An Improved Design Procedure for Small Arrays of Shunt Slots

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Abstract—**An earlier design procedure, valid for arrays of longitudinal slots fed by *air-filled* rectangular waveguide, is generalized to apply to the increasingly used practical case that the waveguide is *dielectric-filled*. This requires abandonment of the model of an equivalent array of loaded dipoles, which results in a surprisingly simpler and more direct derivation of the design equations. The new generalized procedure retains an important feature of the earlier approach in that it includes the effect of external mutual coupling.**

INTRODUCTION

SEVERAL YEARS AGO a procedure was developed which permits the design of linear and planar arrays of longitudinal (shunt) slots fed by air-filled rectangular waveguide [1]. That procedure accounts for external mutual coupling and is particularly useful in the design of small arrays, where the mutual coupling is radically different for peripheral and central slots. It uses the artifice of an equivalent array of loaded dipoles to arrive at one of the principal design equations. This necessitates the assumption that the field distribution in the slot is essentially the same as the current distribution on the equivalent center-fed dipole.

Method of moments techniques have been applied to the problem of determining the electric field distribution in a shunt slot [2]. Despite the fact that the slot is not fed at its center, but rather by transverse electric (TE_{10}) modes which pass under the slot, the results of such studies show that, near resonance, the field distribution is approximately an equiphase *half-cosinusoid*. The phase and the amplitude level, relative to the corresponding quantities for the incident TE_{10} mode, are governed by the slot length, but the amplitude distribution is almost insensitive to length. This behavior is found to occur for a wide range of permittivities of the dielectric which fill the waveguide.

The implication of this finding is that if the waveguide is air-filled, so that resonance occurs at approximately half a free space wavelength, then the electric field distribution in the slot is not much different from the current distribution on a strip dipole, the latter assumed to be placed in free space, with the common length of the slot and its equivalent dipole assumed to fall in a narrow range around the first resonance. This assumption was made in the earlier theory, which invoked an equivalent array of loaded dipoles to model the slot array. However, if the waveguide is filled with a material whose dielectric constant differs sensibly from unity, this assumption is no longer valid, and the equivalent array of loaded dipoles must be abandoned. It is the intent of this paper to show how that can be done while still retaining the key feature of the original design procedure that external mutual coupling be taken into account.

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THE SLOT FIELD DISTRIBUTION

If (x', z') is any point in the aperture of a longitudinal slot, and if the slot is narrow, an excellent approximation to the electric field distribution in the slot can be obtained by matching internal and external longitudinal magnetic fields via the equation

$$H_z^{\mathrm{ext}}(x', z') = H_z^{\mathrm{inc}}(x', z') + H_z^{\mathrm{scat}}(x', z') \tag{1}$$

where the superscripts stand for external, incident, and scattered, respectively. If $H_z^{\mathrm{inc}}(x', z')$ is taken to be a component of an incident TE_{10} mode, thus becoming the known driving function, and if $H_z^{\mathrm{ext}}(x', z')$ and $H_z^{\mathrm{scat}}(x', z')$ are expressed in terms of integrals involving the electric field distribution in the slot, (1) becomes an integral equation in the sought for $E_x(x', z')$. When the slot is partitioned into cells, the method of moments can be employed, using pulse functions and point matching, to deliver the average value of E_x in each cell via matrix inversion.

The various curves in Fig. 1 show typical results of such an exercise, and give the magnitude and phase of $E_x(x', z')$ for an isolated slot which is offset 0.120 in in standard, air-filled X-band waveguide (a = 0.900 in, b = 0.400 in). The frequency used was 9.375 GHz, and the distributions for three slot lengths are displayed. One can see that the amplitude distribution is almost symmetrical for all three lengths, being greatest at resonance, and that the phase distribution is most nearly uniform at resonance, but is nearly so even at lengths 5 percent above and below resonance.

If all three amplitude distributions are normalized to unity, the result is as shown in Fig. 2. One can see that the *shape* of the amplitude distribution is fairly constant over this range of slot lengths; only the level is affected by length. Plotted for comparison is a half-cosinusoid, indicating that the actual field distribution is somewhat broader, but not dramatically so.

