Mitigation of Signal Impairments in Fading Mobile Radio Channels

-Diversity Techniques

Sun Sumei

(sunsm@i2r.a-star.edu.sg)

Diversity Techniques

- Mitigates fading effects by using multiple received/transmitted signals which *experience different fading conditions*.

Space diversity	With multiple antennas
Polarisation diversity	Using differently polarised waves
Frequency diversity	With multiple frequencies
Time diversity	By transmission of the same signal in different times
Angle diversity	Using directive antennas aimed at different directions

- **Signal combining methods.**
  - Maximal Ratio combining (MRC)
  - Equal gain combining (EGC)
  - Selection (switching) combining

- ***Space diversity***
  - o Classified into *micro-diversity* and *macro-diversity.*
    - – **Micro-diversity*:*** Antennas are spaced closely to the order of a wavelength. Effective for fast fading where signal fades in a distance of the order of a wavelength.
      - o **Multiple-input multiple-output (MIMO)** had been widely adopted as both micro-diversity technique and data rate (capacity) enhancement technique
      - o Both **transmit and receive diversity** techniques can be implemented in MIMO.
    - – **Macro (site) diversity*:*** Antennas are spaced wide enough to cope with the topographical conditions (e.g.: buildings, roads, terrain). Effective for shadowing, where signal fades due to the topographical obstructions.
      - o One example is **basestation cooperation** in which multiple basestations transmit cooperatively to the same users

- **PDF of SNR for diversity systems**
  - Needed to analyze the performance of various diversity combining techniques
  - We use an *M*-branch space diversity system as example:
    - *M* antennas are used at the receiver
    - Signal received at each antenna has *Rayleigh distribution*, *independent* of each other, and with the same mean signal and noise power $\Rightarrow$ the same mean SNR for all branches

$$\text{Instantaneous } SNR = \gamma = \frac{E_b}{N_0} \cdot \alpha^2, \qquad \text{Mean } SNR = \Gamma = \overline{\gamma} = \frac{E_b}{N_0} \cdot \overline{\alpha^2}$$

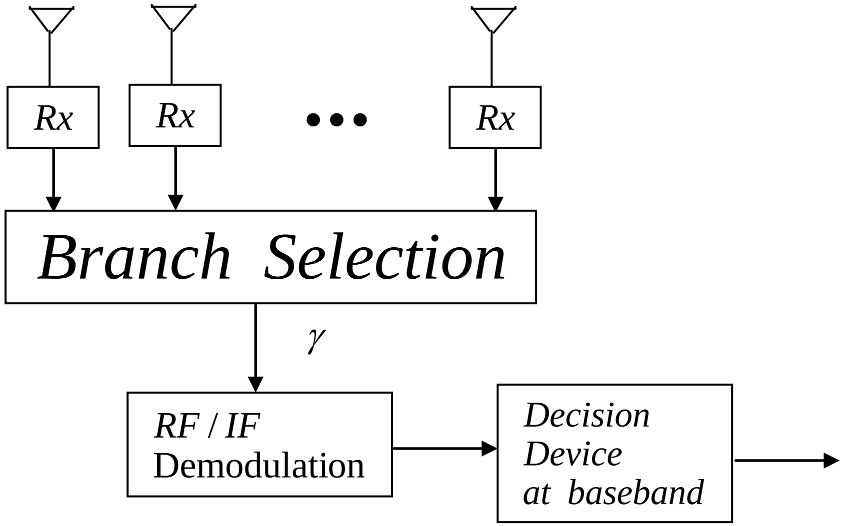
    - Denote the instantaneous SNR at branch *m* by γ_m

$$PDF \;\; of \;\; \gamma_m \Rightarrow p(\gamma_m) = \frac{1}{\Gamma} \cdot e^{-\frac{\gamma_m}{\Gamma}}, \qquad \gamma_m \geq 0$$

- Probability that γ_m takes values less than some threshold *x* is,

$$P\left(\gamma_m \le x\right) = \int_0^x p\left(\gamma_m\right) d\gamma_m = 1 - e^{-\frac{x}{\Gamma}}$$

Selection Diversity



- "*__Branch selection__*" unit selects the branch that has the largest SNR, i.e., the selector output, γ, is written as

$$\gamma = \max\left(\gamma_1, \quad \gamma_2, \quad \cdots \quad \gamma_M\right)$$

- Events in which the selector output SNR, γ, is less than some value, *x*, is exactly the set of events in which each γ_m is simultaneously below *x, i.e.,*

$$\gamma < x \Rightarrow \gamma_1 < x, \quad \gamma_2 < x, \quad \cdots, \text{ and } \gamma_M < x$$

- Since independent fading is assumed in each of the *M* branches,

$$
\begin{aligned}
P(\gamma < x) &= P(\gamma_1 < x, \;\; \gamma_2 < x, \;\; \cdots, \; \gamma_M < x) \\
&= \prod_{m=1}^{M} P(\gamma_m < x) \\
&= \prod_{m=1}^{M} \left[1 - e^{-\frac{x}{\Gamma}} \right] \\
&= \left(1 - e^{-\frac{x}{\Gamma}} \right)^{M}
\end{aligned}
$$

