Maximum Gain (Conjugate Matching)

As shown in the previous lecture, amplifier power gain has the form of $G_T = G_g G_t G_L$ (see (9.7)) where G_t, G_g and G_L are the power gains of the transistor, and input (source) and output (load) matching networks, respectively. Since G_t is predetermined by the parameters of specific transistor, the amplifier total gain may be altered only by the gains of matching circuits. In the section devoted to impedance matching, we have shown that the maximum power is transferred from the source to the load when their impedances provide conjugate match. Because the transistor characteristics are fixed, it is useful to formulate the requirements to the matching networks in terms of the transistor S-parameters.

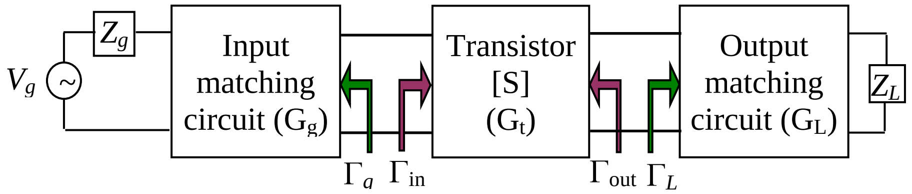


Figure 9.1. A general transistor amplifier circuit

For maximum power transfer from the input matching circuit to the transistor and from the transistor to the output matching network, it is necessary that

$$\Gamma_{in} = \Gamma_g^* \qquad \Gamma_{out} = \Gamma_L^* \tag{10.1}$$

Assuming lossless matching networks, (10.1) will maximise the transducer power gain G_T defined in (9.3):

$$\begin{aligned} G_T &= \frac{1-|\Gamma_g|^2}{|1-\Gamma_g \Gamma_{in}|^2} |S_{21}|^2 \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} = \\ &= \frac{1}{1-|\Gamma_g|^2} |S_{21}|^2 \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2} \end{aligned} \tag{10.2}$$

In a general case of bilateral transistor, Γ_{in} and Γ_{out} affect each other. Therefore the input and output networks should be matched simultaneously. Substituting (10.1) into (9.5) results in the system of coupled equations for Γ_g and Γ_L

$$\begin{aligned} \Gamma_g^* &= S_{11} + \frac{S_{21}S_{12}\Gamma_L}{1-S_{22}\Gamma_L} \\ \Gamma_L^* &= S_{22} + \frac{S_{21}S_{12}\Gamma_g}{1-S_{11}\Gamma_g} \end{aligned} \tag{10.3}$$

The latter equations can be easily separated by substituting one of the equations (10.3) into another. This results in the following quadratic equations for Γ_g and Γ_L

$$\begin{aligned} \Gamma_g^2 - \Gamma_g \frac{1-|S_{22}|^2 - |\Delta|^2 + |S_{11}|^2}{S_{11} - S_{22}^*\Delta} + \frac{S_{11}^* - S_{22}\Delta^*}{S_{11} - S_{22}^*\Delta} = 0 \\ \Gamma_L^2 - \Gamma_L \frac{1-|S_{11}|^2 - |\Delta|^2 + |S_{22}|^2}{S_{22} - S_{11}^*\Delta} + \frac{S_{22}^* - S_{11}\Delta^*}{S_{22} - S_{11}^*\Delta} = 0 \end{aligned} \tag{10.4}$$

The solutions of (10.4) are

$$\Gamma_g = \frac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1}$$
$$\Gamma_L = \frac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2} \qquad (10.5)$$

where
$$B_n = 1 - |S_{3-n,3-n}|^2 - |\Delta|^2 + |S_{nn}|^2$$
$$C_n = S_{nn} - S^*_{3-n,3-n}\Delta \qquad n = 1,2 \qquad (10.6)$$

In the case of unilateral transducer (S_{12}=0), the result is much simpler because from (10.3) we immediately obtain $\Gamma_g = S_{11}^*$ and $\Gamma_L = S_{22}^*$, and

$$G_{TU\max} = \frac{|S_{21}|^2}{\left|1-|S_{22}|^2\right|\left|1-|S_{11}|^2\right|} \qquad (10.7)$$

(compare with (6.40) for the two-port power gains).

