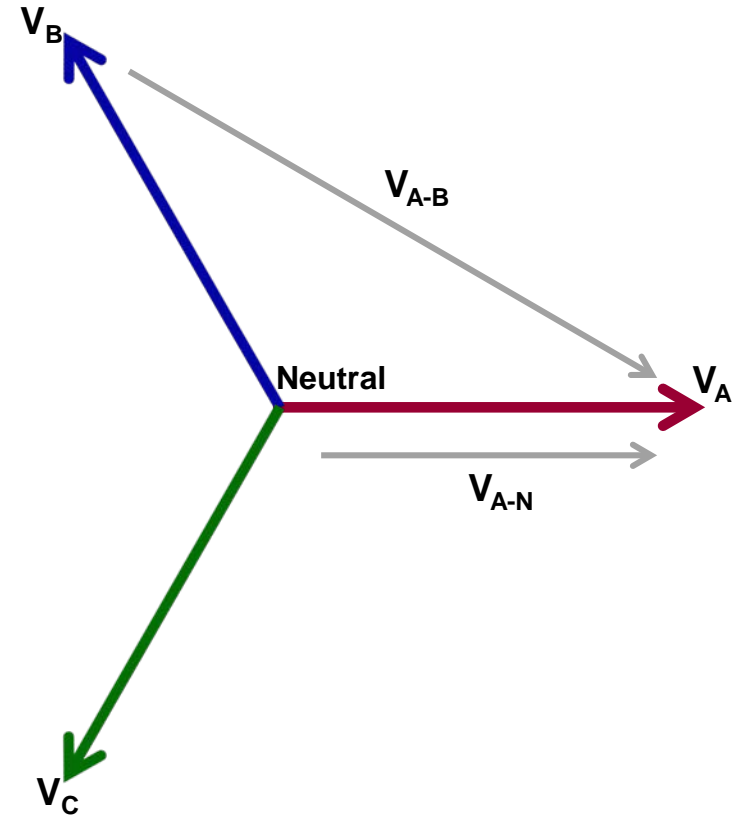
# Three-Phase “AC” Sinusoidal Voltage – The Basics

- The resulting time-varying “rotating” voltage vectors appear as three sinusoidal waveforms
  - Separated by  $120^\circ$
  - Of equal peak amplitude
- Voltage value =  $V_x \sin(\alpha)$ 
  - $V_x$  = magnitude of phase voltage vector
  - $\alpha$  = angle of rotation, in radians
- Three-phase AC voltage is used for a variety of reasons
  - More efficient to generate power with three-phase generators
  - Easier to manufacture high power transformers and motors
  - Better control capability for low power motors



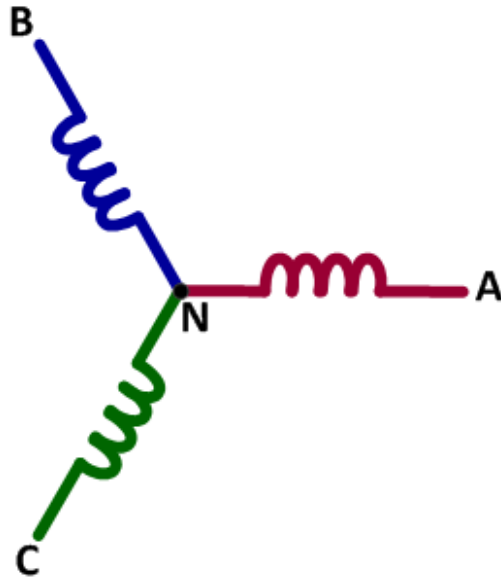
# Three-Phase “AC” Sinusoidal Voltage – The Basics

- Voltages can be measured two ways
  - Line-Line
    - Also referred to as Phase-Phase
    - e.g. from  $V_A$  to  $V_B$ , or  $V_{A-B}$
  - Line-Neutral
    - Neutral must be present and accessible
    - e.g. from  $V_A$  to Neutral, or  $V_{A-N}$
- $V_{L-L}$  conversion to  $V_{L-N}$ 
  - Magnitude:  $V_{L-N} * \sqrt{3} = V_{L-L}$
  - Phase:  $V_{L-N} - 30^\circ = V_{L-L}$

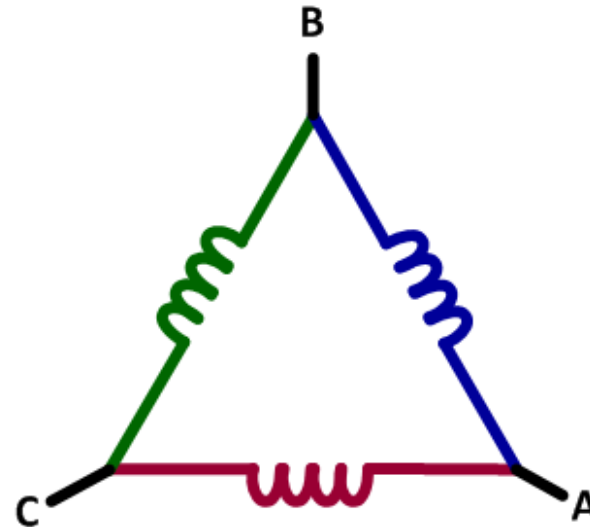


# Three-Phase “AC” Sinusoidal Voltage – The Basics

- Wye (Y) 3-phase Connection
  - Neutral is present in the winding
    - but often is not accessible
  - Most common configuration



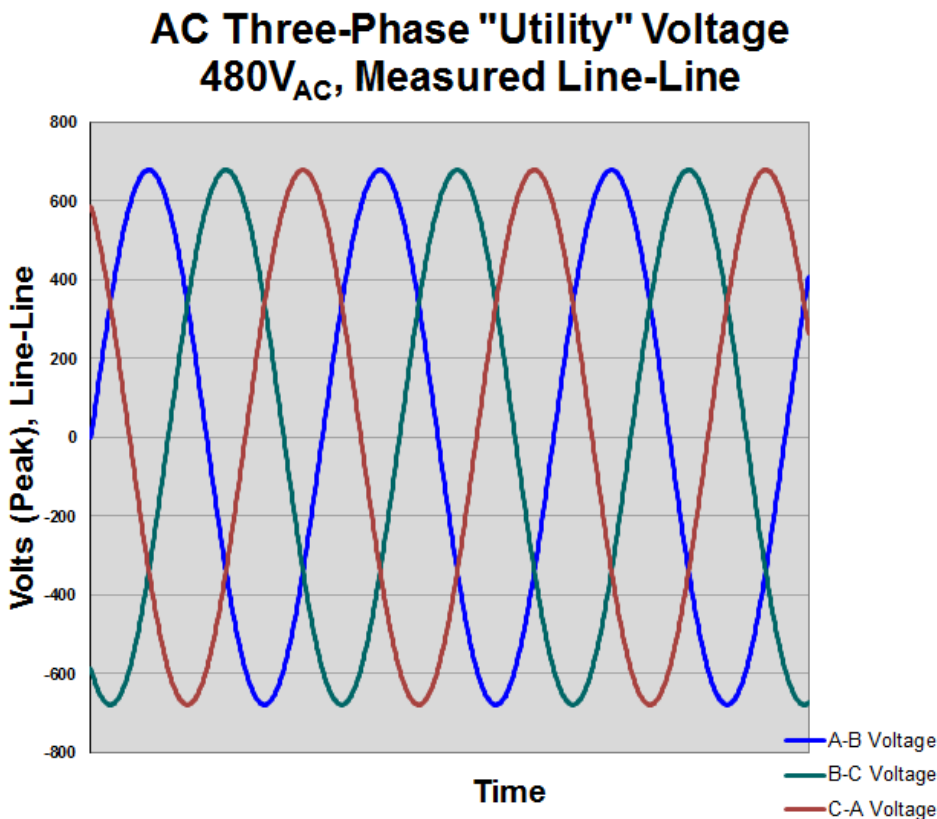
- Delta ( $\Delta$ ) 3-phase Connection
  - Neutral is not present in the winding (in most cases)



# Three-Phase “AC” Sinusoidal Voltage – The Basics (example)

## Line-Line Voltage Measurements

- Important to Know
  - Voltage is stated as “ $V_{AC}$ ”, but this is really  $V_{RMS}$
  - Rated Voltage is always for Line-Line ( $V_{L-L}$ )
    - The three phases are usually referred to as A, B, and C
    - Line-Line is A-B ( $V_{A-B}$ ), B-C ( $V_{B-C}$ ), and C-A ( $V_{C-A}$ )
    - Line-Line is sometimes referred to as Phase-Phase
- $V_{PEAK(L-L)} = \sqrt{2} * V_{L-L}$ 
  - 679V in this case
- $V_{PK-PK(L-L)} = 2 * V_{PEAK(L-L)}$

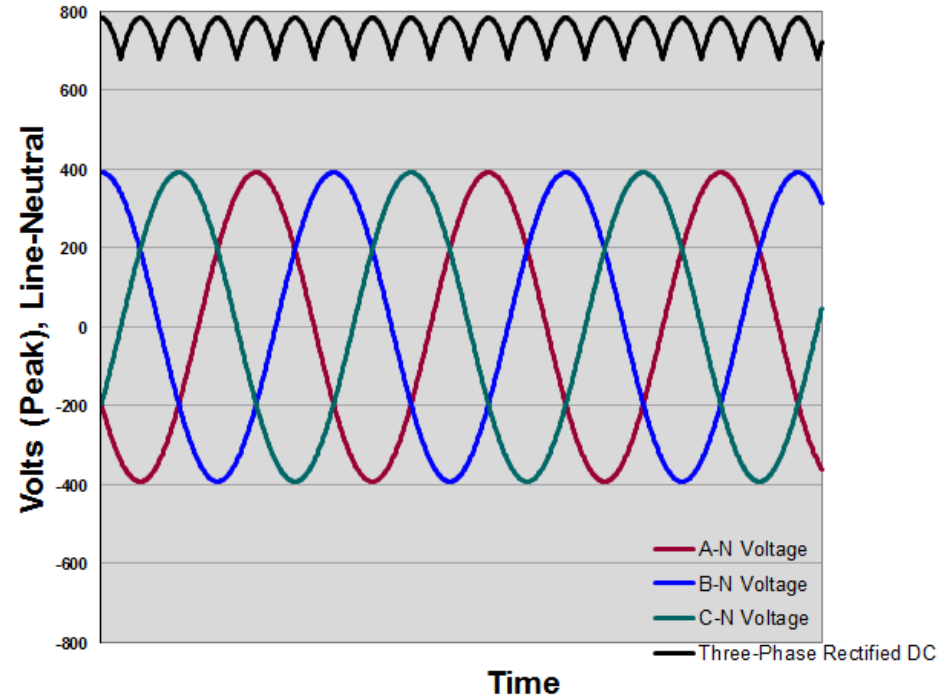


# Three-Phase "AC" Sinusoidal Voltage – The Basics (example)

## Line-Neutral Voltage Measurements

- If a neutral wire is present, three-phase voltages may also be measured Line-Neutral
  - $V_{\text{LINE-NEUTRAL}} (V_{\text{L-N}}) = V_{\text{L-L}} / \sqrt{3}$ 
    - $277V_{\text{AC}} (V_{\text{RMS}})$  in this case
  - $V_{\text{PEAK}} = \sqrt{2} * V_{\text{L-N}}$ 
    - 392 V in this case
  - $V_{\text{PK-PK}} = 2 * V_{\text{PEAK}}$
- If all three phases are rectified and filtered
  - $V_{\text{DC}} = \sqrt{2} * V_{\text{L-N}} * \sqrt{3} = V_{\text{PEAK}} * \sqrt{3}$
  - Practical max filtered DC bus voltage is less than vector sum