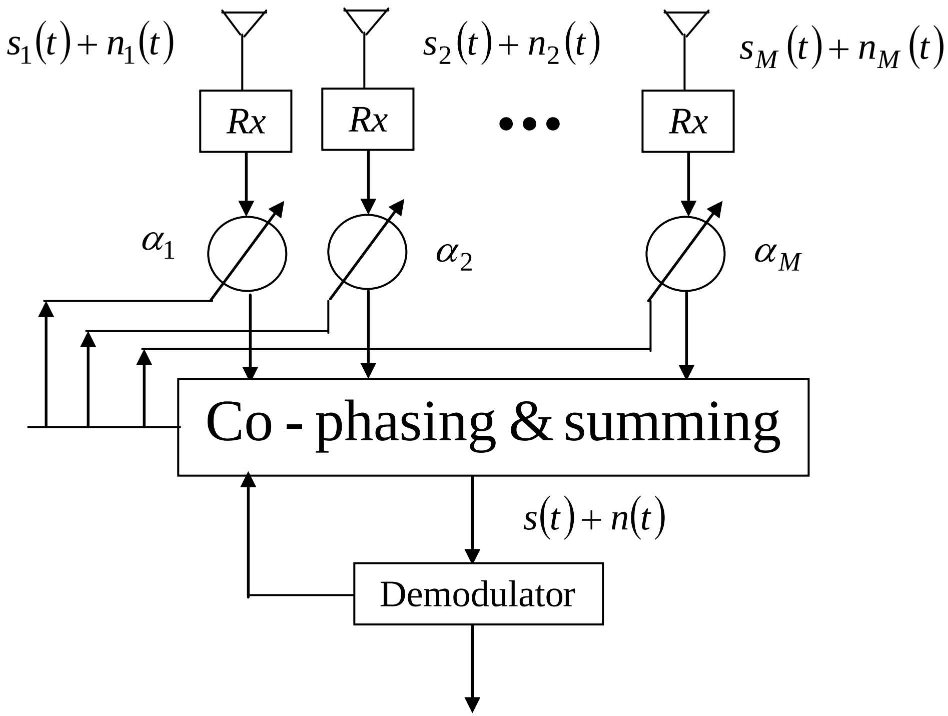
- We hence have

$$
p_\gamma(x) = \frac{d}{dx} P(\gamma < x) = \frac{M}{\Gamma} \cdot e^{-\frac{x}{\Gamma}} \cdot \left(1 - e^{-\frac{x}{\Gamma}} \right)^{M-1}
$$

$$
\Rightarrow \;\; p_\gamma(\gamma) = \frac{M}{\Gamma} \cdot e^{-\frac{\gamma}{\Gamma}} \cdot \left(1 - e^{-\frac{\gamma}{\Gamma}} \right)^{M-1}
$$

- We can then compute the average BER performance with selection diversity.

Maximal Ratio Combining



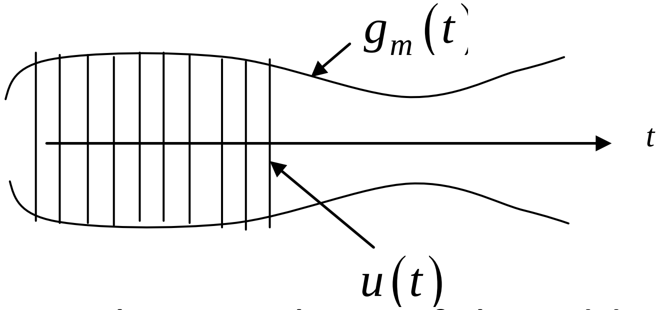
- $s_m(t) = g_m(t) \cdot u(t)$ is complex envelope of signal in the *m-th* branch.
- $g_m(t) = H_m(0,t)$ is the channel coefficient for the *m-th* branch.
- The complex equivalent lowpass signal *u(t)* contains the information common to all branches.

- Assume *u(t)* is normalised to unit mean square envelope such that

$$\left\langle \left|u(t)\right|^2 \right\rangle = \frac{1}{T}\int_0^T \left|u(t)\right|^2 dt = 1$$

$$\frac{1}{2}\left\langle \left|s_m(t)\right|^2 \right\rangle = \frac{1}{2}\left\langle \left|g_m(t)u(t)\right|^2 \right\rangle = \frac{1}{2}\left\langle \left|g_m(t)\right|^2 \left|u(t)\right|^2 \right\rangle = \frac{1}{2}\left\langle \left|g_m(t)\right|^2 \right\rangle \underbrace{\left\langle \left|u(t)\right|^2 \right\rangle}_{=1} = \frac{1}{2}\left\langle \left|g_m(t)\right|^2 \right\rangle .$$

- Assume time variation of $g_m(t)$ is much slower than that of $u(t)$



- Let $n_m(t)$ be the complex envelope of the additive Gaussian noise in the *m-th* branch.

$$\frac{1}{2}\left\langle \left|n_m(t)\right|^2 \right\rangle = N_m$$, $\Rightarrow$ normally N_m's are assumed to be equal for all M branches

- Define SNR of m-*th* branch as

$$\gamma_m = \frac{\frac{1}{2}\left\langle \left| s_m(t) \right|^2 \right\rangle}{\frac{1}{2}\left\langle \left| n_m(t) \right|^2 \right\rangle} = \frac{1}{2}\frac{\left| g_m \right|^2}{N_m}.$$

- The signal component at the output of the "*Co-phasing & summing*" unit is

$$s(t) = \sum_{m=1}^{M} \alpha_m s_m(t)$$

where α_m's are the complex combining weight factors.