Similar results have been obtained for other slot offsets, for other waveguide sizes and frequencies, and for other dielectrics than air filling the guide. All of this prompts the suggestion that a good approximation to the electric field distribution in a longitudinal slot whose length $2l$ is within 5 percent of resonance is

$$E_x(x', z') = \frac{V^s}{w} \cos \frac{\pi z'}{2l} \tag{2}$$

in which $2l$ is the slot length, w is the slot width, and V^s is the slot voltage, measured across the slot at its center. In (2), the Z' origin has been taken in the waveguide cross section which bisects the slot. Equation (2) can be contrasted to the assumption in the earlier theory

$$E_x = \frac{V^s}{w} \sin\left[k_0(l - |z'|)\right] \tag{3}$$

wherein $k_0 = 2\pi/\lambda_0$ is the free space wavenumber. For air-filled guide, and for slots near resonance, the two equations are not appreciably dissimilar. But for a dielectric-filled guide, (2) is clearly

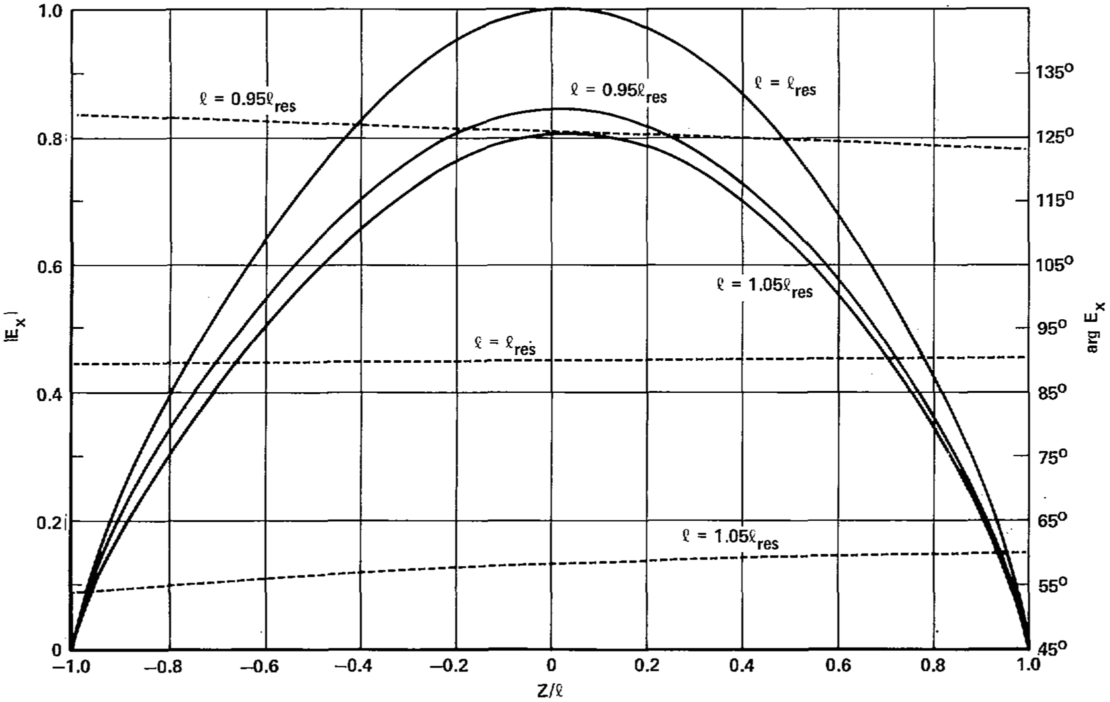

Fig. 1. Electric field distribution versus length for a longitudinal shunt slot. Solid lines, magnitude. Dashed lines, phase.