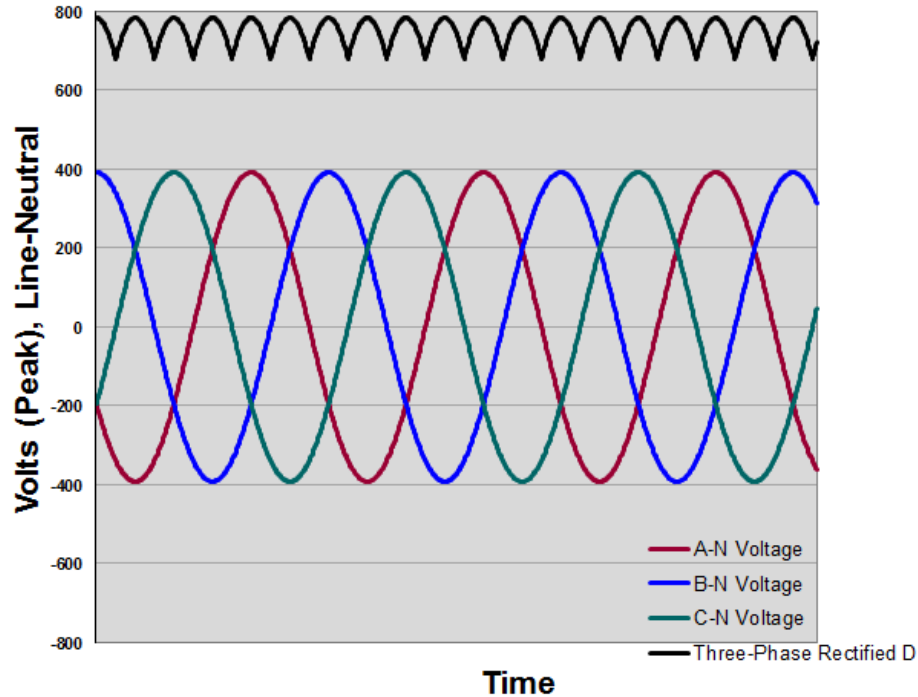
**AC Three-Phase "Utility" Voltage**  
**480V<sub>AC</sub>, Measured Line-Neutral**



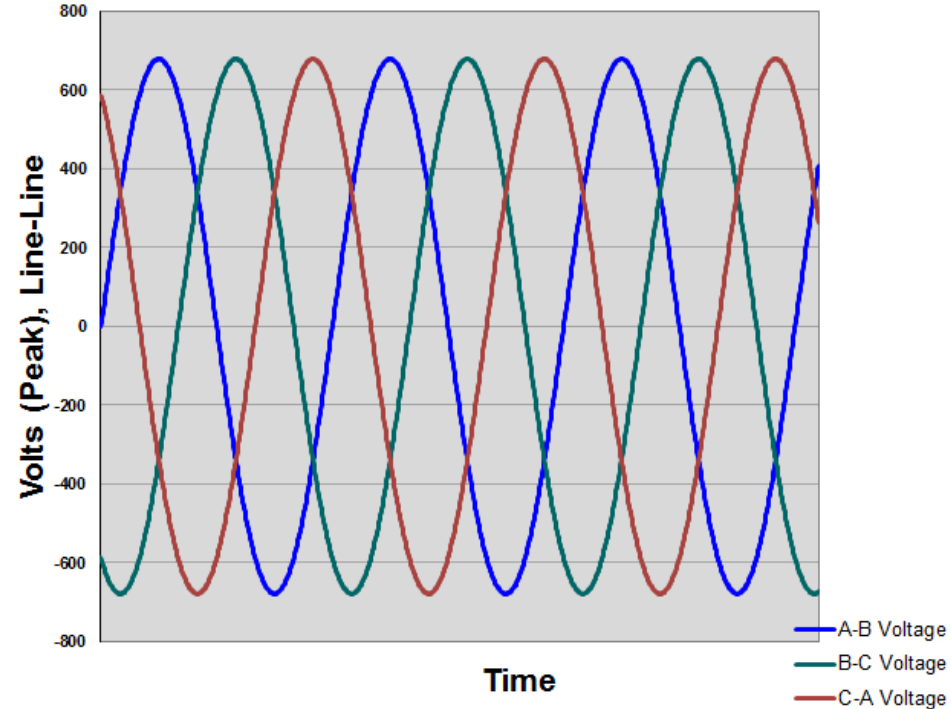
# Three-Phase "AC" Sinusoidal Voltage – The Basics (example)

## Comparison of Line-Neutral and Line-Line Voltage Measurements

**AC Three-Phase "Utility" Voltage  
480V<sub>AC</sub>, Measured Line-Neutral**



**AC Three-Phase "Utility" Voltage  
480V<sub>AC</sub>, Measured Line-Line**



# Utility Voltage Classes, per ANSI C84.1-1989

ANSI is a US Standards Organization, but IEC and other are similar

- Low Voltage, 50V Class (Safety)
  - Commonly used, but not a “utility voltage class”
- Low Voltage, 600V Class (Distribution),  $<1000V_{\text{RMS}}$ 
  - Residential, small commercial, Single-phase
    - 100/110/120V
    - 208V
    - 220/240V
  - Three-phase
    - 380/400V
    - 440/480V
    - 575/600V
    - Max = 690V (600V + 15%)
- Medium Voltage (Generation, Distribution and Subtransmission)
  - 5kV, 15kV, 25kV, 35kV, 69kV “Classes”



# Devices used to measure high voltages

- High Voltage Differential Probes
  - 1kV, 2kV, 6kV safety-rated (isolated)
  - 1% accuracy
  - Excellent CMRR performance
  - DC to 100+ MHz
- High Voltage Passive Probes
  - DC to ~500 MHz
- Differential Amplifiers
  - DA1855A (Teledyne LeCroy
  - CIC Research
- Potential Transformers
  - Not DC rated
  - Limited high frequency response



# “AC” Sinusoidal Line Current



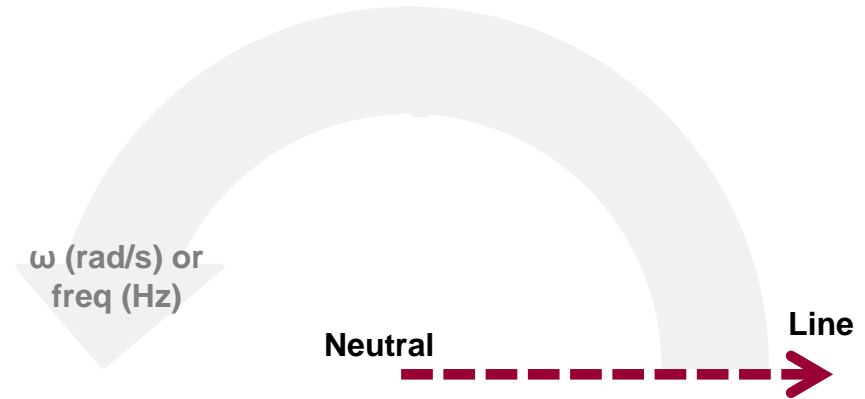
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# “AC” Sinusoidal Current – The Basics

- Other “Names”
  - “Grid”, “Household”, “Line”, “Utility”, “Mains”...
  - Regardless, we mean what comes off the utility wires to your home or business
- Unit Value
  - $I_{\text{RMS}}$  **ALWAYS**, but typically stated as  $I_{\text{AC}}$ 
    - These units/terms are used interchangeably in this context
- # Phases
  - Single-phase (single-conductor)
  - Three-phase (three-conductor)
- Measurement Reference Point
  - Always “Line” current
    - In a Wye (Y) system, all line current flows to Neutral, so line currents are winding currents
    - In a Delta ( $\Delta$ ) system, line currents are terminal currents that flow to two different windings
- Shape
  - Nominally a sinewave, but not a pure sinewave – always contains some distortion in real-world systems
  - Standards specify <5% distortion

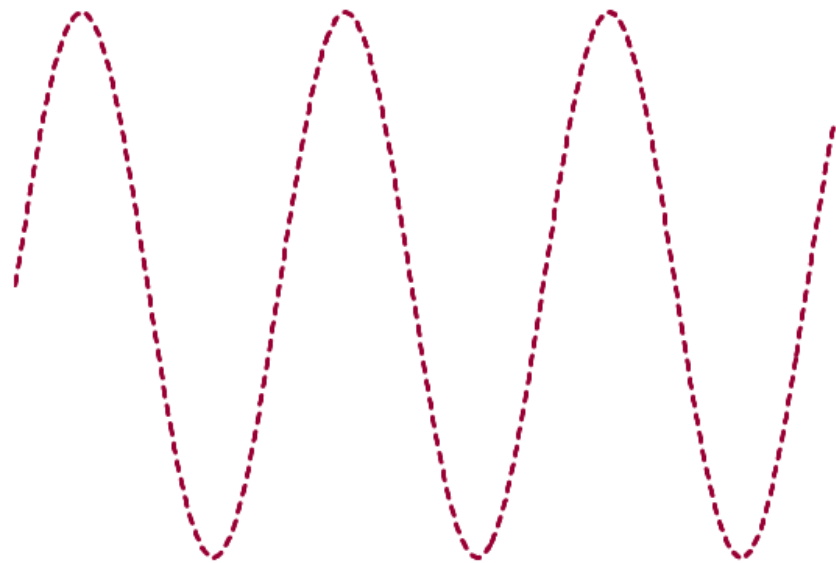
# Single-Phase “AC” Sinusoidal Current – The Basics

- Like voltage, the single-phase current vector rotates at a given frequency
  - Typically, 50 or 60 Hz
- At any given moment in time, the current magnitude is  $I \sin(\alpha)$ 
  - $I$  = magnitude of current vector
  - $\alpha$  = angle of rotation, in radians