- o These factors are changed from instant to instant as channel coefficient for the branch changes over the short term fading

- How should α_m be chosen to achieve ***maximum combiner output SNR*** at each instant?

$$s(t) = u(t) \cdot \sum_{m=1}^{M} \alpha_m g_m, \qquad \mathrm{n}(t) = \sum_{m=1}^{M} \alpha_m n_m(t)$$

- Assuming $n_m(t)$'s are mutually independent (uncorrelated), we have

$$N=\frac{1}{2}\left\langle\left|n(t)\right|^2\right\rangle=\frac{1}{2}\left\langle n(t)n^*(t)\right\rangle=\frac{1}{2}\left\langle\sum_{m=1}^{M}\sum_{l=1}^{M}\alpha_m\alpha_l^* n_m(t)n_l^*(t)\right\rangle=\sum_{m=1}^{M}\left|\alpha_m\right|^2 N_m$$

$$S=\frac{1}{2}\left\langle\left|s(t)\right|^2\right\rangle=\frac{1}{2}\left\langle s(t)s^*(t)\right\rangle=\frac{1}{2}\left\langle\left|u(t)\right|^2\sum_{m=1}^{M}\sum_{l=1}^{M}\alpha_m\alpha_l^* g_m g_l^*\right\rangle=\frac{1}{2}\underbrace{\left\langle\left|u(t)\right|^2\right\rangle}_{=1}\left|\sum_{m=1}^{M}\alpha_m g_m\right|^2=\frac{1}{2}\left|\sum_{m=1}^{M}\alpha_m g_m\right|^2$$

- Instantaneous output SNR, γ,

$$\gamma=\frac{S}{N}=\frac{1}{2}\frac{\left|\sum_{m=1}^{M}\alpha_m g_m\right|^2}{\sum_{m=1}^{M}\left|\alpha_m\right|^2 N_m}$$

- Apply the Schwarz Inequality for complex valued numbers

$$\left|\sum_{k=1}^{M}{a_m}^* b_m\right|^2\le\left(\sum_{m=1}^{M}\left|a_m\right|^2\right)\left(\sum_{m=1}^{M}\left|b_m\right|^2\right)$$

The equality holds if $b_m=Ka_m$ for all *m*, where *K* is an arbitrary complex constant.

o Let

$$a_m = \frac{g_m^*}{\sqrt{N_m}} \quad \& \quad b_m = \alpha_m \sqrt{N_m}$$

o $$\left|\sum_{m=1}^{M} \alpha_m g_m\right|^2 \le \left(\sum_{m=1}^{M} \frac{|g_m|^2}{N_m}\right)\left(\sum_{m=1}^{M} |\alpha_m|^2 N_m\right),$$

Then $$\gamma \le \frac{1}{2}\sum_{m=1}^{M} \frac{|g_m|^2}{N_m},$$

with equality holding if and only if $\alpha_m = K\frac{g_m^*}{\sqrt{N_m}}$, for each m.

- <u>**Optimum weight for each branch has magnitude proportional to the signal magnitude and inversely proportional to the branch noise power level, and has a phase, cancelling out the channel phase**</u>

- This phase alignment allows coherent addition of branch signals $\Rightarrow$ "co-phasing".

$$\Rightarrow \gamma = \sum_{m=1}^{M} \gamma_m \, .$$

- Recall that

$$\gamma_m = \frac{1}{2}\frac{\left|g_m\right|^2}{N_m} = \frac{1}{2}\frac{\left(T_{c,m}^2 + T_{s,m}^2\right)}{N_m},$$

$T_{c,m}^2$ and $T_{s,m}^2$ each has a chi-square distribution.

➢ γ is distributed as chi-square with *2M* degrees of freedom.

$$\Rightarrow p_\gamma(\gamma) = \frac{1}{(M-1)!}\cdot\frac{\gamma^{M-1}}{\Gamma^M}\cdot e^{-\frac{\gamma}{\Gamma}}$$ where Γ is the average SNR per branch.

(i) Schwartz, Bennett and Stein, "Communications systems and techniques", McGraw Hill, New York, 1966, p. 443, Eq.(10-5-20)

(ii) J.G. Proakis, "Digital Communications", Third Edition, Mc Graw Hill, Inc., p. 42, Eq. (2-1-110).

- Average SNR, $\bar{\gamma}$, is simply the sum of the individual $\overline{\gamma_m}$ for each branch, which is Γ,

$$\therefore \bar{\gamma} = \sum_{m=1}^{M} \overline{\gamma_m} = \sum_{m=1}^{M} \Gamma = M\Gamma$$

Example

Consider diversity reception with *M* independent Rayleigh fading branches and maximal ratio combining. Assume equal average SNR per branch.

(i) For *M=2*, find the probability that the SNR at the combiner output is 8 dB above the average branch SNR.

(ii) For M=4, determine the required average E_b / N_0 for DBPSK in order to sustain a 10^{-3} BER.