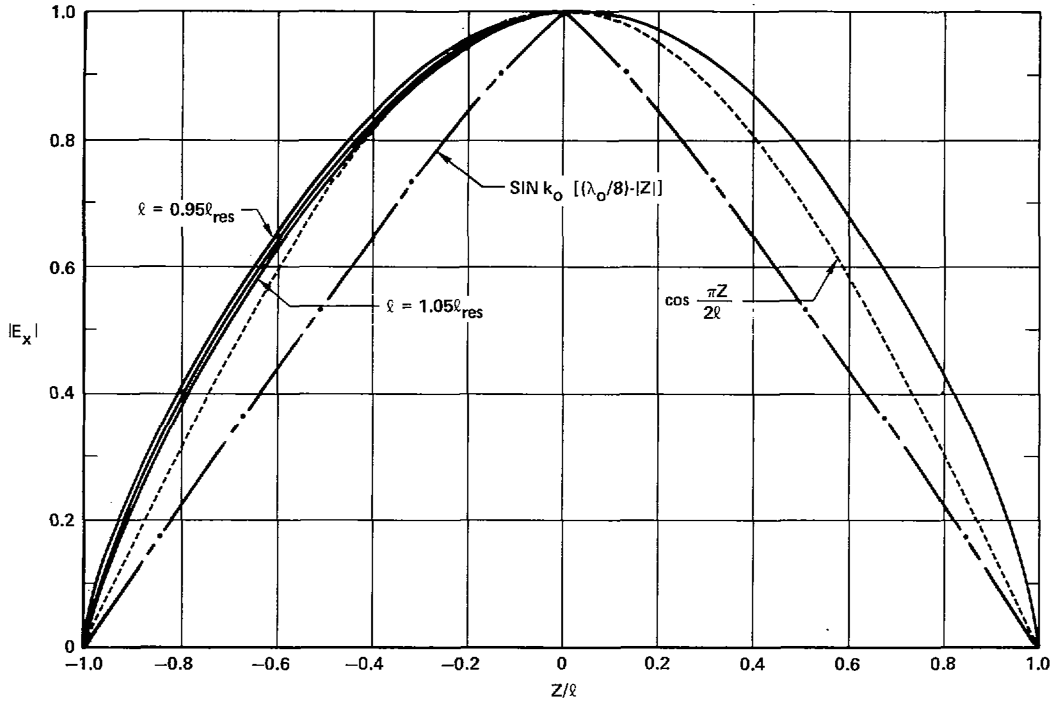

Fig. 2. Normalized electric field magnitude versus length for a longitudinal shunt slot. Comparison to a cosinusoid.

superior. The reason for this is that the resonant length of a shunt slot is significantly affected by the permittivity of the dielectric filling the guide. As an illustration of the error that can occur in using (3) for other than air-filled guide, Fig. 2 also contains a normalized plot of (3) for the case of a dielectric of high enough permittivity to cause the resonant length to be $\lambda_0/4$.

In what follows, (2) will be adopted as being a more realistic approximation than (3) to the true field distribution in the slot under all circumstances. Consistent with the features noted in Figs. 1 and 2, V^s will be taken to be a complex amplitude, with its magnitude and phase dependent on the offset and length of the slot.

THE FIRST DESIGN EQUATION

General relations connecting the backscattered mode amplitude B_{mn} and the forward-scattered mode amplitude C_{mn} to the slot voltage V^s are well known [3]. For a shunt slot, and the symmetric E_x slot field distribution assumed in (2), the TE_{10} mode scattering is symmetric and given by

$$B_{10} = C_{10} = \frac{\int_{\text{slot}} (\mathbf{E}_1 \times \mathbf{H}_2) \cdot dS}{2 \int_{S_1} (\mathbf{E}_{10,t} \times \mathbf{H}_{10,t}) \cdot \mathbf{1}_z \, dS_1} \tag{4}$$

in which $\mathbf{E}_1$ is found in (2) and

$$\mathbf{H}_2 = \mathbf{1}_z j \cos\left(\frac{\pi x'}{a}\right) e^{-j\beta_{10} z'}$$

$$\mathbf{E}_{10,t} = \mathbf{1}_y \frac{\omega\mu_0}{(\pi/a)} \sin\left(\frac{\pi x'}{a}\right) \tag{5}$$

$$\mathbf{H}_{10,t} = \mathbf{1}_x \frac{\beta_{10}}{(\pi/a)} \sin\left(\frac{\pi x'}{a}\right).$$

Reduction of (4) gives

$$B_{10} = \frac{(\pi/a)^2 V^s}{j\omega\mu_0 w \beta_{10} ab} \int_{-l}^{l} \cos\left(\frac{\pi z'}{2l}\right) e^{-j\beta_{10} z'} \, dz' \cdot \int_{\frac{a}{2}+x-\frac{w}{2}}^{\frac{a}{2}+x+\frac{w}{2}} \cos\frac{\pi x'}{a} \, dx'$$

wherein x is the offset of the slot from the centerline of the guide. For w small, this reduces further to

$$B_{10} = -\frac{(\pi/a)^2 V^s}{j\omega\mu_0 \beta_{10} ab} \sin\left(\frac{\pi x}{a}\right) \int_{-l}^{l} \cos\left(\frac{\pi z'}{2l}\right) \cdot \cos(\beta_{10} z') \, dz' \tag{6}$$

where, because of symmetry, only the even part of $e^{-j\beta_{10}z'}$ has been retained. Integration gives

$$B_{10} = -K \frac{(\pi/2kl)\cos(\beta_{10} l)}{(\pi/2kl)^2 - (\beta_{10}/k)^2} \sin(\pi x/a) V^s \tag{7}$$

in which

$$K = \frac{2(\pi/a)^2}{j\omega\mu_0(\beta_{10}/k)(ka)(kb)}, \qquad k = \omega\sqrt{\mu_0 \epsilon}. \tag{8}$$

Equation (7) can be used as a replacement for [1, eq. (6)], serving as the relation between the slot voltage and the backscattered TE_{10} mode amplitude.