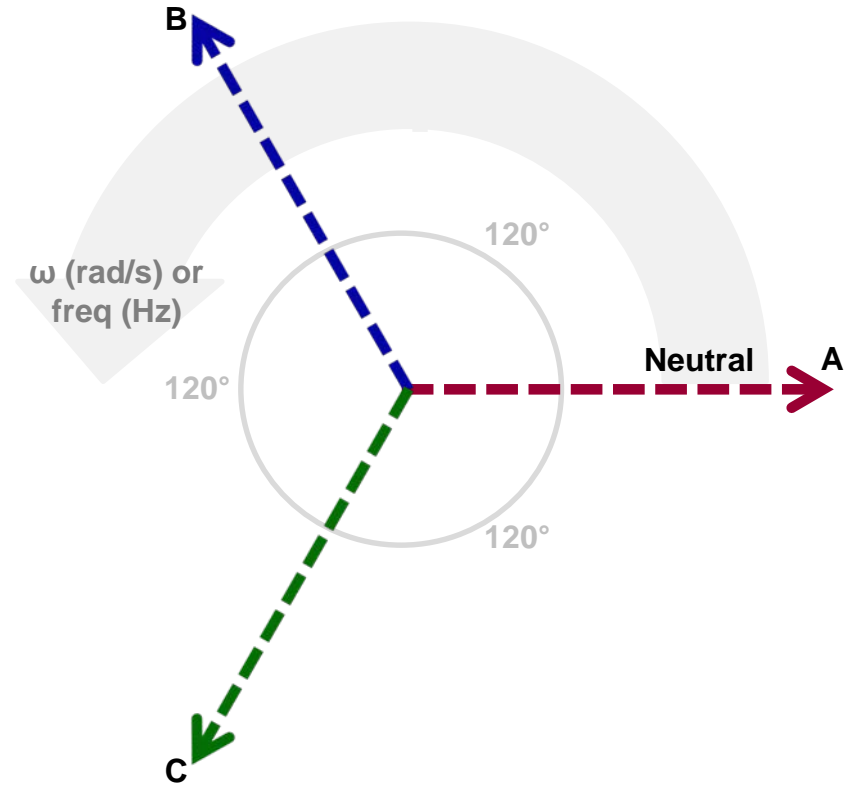
# Single-Phase “AC” Sinusoidal Current – The Basics

- The resulting time-varying “rotating” current vector appears as a sinusoidal waveform



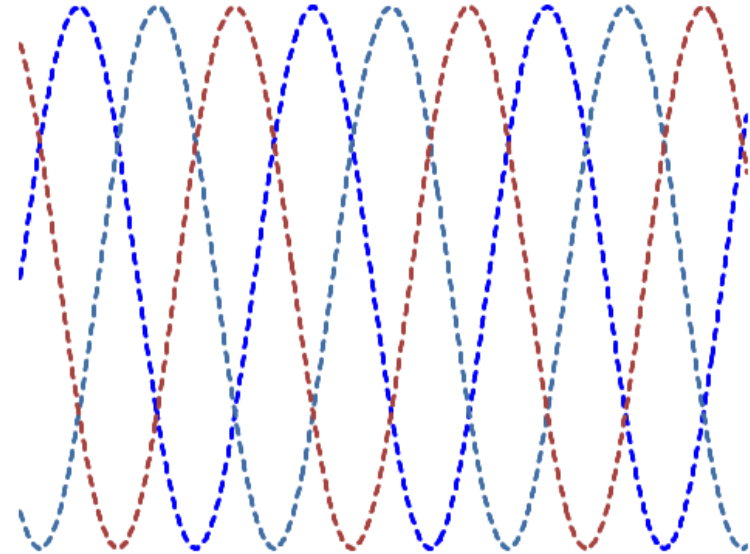
# Three-Phase “AC” Sinusoidal Current – The Basics

- Like voltage, three-phase current has the three different line current vectors that rotate at a given frequency
  - Typically, 50 or 60 Hz for utility-supplied voltage



# Three-Phase “AC” Sinusoidal Current – The Basics

- Like voltage, the resulting time-varying “rotating” current vectors appear as three sinusoidal waveforms
  - Separated by  $120^\circ$
  - Of equal peak amplitude for a balanced load
- Current value =  $I_x \sin(\alpha)$ 
  - $I_x$  = magnitude of line current vector
  - $\alpha$  = angle of rotation, in radians



# Devices used to measure currents

*These devices have frequency response from DC*

- Current Probes
  - 30A, 150A, 500A
  - 1% accuracy
  - DC up to 100 MHz
  - Expensive, but multi-purpose for a wide range of oscilloscope probing requirements
- Specialty Current Transducers
  - e.g. Danisense
  - <1% accuracy
  - DC to ~100 kHz
- Why DC?
  - Low frequencies present at start-up events
  - ????





# Devices used to measure currents, cont'd

*These devices have AC frequency response only*

- Rogowski Coils (e.g. PEM-UK)
  - Frequency response depends on model (lowest ~5-15 Hz typical)
  - Lowest cost
  - Split-core
  - Very small to very high loop sizes
- Pearson Current Transformers (CT)
  - Frequency response depends on model (lowest ~5 Hz typical)
  - Split-core (typical)
  - Built-in shunt resistors for scaled voltage output
- Conventional Turns Ratio CT\*
  - Frequency response typically covers line frequency range with a little margin
  - Scaled output current
  - Need shunt resistor on output to convert to voltage output



\*Note: dangerous open-circuit voltages can occur at the output of these devices – use extreme caution, and avoid operating open-circuited

# Current Sensor Adapter to Teledyne LeCroy Oscilloscope

- Provides ability for third-party current sensor to operate like a Teledyne LeCroy “probe”
  - Programmable
  - Customizable
    - Bandwidth filtering
    - Shunt resistor
  - Converts any linear voltage or current input to output scaled in Amperes
- Simplifies the setup



# Pure Sinewave Power Calculations

For the moment, let's assume the most simple case – a single sinusoidal line voltage and a single sinusoidal line current supplying a linear load.



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# “AC” Power – The Basics

## Single-phase, resistive loads

- Electric Power
  - “The rate at which energy is transferred to a circuit”
  - Units = Watts (one Joule/second)
- For purely resistive loads
  - $P = I^2R = V^2/R = \text{instantaneous } V \cdot I$
  - The current vector and voltage vector are in perfect phase



# “AC” Power – The Basics

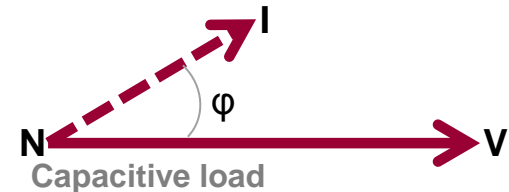
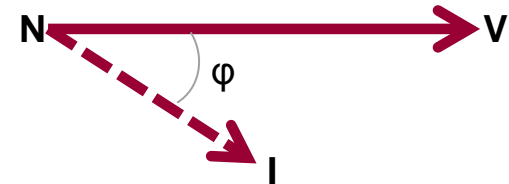
## Introducing Power Factor

- Phase Angle ( $\phi$ )
  - Indicates the angular difference between the current and voltage vectors
    - Degrees:  $-90^\circ$  to  $+90^\circ$
    - or radians:  $-\pi/2$  to  $+\pi/2$
- Power Factor (PF, or  $\lambda$ )
  - $\cos(\phi)$  for *purely sinusoidal waveforms*
  - Unitless, 0 to 1,
    - $1 = V$  and  $I$  in phase, purely resistive load
    - $0 = 90^\circ$  out of phase, purely capacitive or purely inductive load
    - Not typically “signed” – current either leads (capacitive load) or lags (inductive load) the voltage

Purely resistive load

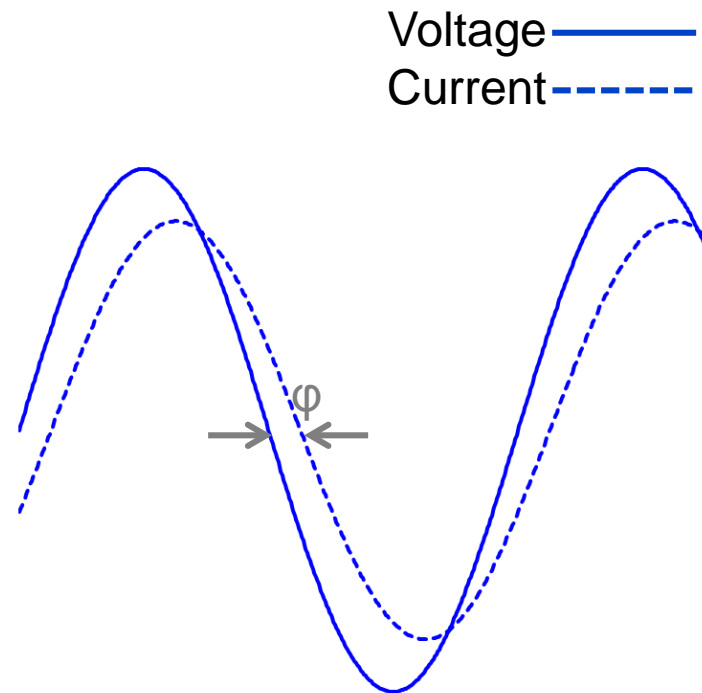


Inductive load



# Phase Angle for Two Sinusoidal Waveforms

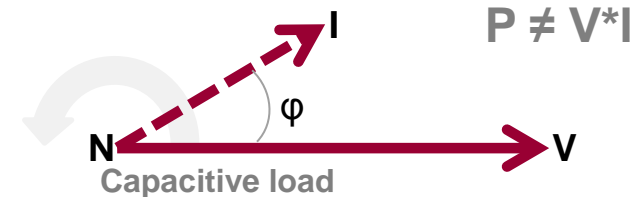
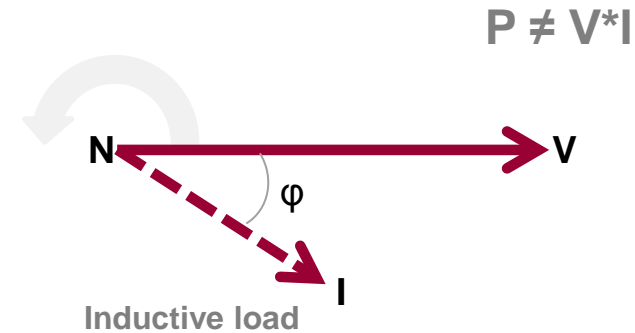
- Phase Angle ( $\phi$ ) can be directly measured between two pure voltage and current sinewaves
- $\phi$  would be 0 for a purely resistive load
  - Therefore,  $\cos(\phi) = 1$
  - In this case, Power in Watts is simply  $V \cdot I$



# “AC” Power – The Basics

## Single-phase, non-resistive loads

- For capacitive and inductive loads
  - $P \neq V \cdot I$  since voltage and current are not in phase
- For inductive loads
  - The current vector “lags” the voltage vector angle  $\phi$
  - Purely inductive load has angle  $\phi = 90^\circ$
- Capacitive Loads
  - The current vector “leads” the voltage vector by angle  $\phi$
  - Purely capacitive load has angle  $\phi = 90^\circ$