Example. Design an amplifier for maximum gain at 2 GHz using a length of the transmission line and a single stub matching section. Calculate and plot return loss and gain in a frequency band 1.5 - 3 GHz. The BJT AT45511 has the following S-parameters (Z_c=50 Ω)

f, GHz	S_{11}	S_{21}	S_{12}	S_{22}
1.5	0.46 ∠-176°	3.61 ∠72°	0.087 ∠44°	0.44∠-43°
1.8	0.46 ∠170°	3.051 ∠64°	.097 ∠45° 0	.43 ∠-45°
2.0	0.46 ∠162°	2.774 ∠59°	.103 ∠45° 0	.42 ∠-47°
2.4	0.47 ∠148°	2.337 ∠50°	.117 ∠46° 0	.42 ∠-51°
3.0	0.5 ∠130°	1.901 ∠36°	.139 ∠45° 0	.41 ∠-59°

Solution

1. Check the transistor stability at 2 GHz by calculating K=1.09>1 and |Δ|=0.103<1. Thus the transistor satisfies the necessary criteria of unconditional stability. At the other listed frequencies, the transistor is potentially unstable only at 1.5 GHz.
2. For the maximum gain at the specified frequency 2 GHz, matching networks should provide the conjugate match. The required Γ_g and Γ_L are calculated with the aid of (10.5) and have the values: Γ_g = 0.782 ∠-159.4° and Γ_L = 0.767 ∠50.1°.
3. Effective gain factors G_t, G_g and G_L for the transistor and input (source) and output (load) matching networks are determined using equation (10.2)

G_g =2.573 (4.105 dB)
G_t =7.695 (8.862 dB)
G_L =0.894 (-0.488 dB)

Then the total transducer gain equals to 10log(G_t*G_g*G_L)=12.479 dB.

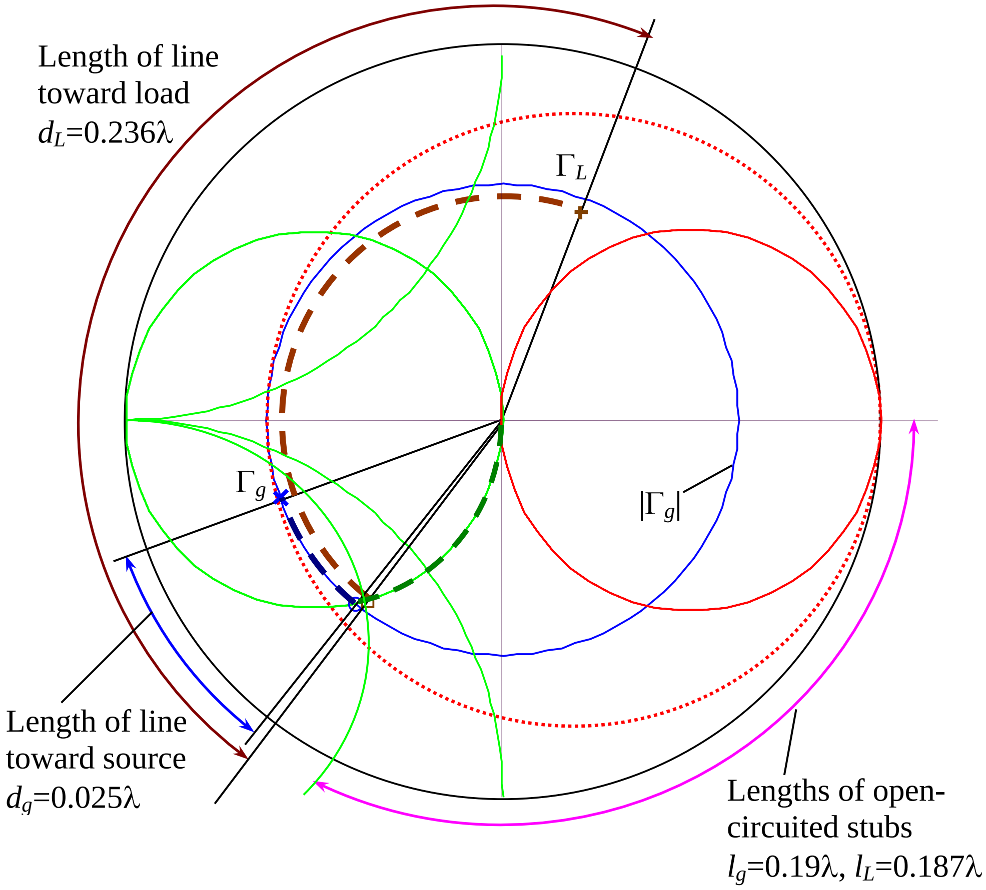


Figure 10.1. Smith chart for design of amplifier matching circuits.

4. The simplest matching networks containing an open-circuited shunt stub and a length of the transmission line can be designed with the aid of the Smith chart (Figure 10.1). For the given Γ_g and Γ_L, the impedances toward source and load are:

$$Z_g = 0.126\text{-}0.179j \qquad \text{and} \qquad Z_L = 0.68+1.947j$$

5. Following the standard procedure, we convert to normalised admittances and find the lengths of lines which transform the normalised admittances y_g, y_L to the circle Re(y)=1. From the two possible solutions, we choose the one which provides Im(y)>0 for the shortest open-circuited stubs being less then λ/4: lengths of open-circuited stubs are: at the source l_g=0.19λ and at the load l_L=0.187λ.