To this point, the causes of the slot voltage V^s have not been identified. For the nth slot in an array of N slots, it will be assumed that the causes are TE_{10} modes passing underneath the slot in both directions, plus external coupling to the other $N-1$ slots in the array. It will be necessary to connect V^s to these causes. But before passing on to that question, it is useful to introduce the concept of an equivalent circuit for the nth slot. Because (2) embodies the assumption that the electric field distribution in the slot is symmetrical, it follows as a consequence that $B_{10} = C_{10}$. This symmetrical scattering is analogous to the scattering from a shunt element on a transmission line. Thus if one models the nth slot in terms of its active admittance Y_n^a on an equivalent transmission line of characteristic conductance G_0, the scattering from Y_n^a is given by [4]

$$B = C = -\frac{1}{2}\frac{Y_n^a}{G_0} V_n \tag{9}$$

with V_n the mode voltage at the juncture where the shunt element Y_n^a is placed.

Equations (7) and (9) can be connected by requiring that B_{10} and B have the same phase at any cross section z and that the backscattered power levels be the same in both cases. Under those conditions, the two equations combine to give

$$\boxed{\frac{Y_n^a}{G_0} = K_1 f_n \frac{V_n^s}{V_n}} \tag{10}$$

in which

$$K_1 = \frac{1}{j(a/\lambda)} \sqrt{\frac{2(k/k_0)}{\eta G_0 (\beta_{10}/k)(ka)(kb)}} \tag{11}$$

with η the impedance of free space, and with

$$f_n = \frac{(\pi/2kl_n)\cos(\beta_{10} l_n)}{(\pi/2kl_n)^2 - (\beta_{10}/k)^2} \sin\left(\frac{\pi x_n}{a}\right). \tag{12}$$

Equation (10) is a principal result of the analysis and will be called the first design equation. It can serve as a replacement for [1, eq. (7)].

THE SECOND DESIGN EQUATION

It is now appropriate to turn to the causes of the slot voltage induced in the nth slot. The first of these is a TE_{10} mode of complex amplitude A_{10} incident from the left ($z = -\infty$). If all other slots are covered with conducting tape, and if the waveguide containing the nth slot is terminated beyond the nth slot in a matched load, the equivalent transmission line circuit consists of the self admittance Y of the slot in parallel with the conductance of the transmission line G_0 The incident wave A on the transmission line causes a backscattered wave B, the two being related by

$$\frac{Y}{G_0} = -\frac{2(B/A)}{1 + (B/A)}. \tag{13}$$

This equation is equally valid if B_{10}^n/A_{10}^n is substituted for B/A. Let this be done; rearrangement gives

$$B_{10}^n = -\frac{\frac{Y}{G_0}(x_n, l_n)}{2 + \frac{Y}{G_0}(x_n, l_n)} A_{10}^n. \tag{14}$$

Equation (14) indicates that if Y/G_0 is measured for an isolated slot as a function of its offset and length, one is able from that knowledge to deduce the backscattering coefficient. Since $B_{10}^n = C_{10}^n$, the forward-scattering coefficient is also readily obtained. And if this procedure is repeated for a TE_{10} mode of complex amplitude D_{10}^n incident from the right ($z = +\infty$), one finds that (14) once again applies, with A_{10}^n replaced by D_{10}^n. Thus the scattering matrix of an isolated shunt slot, viewed as a two-port microwave device, is completely determined if $Y/G_0(x_n, l_n)$ is known.[1]

It will be convenient to consider the total slot voltage V_n^s to be composed of three parts, i.e.,

$$V_n^s = V_{n,1}^s + V_{n,2}^s + V_{n,3}^s \tag{15}$$

in which $V_{n,1}^s$ is due to A_{10}^n, $V_{n,2}^s$ is due to D_{10}^n, and $V_{n,3}^s$ is due to external coupling with the other $N-1$ slots in the array. Then, upon combining (7), (12), and (14), we find that

$$V_{n,1}^s = \frac{1}{Kf_n} \frac{\dfrac{Y}{G_0}(x_n, l_n)}{2 + \dfrac{Y}{G_0}(x_n, l_n)} A_{10}^n. \tag{16}$$

Similarly,

$$V_{n,2}^s = \frac{1}{Kf_n} \frac{\dfrac{Y}{G_0}(x_n, l_n)}{2 + \dfrac{Y}{G_0}(x_n, l_n)} D_{10}^n. \tag{17}$$

The remaining partial slot voltage $V_{n,3}^s$ can be deduced with the aid of the reciprocity theorem, given in the form [5]

$$\int_V (\mathbf{E}^b \cdot \mathbf{J}^a - \mu_0 \mathbf{H}^b \cdot \mathbf{J}_m^a)\, dV = \int_V (\mathbf{E}^a \cdot \mathbf{J}^b - \mu_0 \mathbf{H}^a \cdot \mathbf{J}_m^b)\, dV. \tag{18}$$