# “AC” Power – The Basics

## Introducing Single-phase Real, Apparent and Reactive Power

### ■ Apparent Power

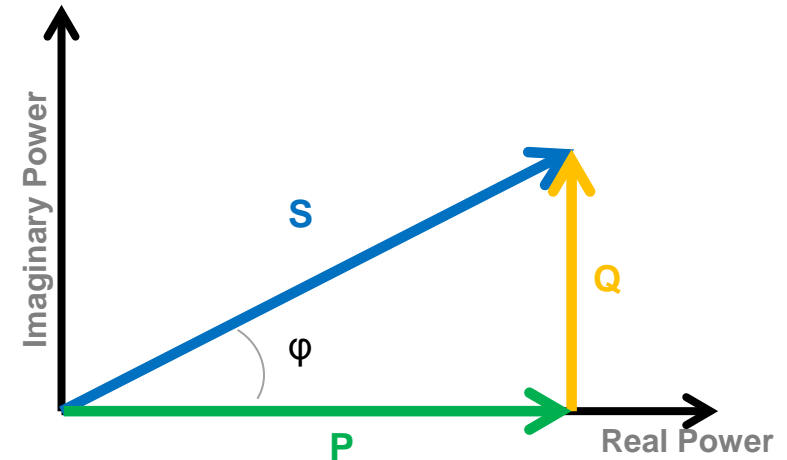
- $|S|$ , in Volt-Amperes, or VA
- $= V_{\text{RMS}} * I_{\text{RMS}}$  for a given power cycle
- Assumes that we can measure “True RMS” values for a given sinusoid

### ■ Real Power

- $P$ , in Watts
- $= |S| * \cos(\varphi)$
- Assumes that we can measure phase angle between two sinusoidal waveforms

### ■ Reactive Power

- $Q$ , in Volt-Amperes reactive, or VAR
- $Q = \sqrt{(S^2 - P^2)}$
- Does not “transfer” to load during a power cycle, just “moves around” in the circuit





# Single-Phase “AC Line” Voltage and Current

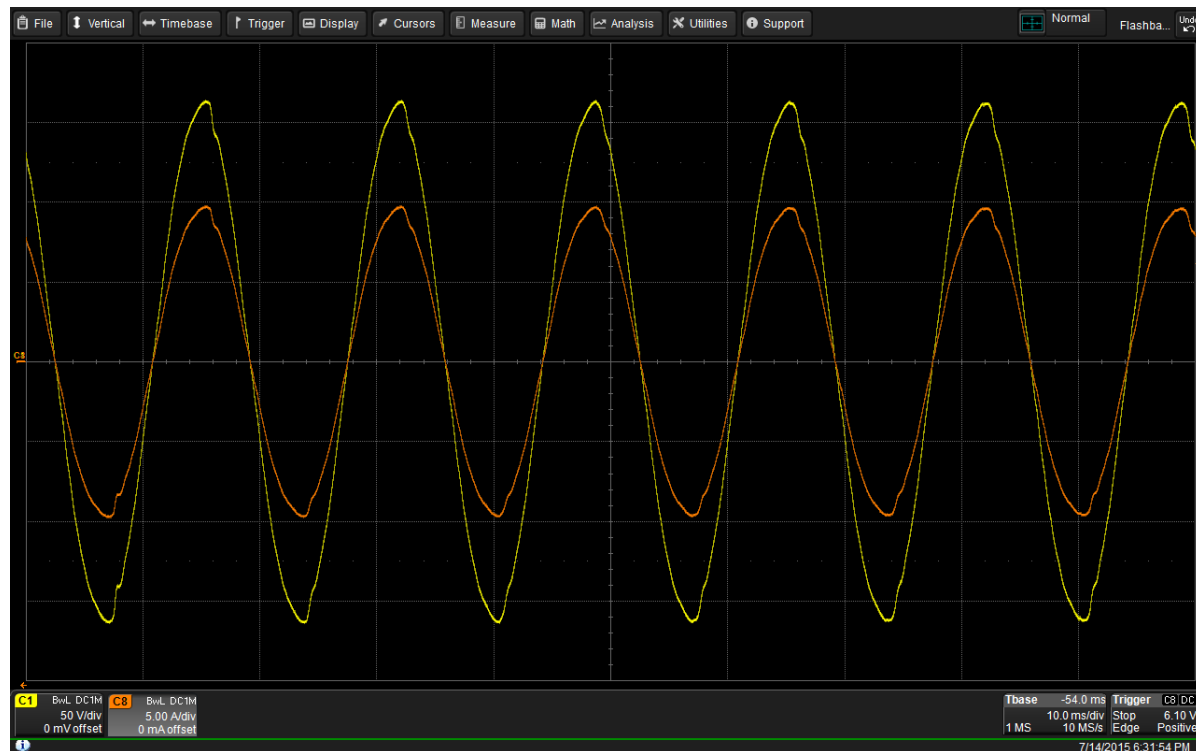
*Supplying a linear load – we’ll call this linear load a “toaster”*

- Acquired with a Teledyne LeCroy HDO8000

- 8ch, 12-bit, 1 GHz



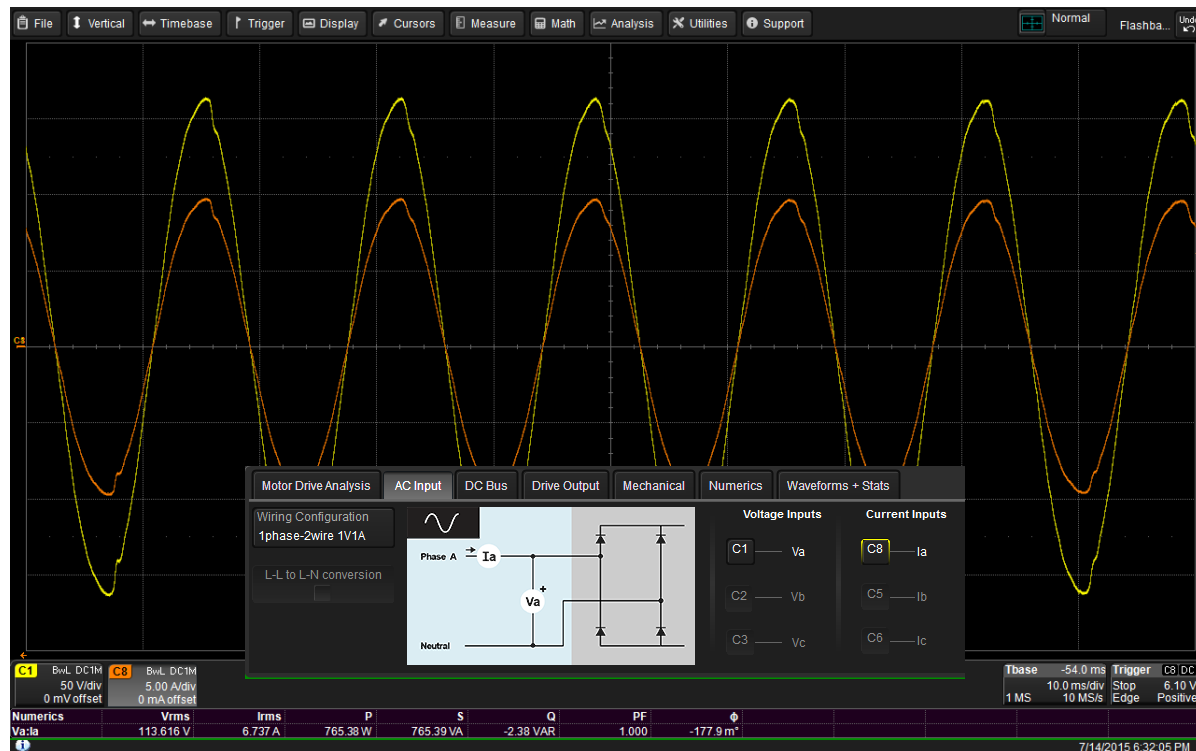
- Note that there is no measurable difference in phase between voltage (C1) and current (C8) signals



# Single-Phase “AC Line” Voltage and Current

*Supplying a linear load – we’ll call this linear load a “toaster”*

- Motor Drive Analyzer Model (based on HDO8000 platform) calculates various power parameters
  - Note the 1.000 Power Factor
  - Negligible Phase Angle ( $\phi$ ), but not non-zero
    - Due to small skew in probes that was not deskewed
- Note that these “AC Line” waveforms supplying a linear load have distortion on them

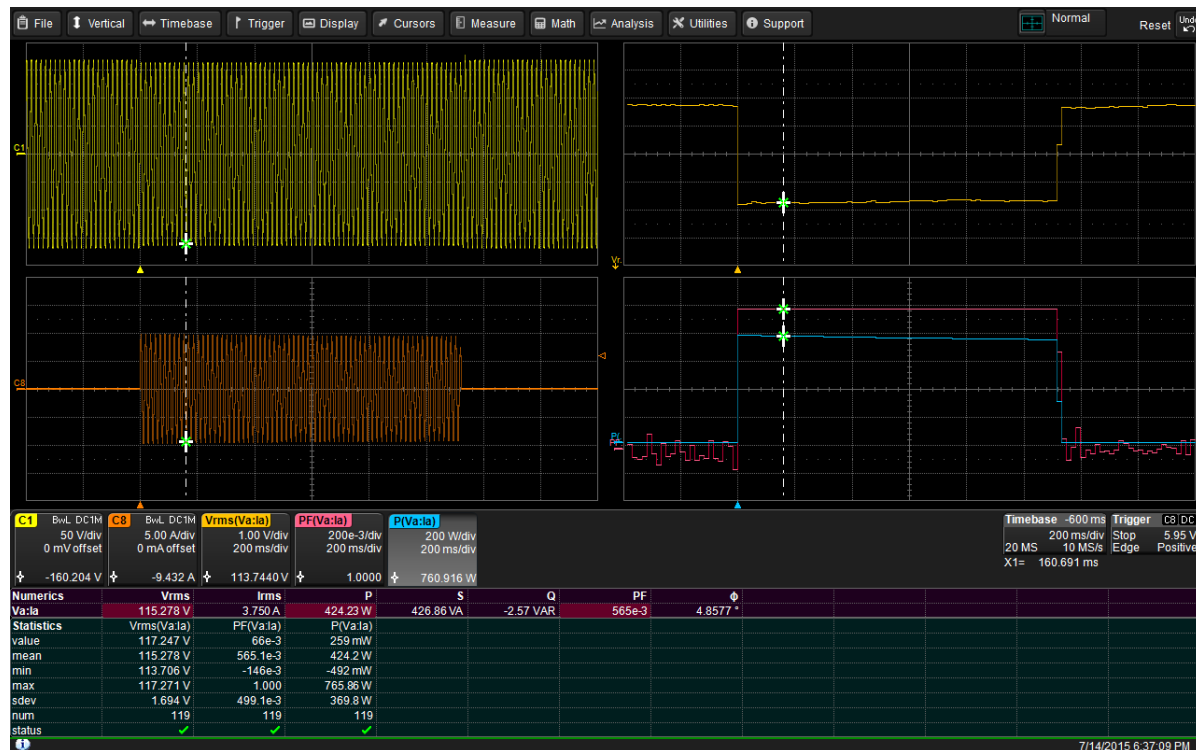


Numerics	Vrms	Irms	P	S	Q	PF	$\phi$
Va:Ia	113.616 V	6.737 A	765.38 W	765.39 VA	-2.38 VAR	1.000	-177.9 m°