<u>Solution:</u>

(i) For 2-branch MRC combining, the pdf of the SNR at combiner output is

$$p_\gamma(\gamma) = \frac{\gamma}{\Gamma^2} e^{-\frac{\gamma}{\Gamma}}$$

The event that the SNR at the combiner output is 8 dB above the average branch SNR is

$$10\log_{10}\left(\frac{\gamma}{\Gamma}\right)=8 \quad \Rightarrow \quad \frac{\gamma}{\Gamma}=10^{0.8}=6.31 \quad \Rightarrow \quad \gamma=6.31\Gamma$$

$$\int_{6.31\Gamma}^{\infty} p_{\gamma}(\gamma)d\gamma=\int_{6.31\Gamma}^{\infty}\frac{\gamma}{\Gamma^2}\cdot e^{-\frac{\gamma}{\Gamma}}d\gamma=\int_{6.31}^{\infty}x\cdot e^{-x}dx$$

Using integration by parts $\int udv = uv - \int vdu$

$$u=x, \quad du=dx, \quad v=-e^{-x} \Rightarrow \frac{dv}{dx}=-e^{-x}(-1)=e^{-x}$$

$$-xe^{-x}\Big|_{6.31}^{\infty}-\int_{6.31}^{\infty}\left(-e^{-x}\right)dx=6.31e^{-6.31}-e^{-x}\Big|_{6.31}^{\infty}=(6.31+1)e^{-6.31}=0.0133$$

(ii) For DBPSK,

$$p_{b|\gamma} = \frac{1}{2} e^{-\gamma}$$

For 4-branch MRC, we have

$$p_\gamma(\gamma) = \frac{1}{(M-1)!} \cdot \frac{\gamma^{M-1}}{\Gamma^M} \cdot e^{-\frac{\gamma}{\Gamma}} \Bigg|_{M=4} = \frac{1}{6} \cdot \frac{\gamma^3}{\Gamma^4} \cdot e^{-\frac{\gamma}{\Gamma}}$$

We hence have

$$p_{b|\gamma} = \frac{1}{2} e^{-\gamma}$$

$$\Rightarrow P_e = \int_0^\infty p_{b|\gamma} p_\gamma(\gamma) d\gamma = \int_0^\infty \frac{1}{2} e^{-\gamma} \cdot \frac{1}{6} \cdot \frac{\gamma^3}{\Gamma^4} \cdot e^{-(\gamma/\Gamma)} d\gamma = \frac{1}{12} \cdot \frac{1}{\Gamma^4} \int_0^\infty e^{-\gamma} \cdot \gamma^3 \cdot e^{-(\gamma/\Gamma)} d\gamma = \frac{1}{12} \cdot \frac{1}{\Gamma^4} \int_0^\infty \gamma^3 \cdot e^{-\frac{1+\Gamma}{\Gamma}\gamma} d\gamma$$

Now we will use the result

$$\int_0^\infty x^n e^{-x} dx = n!$$

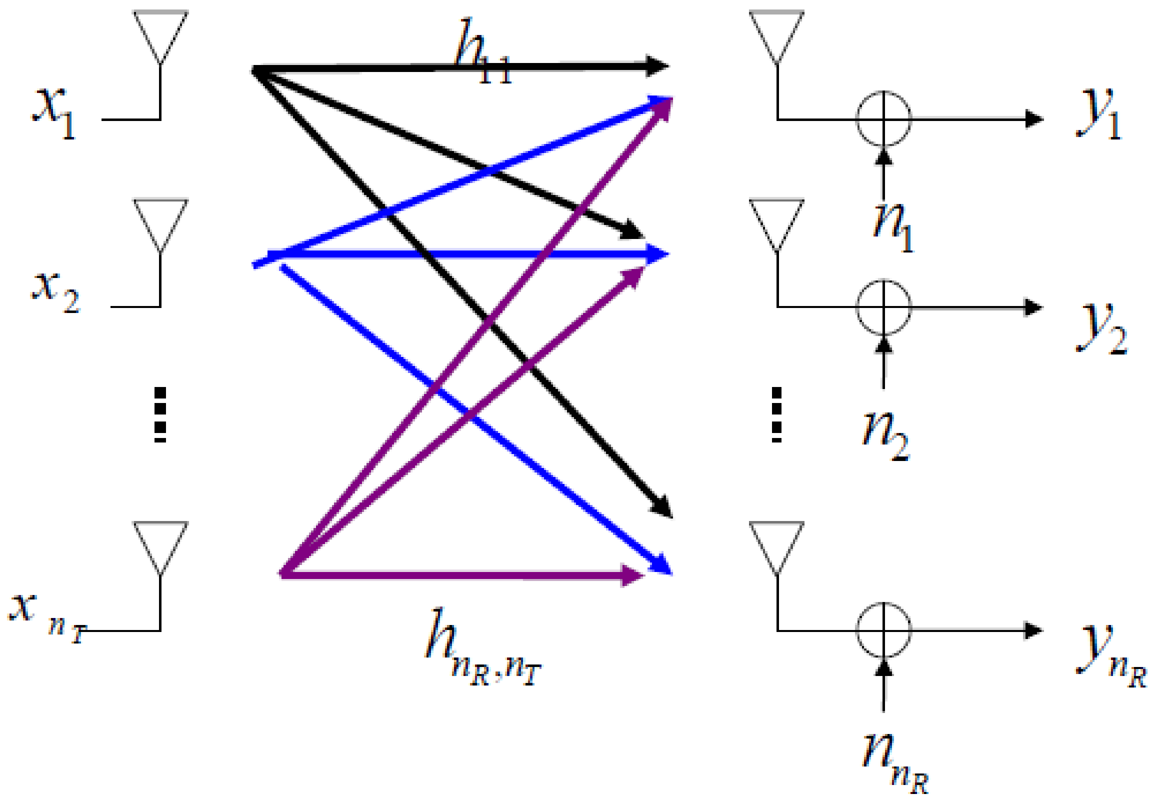
Let

$$x = \gamma\left(\frac{1+\Gamma}{\Gamma}\right) \Rightarrow dx = d\gamma\left(\frac{1+\Gamma}{\Gamma}\right)$$