To see how this can be accomplished, consider the situation shown in Fig. 3. The nth slot is seen to have a displacement x_n from the centerline of its waveguide and a length $2l_n$. Its central point is at the cross section z_n. In the development to follow, the nth slot will be the *only* slot cut in any of the waveguides, all of which are assumed to extend to $\pm\infty$. The position and dimensions that the mth slot would have in the actual array are suggested by the dotted lines in Fig. 3. (In that figure the mth slot is shown as though it were in a different waveguide than the nth slot, but the analysis also applies if they are in the same waveguide). The offset of the virtual mth slot is seen to be x_m from the centerline of its waveguide, its length is $2l_m$, and its central point lies in the cross section z_m. The purpose for the phantom appearance of the mth slot will now be explained.

In the a situation, let a TE_{10} mode of amplitude A^a be incident on the nth slot from $z = -\infty$. A slot voltage $V_n^{s,a}$ develops at the center of the nth slot, and TE_{10} waves B^a, C^a are scattered by the slot. The TE_{10} fields in the $z < z_n - l_n$ are

$$H_z^a = j \cos \frac{\pi x''}{a} \left[A^a e^{-j\beta_{10}(z-z_n)} + B^a e^{j\beta_{10}(z-z_n)}\right]$$

[1] An alternative to measuring Y/G_0 versus offset and length is to deduce it theoretically, using a technique such as the method of moments.

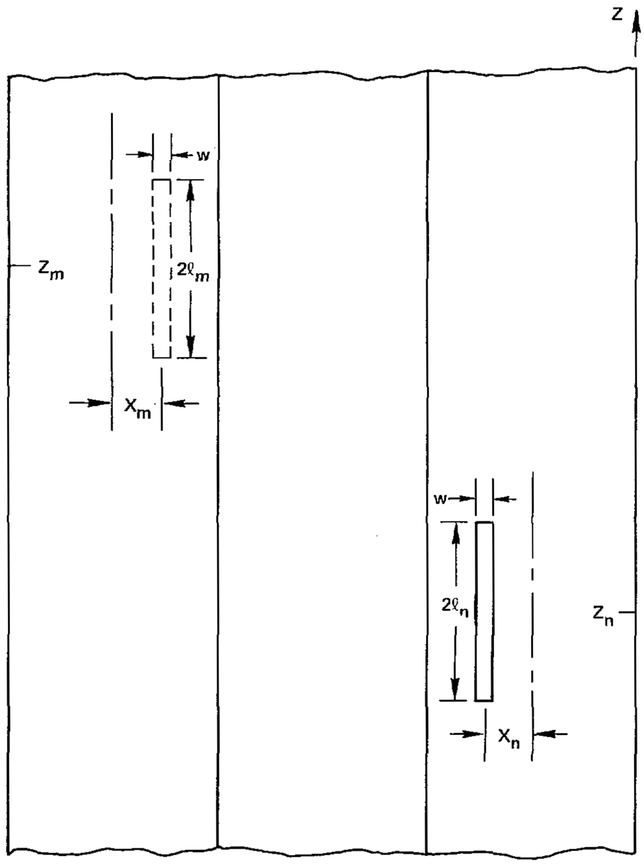

Fig. 3. Positions of two slots in an array.

$$H_x^a = \frac{\beta_{10}}{(\pi/a)} \sin \frac{\pi x''}{a} \left[-A^a e^{-j\beta_{10}(z-z_n)} + B^a e^{j\beta_{10}(z-z_n)}\right]$$

$$E_y^a = \frac{\omega\mu_0}{(\pi/a)} \sin \frac{\pi x''}{a} \left[A^a e^{-j\beta_{10}(z-z_n)} + B^a e^{j\beta_{10}(z-z_n)}\right] \tag{19}$$

in which x'' is measured from the sidewall. We shall place equivalent sources

$$\mathbf{K}^a = \mathbf{1}_z \mathbf{x} \mathbf{H}^a \qquad \mathbf{K}_m^a = -\mu_0^{-1} \mathbf{1}_z \mathbf{x} \mathbf{E}^a \tag{20}$$

in the cross section $z = z_1$, with $z_1 < z_n - l_n$. This will erase all the a fields and sources in $z < z_1$ without affecting either the fields in $z > z_1$, or those in the half-space outside the slot. These equivalent sources are given by

$$\mathbf{K}^a = \mathbf{1}_y \frac{\beta_{10}}{(\pi/a)} \sin \frac{\pi x''}{a} \left[-A^a e^{-j\beta_{10}(z_1-z_n)} + B^a e^{j\beta_{10}(z_1-z_n)}\right] \tag{21}$$

$$\mathbf{K}_m^a = \mathbf{1}_x \frac{\omega}{(\pi/a)} \sin \frac{\pi x''}{a} \left[A^a e^{-j\beta_{10}(z_1-z_n)} + B^a e^{j\beta_{10}(z_1-z_n)}\right].$$

All the other a sources are electric-type and flow in the waveguide walls in $z > z_1$ and in the ground plane.[2] Since $\mathbf{E}_{\text{tang}}^b$ will be zero at all such points, regardless of the choice of the b-sources, a glance at (18) indicates that these other a sources need not be identified.