# Single-Phase “AC Line” Voltage and Current

*Supplying a linear load – we’ll call this linear load a “toaster”*

- Motor Drive Analyzer Model (based on HDO8000 platform) calculates various power parameters
  - Note the 1.000 Power Factor
  - Negligible Phase Angle ( $\phi$ ), but not non-zero
    - Due to small skew in probes that was not deskewed
- Note that these “AC Line” waveforms supplying a linear load have distortion on them



# Conclusions – AC Line Voltage, Current, Power

- The AC line voltage ratings are very different from the typical voltages many engineers are familiar with
- Three-phase systems are a complex mix of magnitude, phase, and rotation and introduce additional measurement challenges
- Current phase lag/lead compared to voltage phase results in more than one type of “power” calculation (Real, Apparent, Reactive)
- Line “power” cannot be thought of as a simple  $V \cdot I$  calculation

# Distorted Waveform (e.g. PWM) Power Calculations


Distorted voltage and current waveforms are comprised of multiple frequencies, and the simple techniques that are used to measure power for pure single-frequency sinusoidal signals cannot be used for these waveforms



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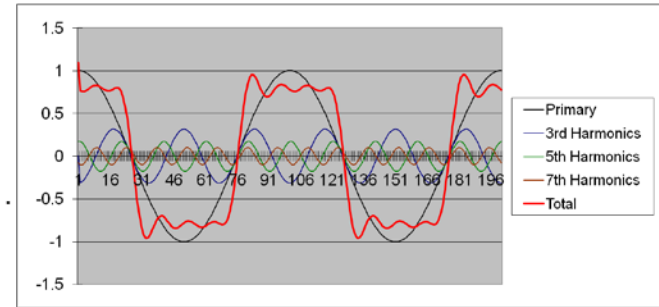
# “Distorted” Waveforms are Complex Sums of Sinusoids

- Any “distorted” (e.g. PWM) waveform is composed of different amplitudes of odd integer sinewave harmonics (“orders”)



Square Wave Harmonics

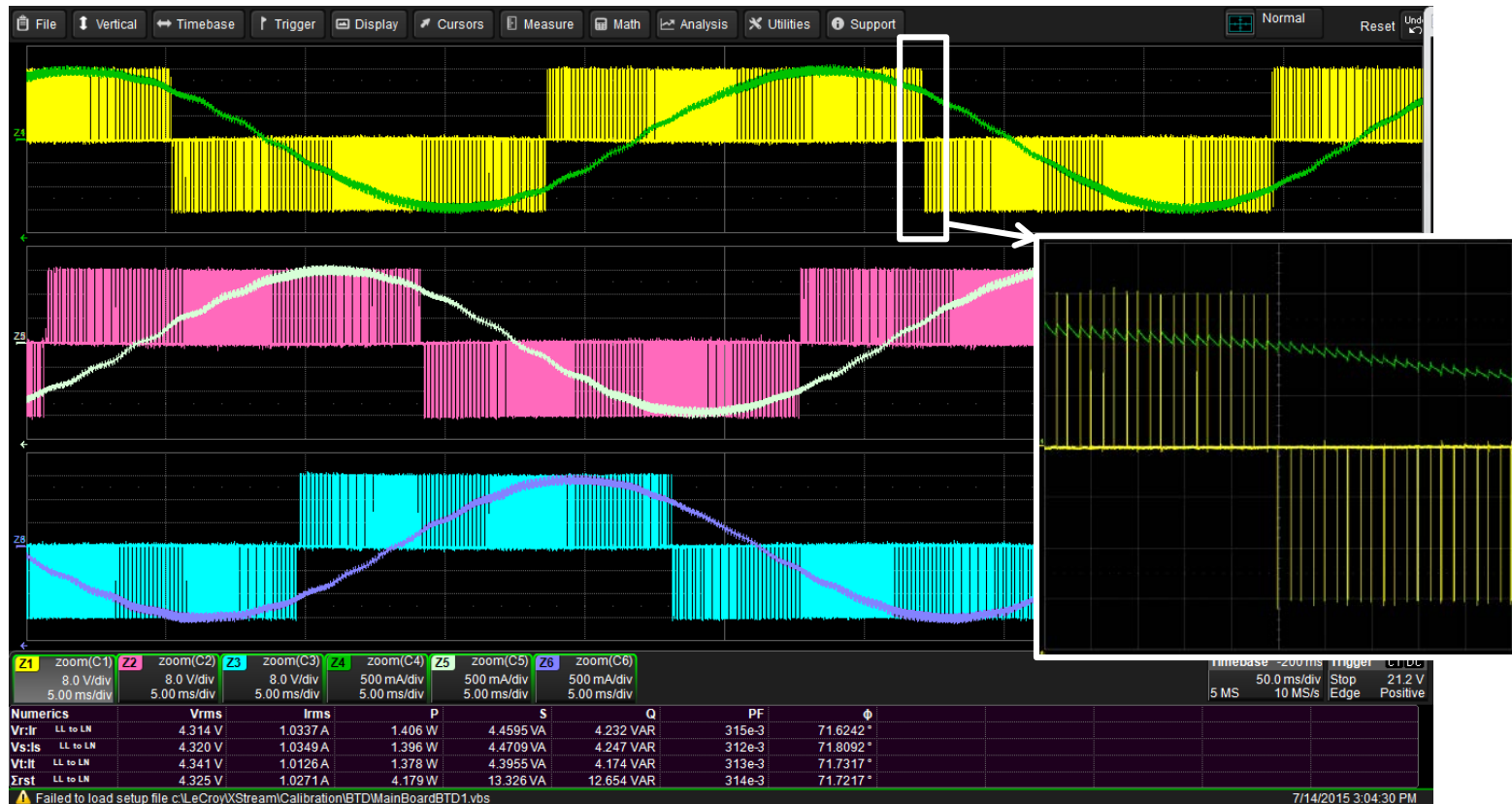
$$x_{\text{square}}(t) = \frac{4}{\pi} \sum_{k=1}^{\infty} \frac{\sin((2k-1)2\pi ft)}{(2k-1)}$$
$$= \frac{4}{\pi} \left( \sin(2\pi ft) + \frac{1}{3} \sin(6\pi ft) + \frac{1}{5} \sin(10\pi ft) + \dots \right)$$



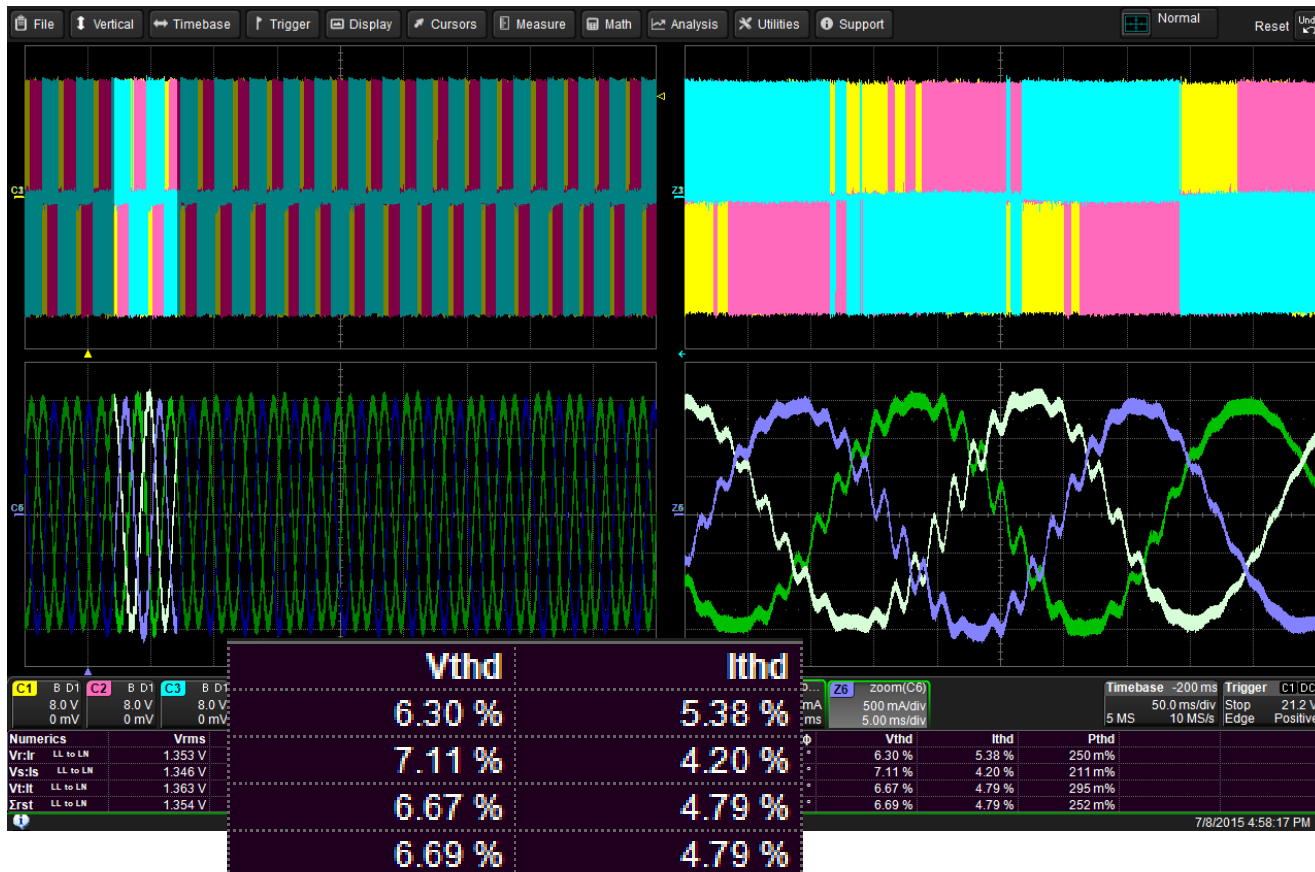
- The voltage and current waveforms will have different magnitude contributions from different harmonic orders
- The phase relationships between voltage and current waveforms for different harmonic orders is not a constant
- Therefore, there is no practical method to measure phase angle between a voltage and current signal to calculate real power from apparent power