$$\therefore P_e = \frac{1}{12}\cdot\frac{1}{\Gamma^4}\int_0^\infty \gamma^3 \cdot e^{-\frac{1+\Gamma}{\Gamma}\gamma} d\gamma = \frac{1}{12}\cdot\frac{1}{(1+\Gamma)^4}\cdot\int_0^\infty x^3 \cdot e^{-x} dx = \frac{1}{12}\cdot\frac{1}{(1+\Gamma)^4}\cdot(3!) = \frac{1}{2}\cdot\frac{1}{(1+\Gamma)^4} = 10^{-3}.$$

$$\Rightarrow \Gamma = 3.728 = 5.72\text{dB}$$

Multiple-Input Multiple-Output (MIMO) and Space-Time Coding



$$\mathbf{h} = \begin{bmatrix} h_{1,1} & h_{1,2} & \cdots & h_{1,n_T} \\ h_{2,1} & h_{2,2} & \cdots & h_{2,n_T} \\ & \vdots & \vdots & \\ h_{n_R,1} & h_{n_R,2} & \cdots & h_{n_R,n_T} \end{bmatrix}, \qquad \mathbf{y} = \mathbf{h}\mathbf{x} + \mathbf{n}$$

- Multiple-input multiple-out (MIMO) refers to a communication system in which both the transmitter and the receiver deploy multiple antennas
  - We denote the number of antennas at the TX and RX by n_T and n_R, respectively
  - Each transmit-receive antenna pair forms one channel, with its coefficient denoted by h_{n_r,n_t}
  - The MIMO channel can be represented by an $n_R \times n_T$ matrix, with its $(n_r, n_t)^{th}$ element h_{n_r,n_t}, corresponding to receive antenna n_r, and transmit antenna n_t
  - In a MIMO system, signals are transmitted from the n_T antennas simultaneously, and received by the n_R antennas simultaneously

– The received signal at each antenna is a superposition of channel-weighted transmitted signals, e.g.,

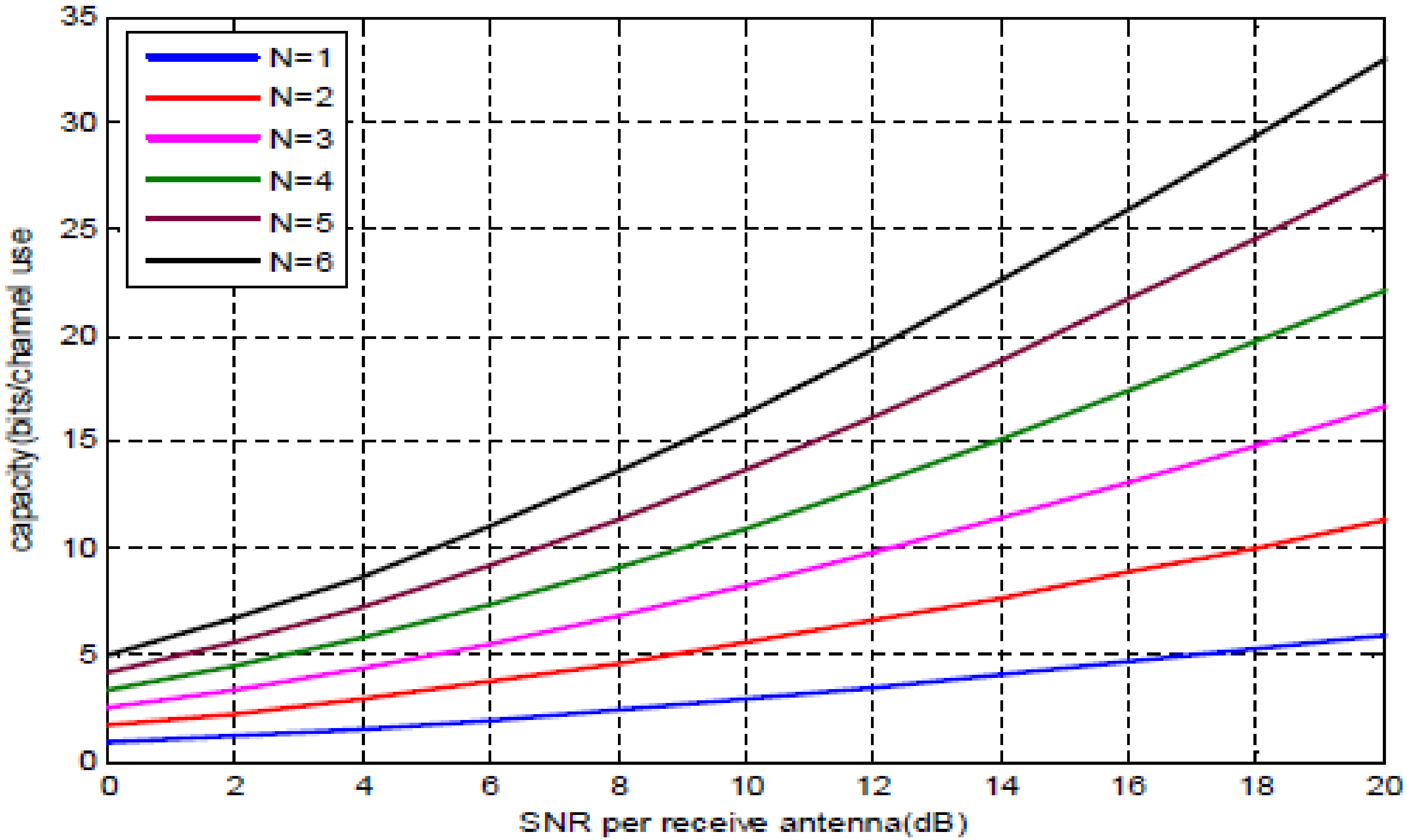
$$y_{n_r} = \sum_{n_t=1}^{n_T} h_{n_r,n_t} x_{n_t}$$