[2] The upper broad wall, in which the nth slot is cut, is assumed to be imbedded in an infinite, perfectly conducting ground plane. The waveguide is infinitely long in $z > z_n + l_n$ and unobstructed. There are no other slots cut in any of the waveguides.

In the b-situation, once again only the nth slot will be present, but there will be no sources at $z = -\infty$ to cause an incident TE_{10} wave. Instead, the sources will be magnetic current sheets placed in the outer half space, skin tight against the ground plane, lying in areas whose projections on the ground plane correspond to the regions occupied by each of the other slots in the actual array. These magnetic sources will be chosen so as to contribute to the external field precisely as would the actual electric fields in the actual slots. They will excite the nth slot externally, the result being that a TE_{10} wave of amplitude B^b propagates in the $-z$ direction in the waveguide containing the nth slot. This wave passes through the cross section $z = z_1$.

When the reciprocity theorem (18) is applied to this pair of situations, we find that

$$\int_{S_1} (\mathbf{E}^b \cdot \mathbf{K}^a - \mu_0 \mathbf{H}^b \cdot \mathbf{K}_m^a)\, dS_1 = -\sum_{m=1}^{N}{}' \int_{S_m} \mu_0 \mathbf{H}_{\text{ext}}^a \cdot \mathbf{K}_{m,\text{ext}}^b\, dS_m \tag{22}$$

in which S_1 is the waveguide cross section at $z = z_1$ and S_m is the surface area normally occupied by the mth slot. The prime on the summation sign means that the term $m = n$ is excluded.

The components of the TE_{10} mode of amplitude B^b are given by

$$\begin{aligned} H_z^b &= jB^b \cos \frac{\pi x''}{a}\, e^{j\beta_{10}(z-z_n)} \\ H_x^b &= \frac{\beta_{10}}{(\pi/a)} B^b \sin \frac{\pi x''}{a}\, e^{j\beta_{10}(z-z_n)} \\ E_y^b &= \frac{\omega\mu_0}{(\pi/a)} B^b \sin \frac{\pi x''}{a}\, e^{j\beta_{10}(z-z_n)}. \end{aligned} \tag{23}$$

Using (21) and (23) in the left side of (22), we find that it has the value $-[\beta_{10}\omega\mu_0 ab/(\pi/a)^2] A^a B^b$. As a result, (22) becomes

$$B^b = \frac{(\pi/a)^2}{\beta_{10}\omega ab} \sum_{m=1}^{N}{}' \int_{S_m} \frac{\mathbf{H}_{\text{ext}}^a}{A^a} \cdot \mathbf{K}_{m,\text{ext}}^b\, dS_m. \tag{24}$$

To evaluate the integrals contained in (24), it is necessary to find first an expression for $\mathbf{H}_{\text{ext}}^a$ along the ground plane. The electric field distribution in the nth slot has been assumed to be in the form (2). The fields produced in the half-space by this slot excitation are the same as those from the doubled magnetic current sheet

$$\mathbf{K}_m^a = \mathbf{1}_z 2\mu_0^{-1} \frac{V_n^{s,a}}{w} \cos \frac{\pi \zeta_n}{2l_n} \tag{25}$$

where now local coordinates (ξ_n, η_n, ζ_n) have been erected with origin at the center of the nth slot.

The electric vector potential function due to this doubled magnetic current sheet has only a z component, given by

$$\begin{aligned} F_z(\xi_n, \eta_n, \zeta_n) &= \int_{-l_n}^{l_n} \int_{-\frac{w}{2}}^{\frac{w}{2}} \frac{2\mu_0^{-1} V_n^{s,a}}{w} \cdot \cos\left(\frac{\pi\zeta_n'}{2l_n}\right) \frac{e^{-jk_0 R}}{4\pi\mu_0^{-1} R} d\xi_n'\, d\zeta_n' \\ &= \frac{V_n^{s,a}}{2\pi} \int_{-l_n}^{l_n} \cos\left(\frac{\pi\zeta_n'}{2l_n}\right) \frac{e^{-jk_0 R}}{R}\, d\zeta_n' \end{aligned} \tag{26}$$

wherein R is the distance from a point $(0, 0, \zeta_n')$ on the axis of the nth slot to an arbitrary point (ξ_n, η_n, ζ_n), measured in local coordinates.