# Example – PMSM Three-Phase Voltage and Current

*What appears to be a sinusoidal AC current even has a “sawtooth” shape...*

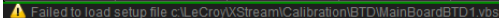


# During Overload Conditions, Distortion Can Increase Greatly





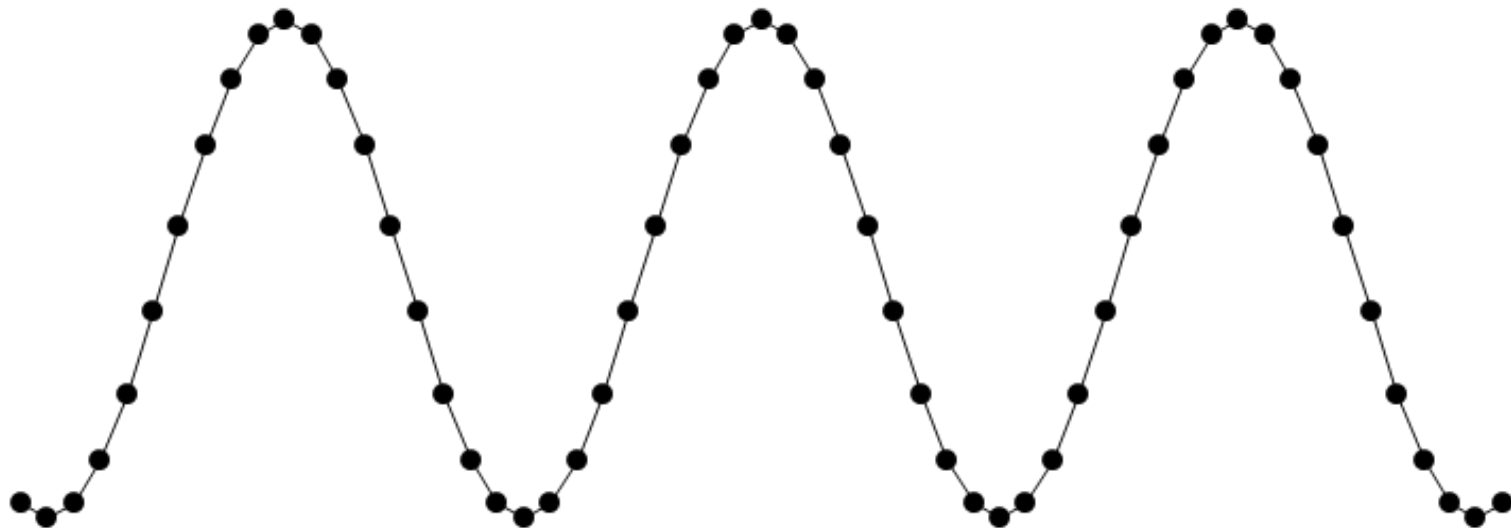
*These have even more inherent distortion than PMSM waveforms...*



# Digital Sampling Technique for Power Calculations

*Required for distorted waveforms, but also works for sinusoidal waveforms*

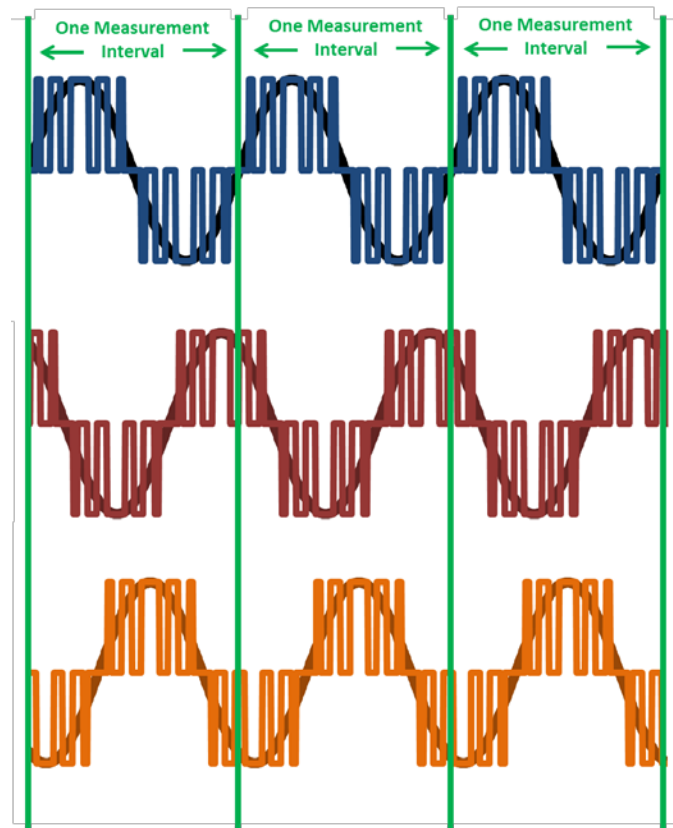
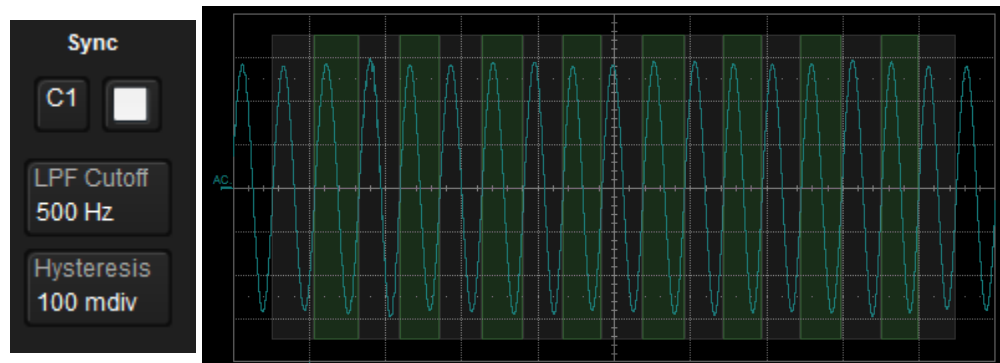
- A “digital” acquisition system samples the analog signal at a given rate (the “sample rate”)



# A Calculation Period is Determined for all Digitally Sampled Signals

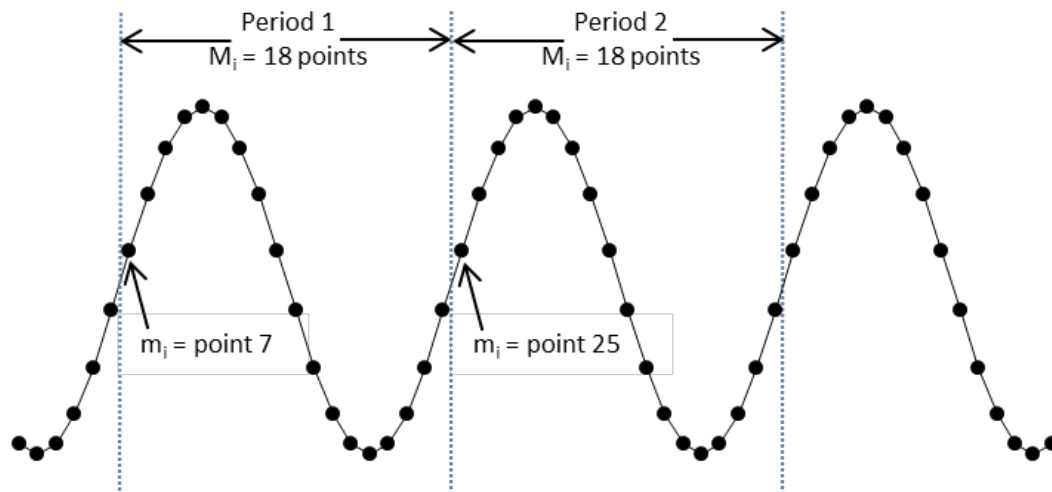
*The selected Sync signal determines the measurement period*

- An acquired digitally sampled signal is chosen to be the reference “Sync” signal
- A low pass filter (LPF) is applied to this signal
- A Hysteresis (band) value is set
- A software algorithm determines a zero-crossing point on the LPF signal, ignoring crossings that occur within the Hysteresis band



# Calculated Per-cycle Values from Digitally Sampled Data

- The digitally samples in each signal are now grouped into measurement periods (cycles), as determined by the Sync signal.
- For a given cycle index  $i$ ...
- the digitally sample voltage waveform is represented as having a set of sample points  $j$  in cycle index  $i$ ...
- For a given cycle index  $i$ , there are  $M_i$  sample points beginning at  $m_i$  and continuing through  $m_i + M_i - 1$ .
- Voltage, current, power, etc. values are calculated on each cycle index  $i$  from 1 to  $N$  cycles.



## Example

- Period 1 is cycle index  $i = 1$
- There is a set of  $j$  sample points in Period 1, beginning with point 7 and ending with point 24
- All Period 1 voltage, current and power calculations are made with this set of points
- Period 2 is cycle index  $i = 2$
- There is a set of  $j$  sample points in Period 2, beginning with point 25 and ending with point 42
- All Period 2 voltage, current and power calculations are made with this set of points
- And so on through Period  $N$

# Formulas Used for Per-cycle Digitally Sampled Calculations

*“Mean” values are calculated from the per-cycle data set*

	Per-Cycle Calculated Values	Mean Calculated Values
<b>V<sub>RMS</sub></b>	$Vrms_i = \sqrt{\frac{1}{M_i} \sum_{j=m_i}^{m_i+M_i-1} V_j^2}$	$Vrms = \frac{1}{N} \sum_{i=1}^N Vrms_i$
<b>I<sub>RMS</sub></b>	$Irms_i = \sqrt{\frac{1}{M_i} \sum_{j=m_i}^{m_i+M_i-1} I_j^2}$	$Irms = \frac{1}{N} \sum_{i=1}^N Irms_i$
<b>Real Power (P, in Watts)</b>	$P_i = \frac{1}{M_i} \sum_{j=m_i}^{m_i+M_i-1} V_j * I_j$	$P = \frac{1}{N} \sum_{i=1}^N P_i$
<b>Apparent Power (S, in VA)</b>	$S_i = Vrms_i * Irms_i$	$S = \frac{1}{N} \sum_{i=1}^N S_i$
<b>Reactive Power (Q, in VAR)</b>	$\text{magnitude } Q_i = \sqrt{S_i^2 - P_i^2}$ <p><i>sign of Q<sub>i</sub> is positive if the fundamental voltage vector leads the fundamental current vector</i></p>	$Q = \frac{1}{N} \sum_{i=1}^N Q_i$

# Formulas Used for Per-cycle Digitally Sampled Calculations

*“Mean” values are calculated from the per-cycle data set*

	Per-Cycle Calculated Values	Mean Calculated Values
<b>Power Factor (<math>\lambda</math>)</b>	$\lambda_i = \frac{P_i}{S_i}$	$\lambda = \frac{1}{N} \sum_{i=1}^N \lambda_i$
<b>Phase Angle (<math>\phi</math>)</b>	$\text{magnitude } \phi_i = \cos^{-1} \lambda_i$ <p><i>sign of <math>\phi_i</math> is positive if the fundamental voltage vector leads the fundamental current vector</i></p>	$\phi = \frac{1}{N} \sum_{i=1}^N \phi_i$

# Conclusions – Distorted Waveform Power Measurements

- Textbook descriptions of power calculations typically assume sinusoidal waveforms for single-phase systems (one voltage, one current).
- The output of a power electronics converter/inverter is a distorted waveform that requires different power calculation methodologies than most engineers are familiar with
  - There is no practical way to measure phase angle between distorted voltage and current waveforms
  - Digital sampling techniques are required
  - These digital sampling techniques also work for pure sinusoids