– If $n_T=1$, we call it SIMO, standing for *single-input multiple-output*. Then receive diversity can be implemented;

– If $n_R=1$, we call it MISO, *multiple-input single-output*. Then transmit diversity can be implemented by using, e.g., **space-time coding**.

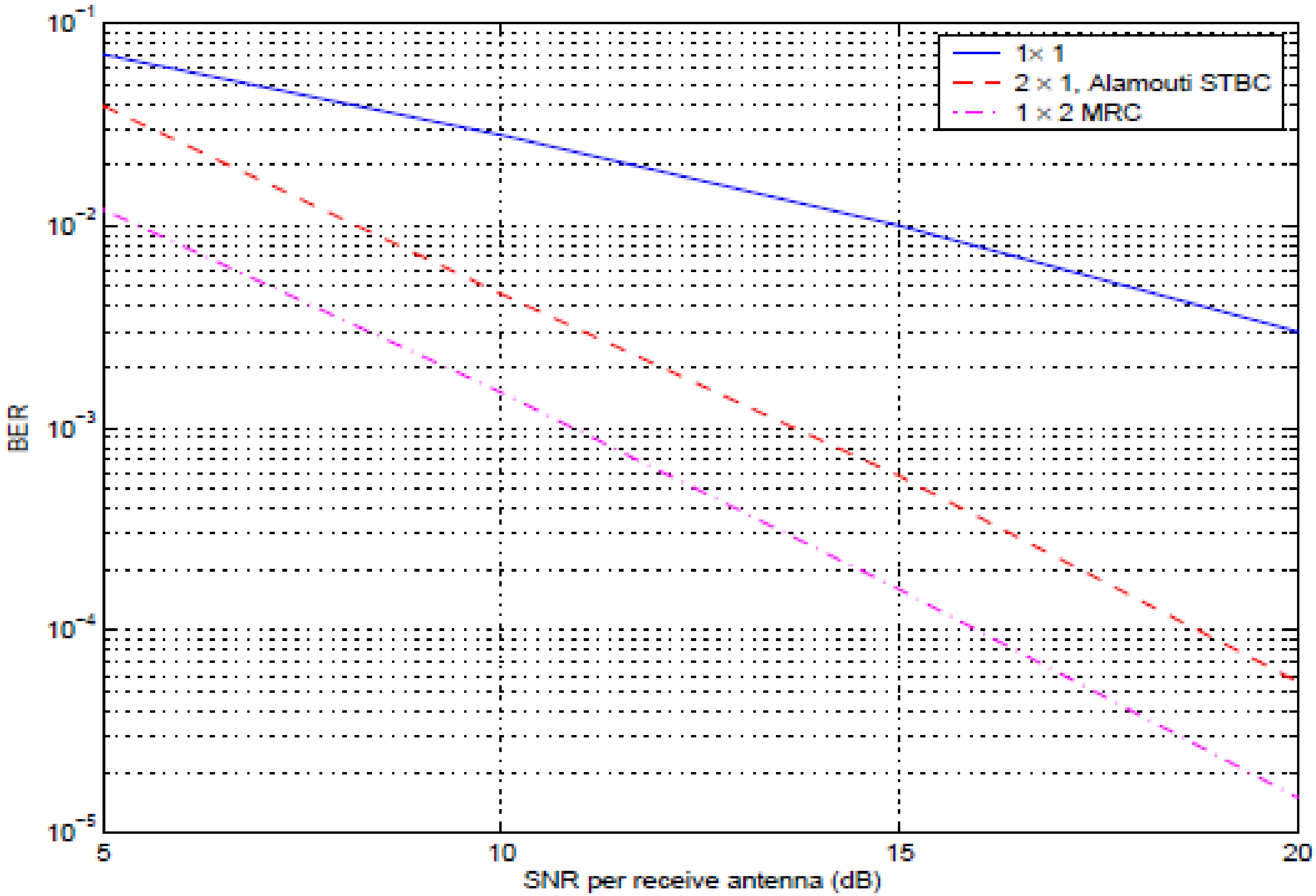
- The "modern" MIMO system was first invented at Bell Labs, by G. J. Foschini, *et. al.,* and published in 1998
- A MIMO system can be deployed to enhance the capacity, i.e., data rate of the system, using the same bandwidth. Hence **spectral efficiency** can be improved.