Because $j\omega\mu_0 \mathbf{H} = \nabla \times \nabla \times \mathbf{F}$, it follows that

$$\begin{aligned} H_{z,\text{ext}}^a(\xi_n, \eta_n, \zeta_n) &= \frac{1}{j\omega\mu_0}\left(\frac{\partial^2}{\partial \zeta_n^2} + k_0^2\right) F_z(\xi_n, \eta_n, \zeta_n) \\ &= \frac{V_n^{s,a}}{2\pi j\omega\mu_0} \int_{-l_n}^{l_n} \cos\left(\frac{\pi\zeta_n'}{2l_n}\right) \left[\frac{\partial^2}{\partial\zeta_n^2} + k_0^2\right] \frac{e^{-jk_0 R}}{R}\, d\zeta_n'. \end{aligned} \tag{27}$$

Since $\mathbf{K}_{m,\text{ext}}^b = \mathbf{1}_z(\mu_0^{-1} V_m^{s,b}/w) \cos(\pi\zeta_m/2l_m)$ in terms of local coordinates at the mth slot, if we combine (24) and (27) and perform the ξ_m' integration, the result is that

$$B^b = \frac{(\pi/a)^2}{j\beta_{10}(2\pi ab)(\omega\mu_0)^2} \frac{V_n^{s,a}}{A^a} \sum_{m=1}^{N}{}' V_m^{s,b} \int_{-lm}^{lm} \cos\left(\frac{\pi\zeta_m'}{2l_m}\right) \cdot \left\{ \int_{-l_n}^{l_n} \cos\left(\frac{\pi\zeta_n'}{2l_n}\right) \left[\frac{\partial^2}{\partial\zeta_n^2} + k_0^2\right] \frac{e^{-jk_0 R}}{R}\, d\zeta_n' \right\} d\zeta_m' \tag{28}$$

where now R is the distance from a point $(0, 0, \zeta_n')$ on the centerline of the nth slot to a point $(0, 0, \zeta_m')$ on the centerline of the mth slot. Thus R depends on the offsets of the two slots, as well as their relative positions in the array.

The ratio $V_n^{s,a}/A^a$ is given by (16). Further, $V_m^{s,b}$ is the actual slot voltage in the mth slot, and we can now delete the superscript b from that quantity. Finally, B^b can be related to the slot voltage $V_{n,3}^s$ induced at the nth slot due to mutual coupling via (7) in the form $B_b = -Kf_n V_{n,3}^s$. When these changes are incorporated into (28), we find that

$$V_{n,3}^s = -j(\beta_{10}/k)(k_0 b)(a/\lambda)^3 \frac{1}{f_n^2} \frac{\frac{Y}{G_0}(x_n, l_n)}{2 + \frac{Y}{G_0}(x_n, l_n)} \cdot \sum_{m=1}^{N}{}' V_m^s g_{mn}(x_m, l_m, x_n, l_n) \tag{29}$$

in which, after two integrations by parts,

$$g_{mn} = \int_{-k_0 l_m}^{k_0 l_m} \cos\left(\frac{z_m'}{4l_m/\lambda_0}\right) \left\{ \frac{1}{(4l_n/\lambda_0)} \cdot \left[\frac{e^{-jk_0 R_1}}{k_0 R_1} + \frac{e^{-jk_0 R_2}}{k_0 R_2}\right] + \left[1 - \frac{1}{(4l_n/\lambda_0)^2}\right] \cdot \int_{-k_0 l_n}^{k_0 l_n} \cos\left(\frac{z_n'}{4l_n/\lambda_0}\right) \frac{e^{-jk_0 R}}{k_0 R} dz_n' \right\} dz_m'. \tag{30}$$

In (30), the substitute variable $z = k_0 \zeta$ has been introduced. R is the distance from the point $P_m(0, 0, \zeta_m')$, measured in local coordinates at the mth slot, to the point $P_n(0, 0, \zeta_n')$, measured in local coordinates at the mth slot. R_1 is the distance from P_m to $P_{n,1}(0, 0, l_n)$ and R_2 is the distance from P_m to $P_{n,2}(0, 0, -l_n)$.[3]

[3] For the case of air-filled waveguides feeding the slots, if $E_x = (V^s/w) \sin [k_0(l - |z|)]$ is used as an approximation to (2), g_{mn} reduces to a single integration which is an economy in computer costs. In that case, the two design equations of this paper reduce to those of [1], but with the advantage that introduction of the model of an equivalent array of loaded dipoles was not necessary.