# Three-Phase Power Calculations

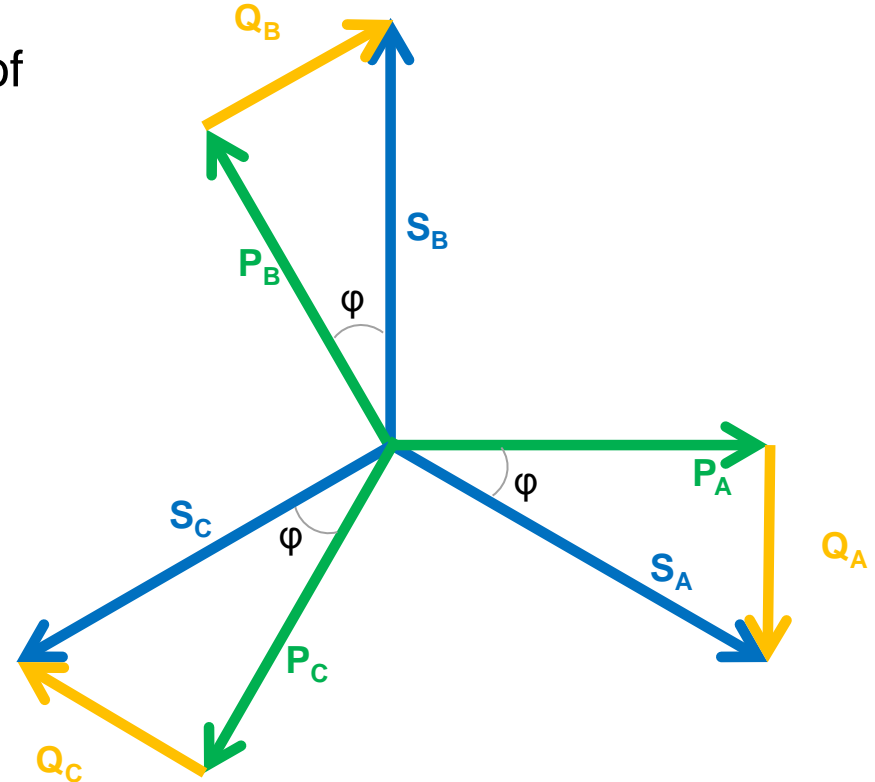


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# Three-Phase Power Calculations

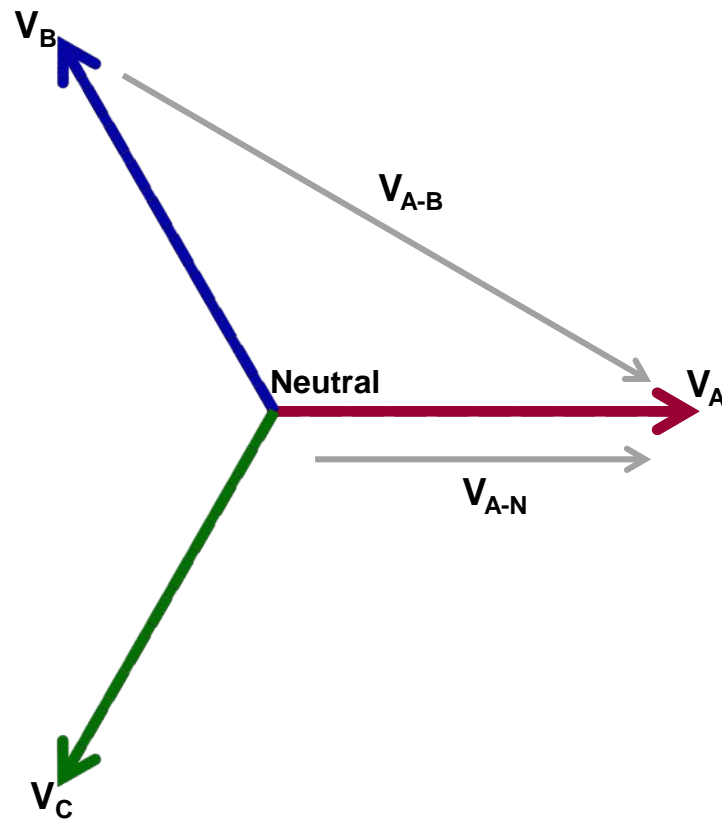
- In general, three-phase power calculations are a simple summation of the individual phase power calculations, and should be balanced across all three phases
  - $P_{\text{Total}} = P_A + P_B + P_C$
  - $S_{\text{Total}} = S_A + S_B + S_C$
  - $Q_{\text{Total}} = Q_A + Q_B + Q_C$
- However, there are exceptions:
  - Line-Line voltage measurement
    - Voltages and currents out-of-phase
  - Delta windings
    - Terminal currents vs. coil currents
  - 2 wattmeter method
    - Summation, but not balance



# Three-Phase Line-Line Voltage Measurement

*Requires Magnitude and Phase Conversion to Line-Neutral Basis*

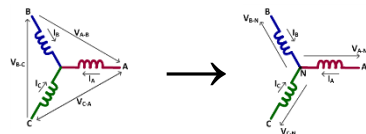
- Mathematically,  $V_{A-B}$  and  $V_{A-N}$  are related as follows:
  - Magnitude  $V_{A-B} = V_{A-N} * \sqrt{3}$
  - Phase  $V_{A-B} = V_{A-N} - 30^\circ$
- This is true for all three phases



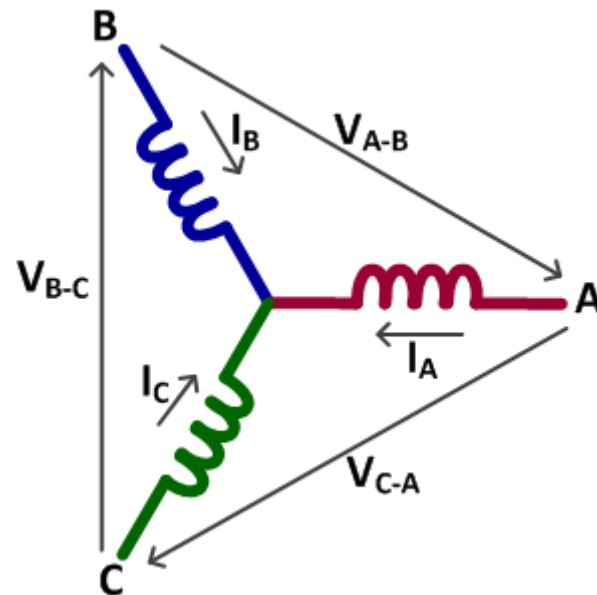
# Three-Phase Line-Line Voltage Measurement

*Requires Magnitude and Phase Conversion to Line-Neutral Basis*

- Voltage is often measured L-L
  - Neutral point may not be accessible, or
  - Customer may prefer L-L probing
- Current is measured L-N
- L-L voltages must be transformed to L-N reference



- Then, calculations are straightforward, as described earlier
  - $P_{\text{Total}} = P_A + P_B + P_C$
  - $S_{\text{Total}} = S_A + S_B + S_C$
  - $Q_{\text{Total}} = Q_A + Q_B + Q_C$

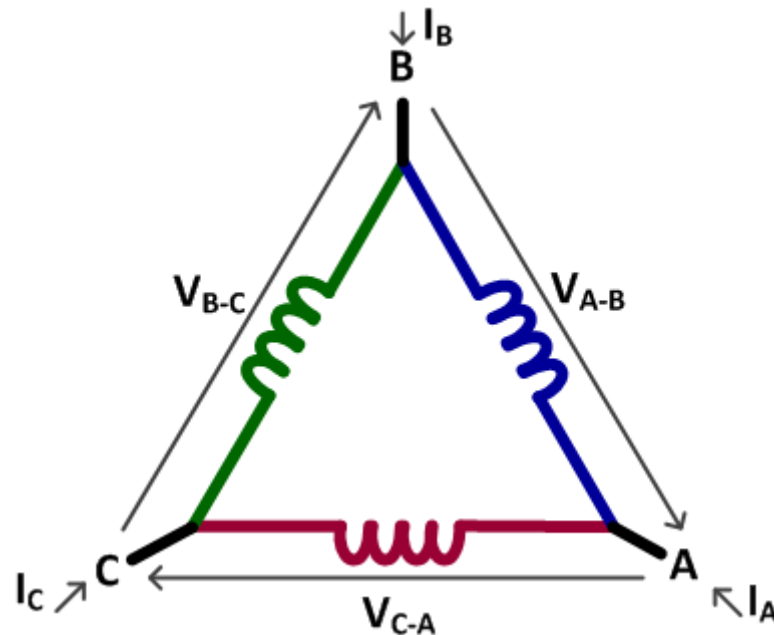


# Delta Windings

## Line-Line Voltage Sensing – 3 Voltages and 3 Currents with a Delta ( $\Delta$ ) Winding

- Voltage is measured L-L
  - Neutral point is not accessible
- Current measured is terminal current
  - These currents are not the “coil” currents
- Therefore, power balance across all three-phase may not be achieved
- However, total three-phase power is still calculated as described earlier

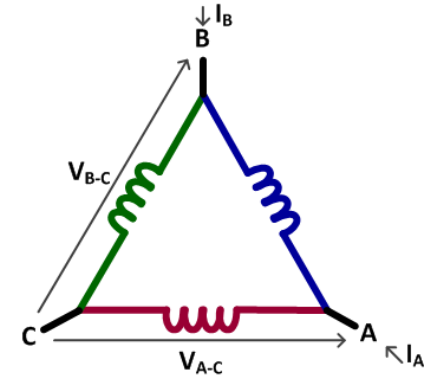
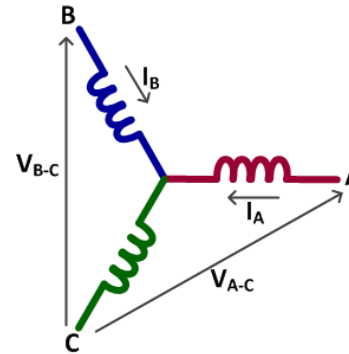
- $P_{\text{Total}} = P_A + P_B + P_C$
- $S_{\text{Total}} = S_A + S_B + S_C$
- $Q_{\text{Total}} = Q_A + Q_B + Q_C$



# Two Wattmeter Three-Phase Power Measurements

## Line-Line Voltage Sensing – 2 Voltages, 2 Currents with Wye (Y or Star) or Delta ( $\Delta$ ) Winding

- This is referred to as the “2 Wattmeter Method”
- Voltage is measured L-L on two phases
  - Note that the both voltages are measured with reference to C phase
- Current is measured on two phases
  - The two that flow into the C phase
- Mathematical assumptions:
  - $\Sigma(I_A + I_B + I_C) = 0$
  - $\Sigma(V_{A-B} + V_{B-C} + V_{C-A}) = 0$
- This is a widely used and mathematically valid method for a balanced three-phase system