- A MIMO system can also be deployed to implement the diversity techniques, hence enhance the **robustness against fading**.
- Since its invention, MIMO has been quickly adopted in a number of wireless communication systems
  - Cellular communications: Long Term Evolution (LTE), LTE-Advanced
  - WiFi: IEEE 802.11n, IEEE 802.11ac
  - WiMAX: IEEE IEEE 802.16e, 802.16m



Ergodic Capacity of NXN Circularly Symmetric Complex Gaussian (CSCG) MIMO

(Almost a linear increase with the number of antennas, *N*)



BER of 1X1, 1X2 MRC, and 2X1 STBC in Rayleigh fading channel, BPSK signal

(STBC: space-time block code)

Space-Time Block Coding (STBC)

- STBC was first introduced by S. M. Alamouti, in 1998
  - *S. M. Alamouti, "A* Simple Transmit Diversity Technique for Wireless Communications," IEEE Journal on Selected Areas in Communications, Vol 16, No. 8, Oct 1998
  - An **orthogonal STBC** was introduced for a 2-transmit antenna system
    - ♦ This code is now commonly referred to as "***Alamouti code*".**
- Orthogonal STBC was then systematically studied by V. Tarokh, *et. al*.
- Quasi-orthogonal STBC was proposed by H. Jafarkhani, *et. al*.
  - Introduces both diversity gain and transmission rate gain
- The most successful STBC adoption in standard and commercial systems is the Alamouti code

Alamouti STBC

- Implemented with two transmit antennas
- Suppose the transmitted signal sequence is denoted as

$$s_0, s_1, s_2, s_3, \cdots$$

wth the average signal power normalized to 1, i.e.,

$$\left\langle \left| s_i \right|^2 \right\rangle = 1$$

- The Alamouti STBC will group 2 consecutive symbols, say, s_0 and s_1, and transmit them using the two antennas in the following manner

	Antenna 0	Antenna 1
1st Symbol Interval	s_0	s_1
2nd Symbol Interval	$-s_1^*$	s_0^*

- Suppose we have 1 receive antenna
- The received signal during the 2 symbol intervals will be

1st Symbol Interval	$y_0 = h_0 s_0 + h_1 s_1 + n_0$
2nd Symbol Interval	$y_1 = -h_0 s_1^* + h_1 s_0^* + n_1$

where h_0 and h_1 are the channel coefficients for the 2 antenna pairs (Tx1-RX, and Tx2-RX), and n_0 and n_1 are the AWGN noise in symbol interval 1 and 2, respectively.

– h_0 and h_1 can be assumed to be identically and independently distributed (iid), and their envelop $r=|h_0|$ and $r=|h_1|$ follow Rayleigh distribution, with pdf

$$pdf(r) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right), \qquad r \geq 0$$

– n_0 and n_1 are assumed to be iid with mean 0, and variance σ_n^2

- We can re-write the received signal in the 2nd symbol interval as

$$y_1^* = h_1^* s_0 - h_0^* s_1 + n_1^*$$

- Hence the decision statistic for s_0 and s_1 are derived from y_0 and y_1^* as

$$\tilde{s}_0 = \frac{1}{\left\| h_0 \right|^2 + \left| h_1 \right|^2 \Big|} \left(h_0^* y_0 + h_1 y_1^* \right) = s_0 + \underbrace{\frac{h_0^* n_0 + h_1 n_1^*}{\left\| h_0 \right|^2 + \left| h_1 \right|^2 \Big|}}_{\tilde{n}_0}$$

$$\tilde{s}_1 = \frac{1}{\left\| h_0 \right|^2 + \left| h_1 \right|^2 \Big|} \left(h_1^* y_0 - h_0 y_1^* \right) = s_1 + \underbrace{\frac{h_1^* n_0 - h_0 n_1^*}{\left\| h_0 \right|^2 + \left| h_1 \right|^2 \Big|}}_{\tilde{n}_1}$$

$\tilde{n}_0$ and $\tilde{n}_1$ are still zero-mean Gaussian.

- The received signal y_0 and y_1^* can also be written in matrix representation, as

$$\underbrace{\begin{bmatrix} y_0 \\ y_1^* \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} h_0 & h_1 \\ h_1^* & -h_0^* \end{bmatrix}}_{\mathbf{H}} \underbrace{\begin{bmatrix} s_0 \\ s_1 \end{bmatrix}}_{\mathbf{s}} + \underbrace{\begin{bmatrix} n_0 \\ n_1^* \end{bmatrix}}_{\mathbf{n}}$$

i.e.,

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n}$$

- Noting that **H** is an ***orthogonal matrix,***, i.e.,

$$\mathbf{H}^H\mathbf{H} = \left\| h_0 \right|^2 + \left| h_1 \right|^2 \left| \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \left(\left| h_0 \right|^2 + \left| h_1 \right|^2 \right) \mathbf{I}$$

We can write the signal detection as a matrix operation:

$$\tilde{\mathbf{s}} = \begin{bmatrix} \tilde{s}_0 \\ \tilde{s}_1 \end{bmatrix} = \frac{\mathbf{H}^H}{\left(\left| h_0 \right|^2 + \left| h_1 \right|^2 \right)} \mathbf{y} = \mathbf{s} + \tilde{\mathbf{n}}$$

With

$$\tilde{\mathbf{n}} = \begin{bmatrix} \tilde{n}_0 \\ \tilde{n}_1 \end{bmatrix} = \begin{bmatrix} \dfrac{h_0^* n_0 + h_1 n_1^*}{\left\| h_0 \right|^2 + \left| h_1 \right|^2 \right|} \\ \dfrac{h_1^* n_0 - h_0 n_1^*}{\left\| h_0 \right|^2 + \left| h_1 \right|^2 \right|} \end{bmatrix}$$

This is the same as the previous result.