6. Since both Z_g and Z_L lie outside the circle Re(Z)=1, the lengths of transmission line at the load and source are shorter than λ/4: d_L=0.236λ and d_g=0.025λ.

7. A complete amplifier RF circuit is shown in Figure 10.2. To calculate frequency response of the matched amplifier, we need to obtain S-matrix for the entire network encompassed

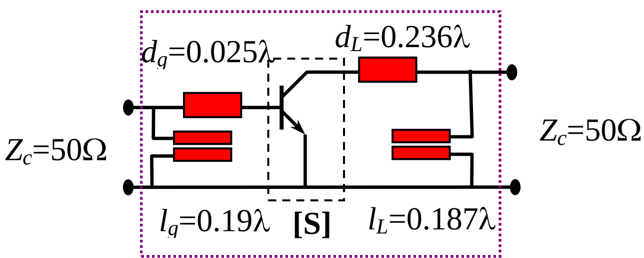


Figure 10.2. RF circuit of the amplifier with matching network

by the dotted line. This can be accomplished in the following 4 steps.

7.1. Move the reference planes toward source and load

$$\mathbf{S}_1 = \begin{bmatrix} e^{-j\theta_g} & 0 \\ 0 & e^{-j\theta_L} \end{bmatrix} \mathbf{S} \begin{bmatrix} e^{-j\theta_g} & 0 \\ 0 & e^{-j\theta_L} \end{bmatrix} \tag{10.8}$$

where $\theta_{g,L} = \beta d_{g,L}$

7.2. Convert the matrix [S_1] into [Y_1]-matrix.

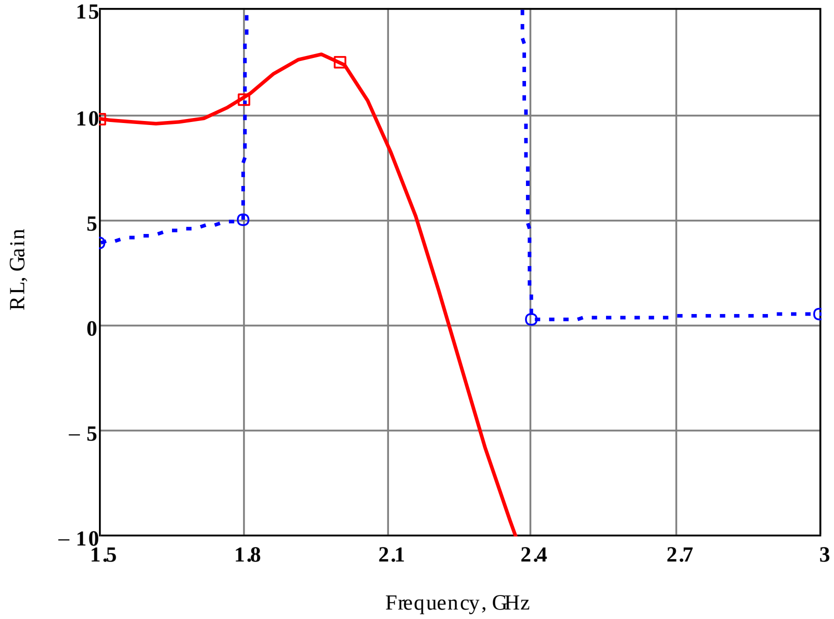
7.3. Add the reactances of shunt open-circuited stubs

$$\left[\mathbf{Y}_{amp}\right] = \mathbf{Y}_1 + \begin{bmatrix} j\tan \beta l_g & 0 \\ 0 & j\tan \beta l_L \end{bmatrix} \tag{10.9}$$

7.4. Convert the matrix [Y_{amp}] back into the matrix [S_{amp}].

8. The matrix [S_{amp}] gives S-parameters of the amplifier including matching networks. Matching circuits are strongly frequency dependent whilst the transistor parameters slowly vary with frequency. Therefore it is worthwhile to interpolate the transistor S-parameters and perform the detailed analysis across the whole specified frequency band.

9. Finally, the stability parameters, Return Loss, RL=-20log(|S_{11}|) and Gain=20log(|S_{21}|) are computed for the complete amplifier network. The results of computations are shown in Figure 10.3 where K=1.09>1 and |Δ|=0.657<1.



f,GHz	RL	Gain dB
1.5	3.786	9.772
1.8	4.952	10.7
2	315.68	12.479
2.4	0.237	−12.221
3	0.475	−15.237

Figure 10.3. Return Loss (blue) and power gain (red) for the amplifier in Figure 10.2