Equation (29) is the external mutual coupling term for an array of waveguide-fed longitudinal shunt slots. It completes the development of expressions for the three partial slot voltages and opens the way to a formulation of the second design equation.

For a resonantly spaced array (slots $\lambda_g/2$ on centers in a common waveguide) we have for the nth slot

$$\sum_{i=n}^{M} \frac{Y_i^a}{G_0} = \frac{A_{10}^n - (B_{10}^n + D_{10}^n)}{A_{10}^n + (B_{10}^n + D_{10}^n)},$$

$$\sum_{i=n+1}^{M} \frac{Y_i^a}{G_0} = \frac{(A_{10}^n + C_{10}^n) - D_{10}^n}{(A_{10}^n + C_{10}^n) + D_{10}^n} \tag{31}$$

in which Y_i^a/G_0 is the active admittance of the ith slot, there being M slots in the waveguide which contains the nth slot. The difference of the two expressions in (31) yields

$$\frac{Y_n^a}{G_0} = \frac{A_{10}^n - (B_{10}^n + D_{10}^n)}{A_{10}^n + (B_{10}^n + D_{10}^n)} - \frac{(A_{10}^n + C_{10}^n) - D_{10}^n}{(A_{10}^n + C_{10}^n) + D_{10}^n}$$

$$= -\frac{2B_{10}^n}{A_{10}^n + D_{10}^n + B_{10}^n} \tag{32}$$

since $B_{10}^n = C_{10}^n$. If we use (7) and (15)–(17) to make substitutions in (32), rearrangement gives

$$\frac{Y_n^a}{G_0} = \frac{2f_n^2(x_n, l_n)}{\dfrac{2f_n^2(x_n, l_n)}{\dfrac{Y}{G_0}(x_n, l_n)} + j(\beta_{10}/k)(k_0 b)(a/\lambda)^3 \sum_{m=1}^{N}{}' \dfrac{V_m^s}{V_n^s} g_{mn}(x_m, l_m, x_n, l_n)}. \tag{33}$$

Equation (33) is the second design equation and can serve as a substitute for [1, eq. (8)]. Together with (10) it permits the design of shunt slot arrays in either dielectric-filled or air-filled waveguide, including external mutual coupling.

DESIGN PROCEDURE

The use of these equations is similar to that of the earlier pair found in [1]. One assumes a set of original lengths and offsets for all the slots in the array (zero offset and resonant length for each is a satisfactory initial assumption). With these lengths and offsets, one computes starting values of

$$\sum_{m=1}^{N}{}' \frac{V_m^s}{V_n^s} g_{mn}(x_m, l_m, x_n, l_n) \tag{34}$$

for every value of n, using the desired slot voltage distribution V_m^s/V_n^s, known from pattern requirements.

With this information in hand, one undertakes a computer search to find the couplet (x_n, l_n) which will make the denominator of (33) pure real. What one finds is that there is a continuum of couplets (x_n, l_n) which will satisfy this condition. Similarly, for the pth slot there is a continuum of couplets which will make the denominator of (33) pure real (with p replacing n). But for a given acceptable couplet (x_n, l_n), there is only one acceptable couplet (x_p, l_p) which will also satisfy

$$\frac{Y_p^a/G_0}{Y_n^a/G_0} = \frac{f_p}{f_n}\frac{V_p^s}{V_n^s}\frac{V_n}{V_p} \tag{35}$$

an equation which is formed by taking the ratio of the first design equation (10) for the pth and nth slots. Thus for each acceptable couplet (x_n, l_n), there is a family of acceptable couplets, one couplet for each slot in the array, which makes each active admittance pure real and also satisfies (35). Which family to pick depends on the specification laid down on the sum of the active admittances. For example, in a linear array, the sum of the normalized active admittances would be required to be unity if an input match were specified.

This procedure must be iterated because the chosen family of couplets will undoubtedly not agree with the original guess for slot lengths and offsets. One needs to recompute the sums in (34) with the new (x_n, l_n) values and keep cycling through this procedure until the last set of values is closer to the penultimate set than the achievable machining tolerances. Experience has shown that convergence normally occurs in three or four iterations.

CONCLUSION

A pair of design equations has been developed for shunt slot arrays, with account taken of external mutual coupling. These equations are an improvement on an earlier pair in that they result from a more realistic assumption for the electric field distribution in the slot. This is particularly important for arrays fed by dielectric-filled waveguides since in that case the earlier pair of design equations can be in serious error. The new derivation is also more direct since it avoids the artifice of an equivalent array of loaded dipoles.

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