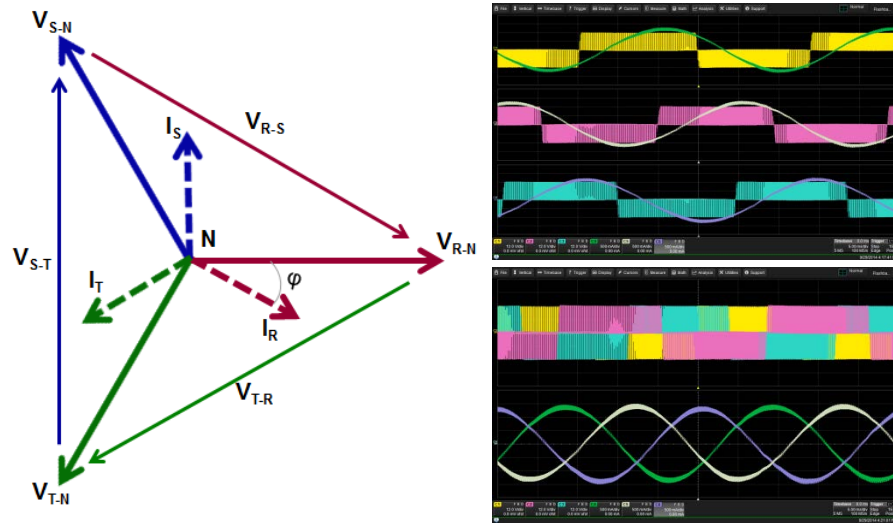


# Two Wattmeter Voltage and Current Associations

## Comparison of Teledyne LeCroy with Yokogawa Power Analyzers

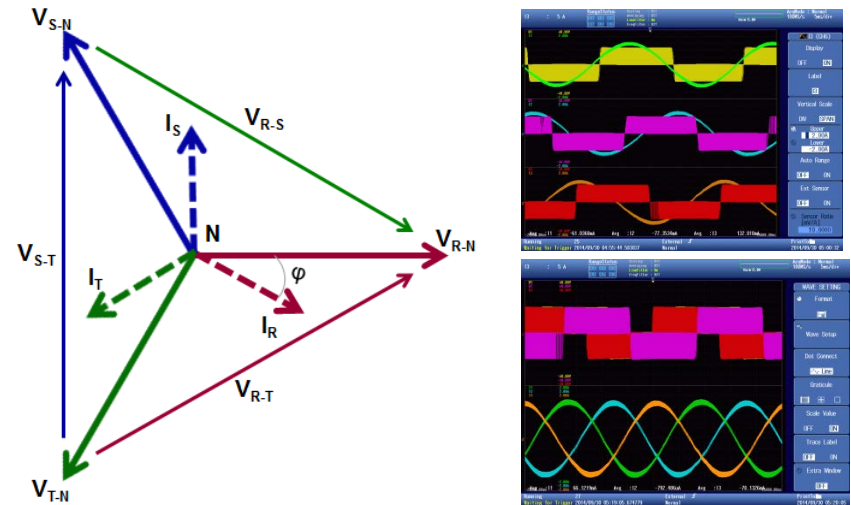
### Teledyne LeCroy

- Benefit - Visually, each L-L voltage capture will display 120° out of phase, as a user would expect
- Drawback – requires a re-connection when changing from a 3 wattmeter to a 2 wattmeter method



### Yokogawa

- Drawback - Visually, the L-L voltage captures will not be 120° out of phase, and this could cause confusion when viewing the waveforms
- Benefit – no re-connection required when changing to a 2 wattmeter method (2V2A)



# Measurement Example



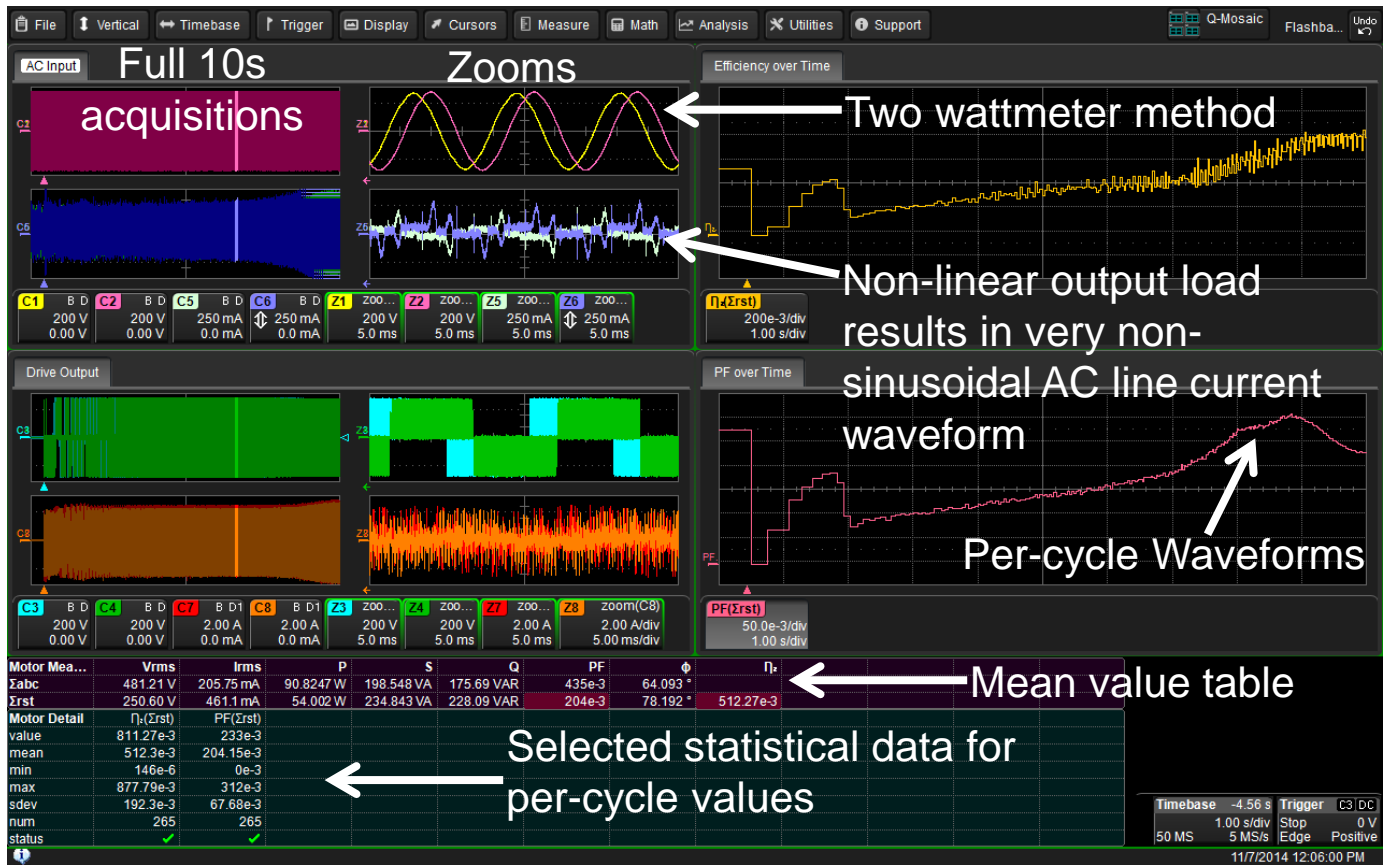
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# Three-Phase Power Measurements

480V drive, AC Input, Drive Output, Power + Other calculations displayed over time

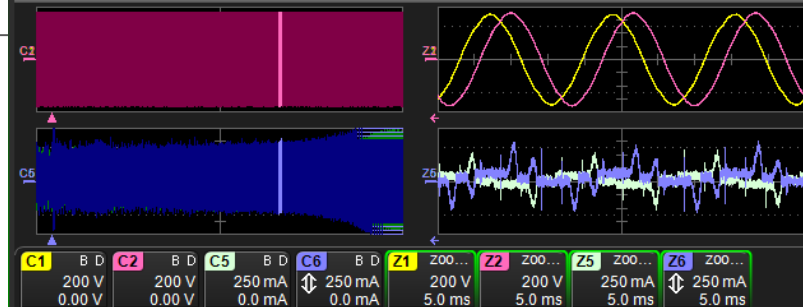
AC Line  
Input

Drive  
PWM  
Output

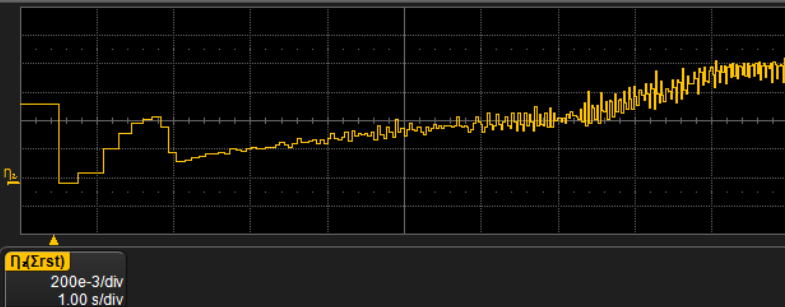




# AC Input



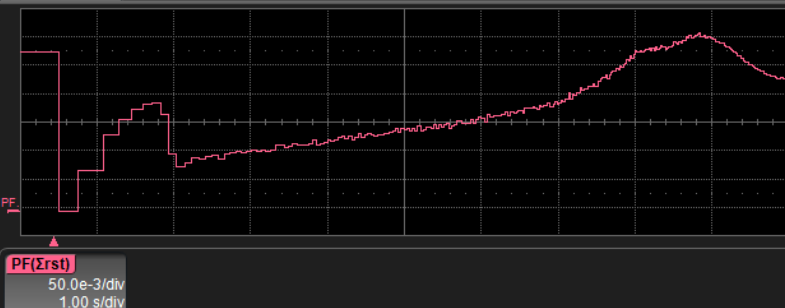
# Efficiency over Time



# Drive Output



# PF over Time



Motor Mea...	Vrms	Irms	P	S	Q	PF	$\phi$	$\eta$
$\Sigma abc$	481.21 V	205.75 mA	90.8247 W	198.548 VA	175.69 VAR	435e-3	64.093 °	
$\Sigma rst$	250.60 V	461.1 mA	54.002 W	234.843 VA	228.09 VAR	204e-3	78.192 °	512.27e-3
Motor Detail	$\eta_r(\Sigma rst)$	$PF(\Sigma rst)$						
value	811.27e-3	233e-3						
mean	512.3e-3	204.15e-3						
min	146e-6	0e-3						
max	877.79e-3	312e-3						
sdev	192.3e-3	67.68e-3						
num	265	265						
status	✓	✓						

Timebase -4.56 s Trigger C3 DC  
 1.00 s/div Stop 0 V  
 50 MS 5 MS/s Edge Positive  
 11/7/2014 12:06:00 PM

Questions?



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