- We hence have shown that the Alamouti code is an ***orthogonal code.***
- With Alamouti code, 2 symbols are transmitted over 2 transmit antennas, in 2 symbol intervals. Therefore, Alamouti code is a "***rate-1"*** code.
- Decoding of Alamouti STBC at receiver is achieved with ***"linear"*** processing, hence simple.

- $\tilde{\mathbf{n}}$ is still Gaussian, with mean 0, and the variance of $\tilde{n}_0$ and $\tilde{n}_1$ are

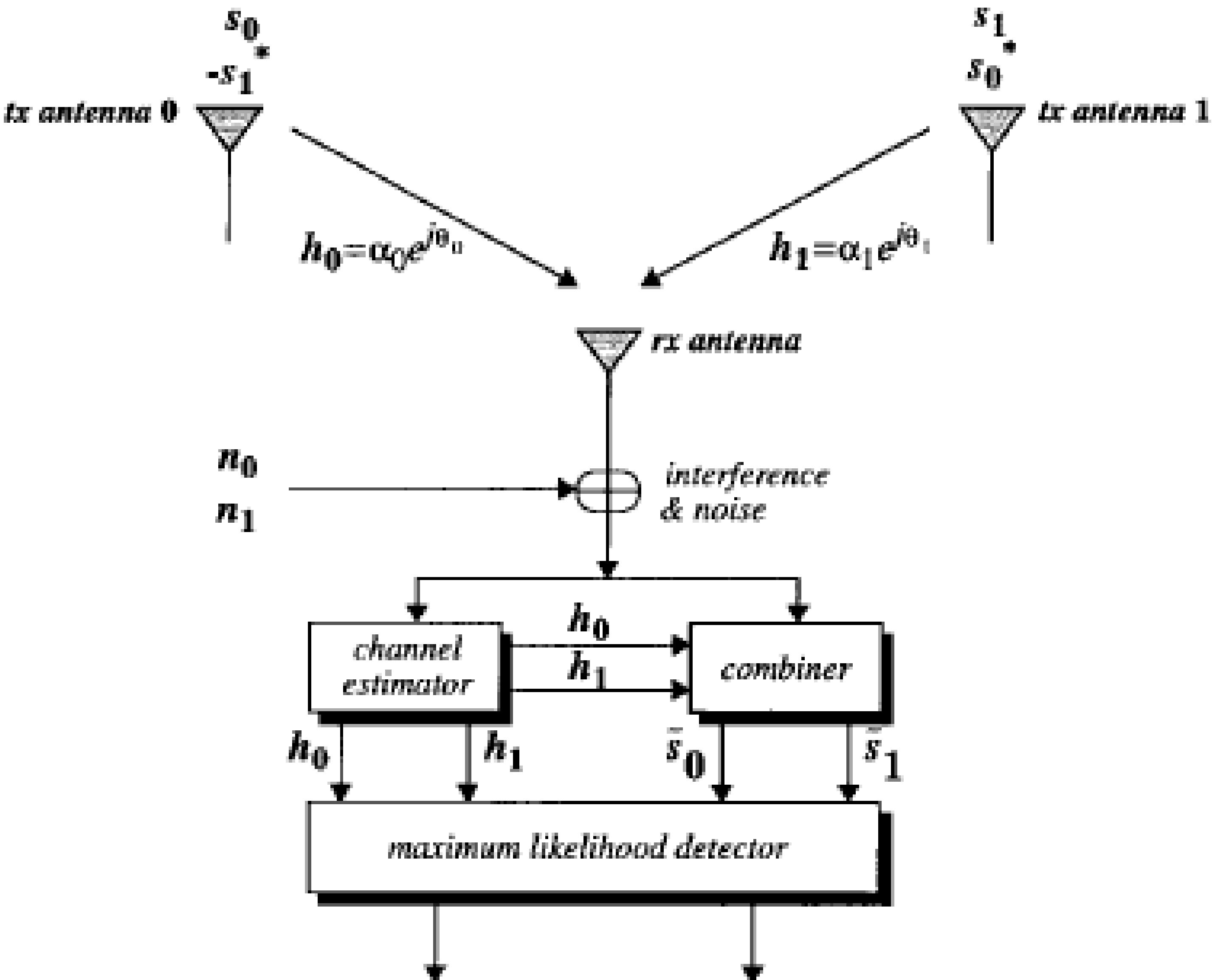
$$\left\langle \left|\tilde{n}_0\right|^2 \right\rangle = \left\langle \left|\tilde{n}_1\right|^2 \right\rangle = \frac{\sigma_n{}^2}{\left|\left|h_0\right|^2 + \left|h_1\right|^2\right|}$$

- We therefore have the receiver output SNR for $\tilde{s}_0$ and $\tilde{s}_1$ as

$$\gamma = \frac{\left|\left|h_0\right|^2 + \left|h_1\right|^2\right|}{\sigma_n{}^2} = \gamma_0 + \gamma_1$$

which is the same as the MRC combining SNR.

- Hence the Alamouti STBC with two transmit antennas and one receive antennas can achieve the same performance as the MRC combining
- γ follows chi-square distribution with order 4.



- *Taken from S. M. Alamouti,* "A Simple Transmit Diversity Technique for Wireless Communications," IEEE Journal on Selected Areas in Communications, Vol 16, No. 8, Oct 1998