

Application of Aggressive Space Mapping to the Synthesis of Composite Right/Left Handed (CRLH) Transmission Lines Based on Complementary Split Ring Resonators (CSRRs)

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Abstract— In this paper we report the synthesis of metamaterial transmission lines by means of aggressive space mapping (ASM). Specifically, it is demonstrated that ASM can be applied to the automated layout generation of composite right/left handed (CRLH) microstrip lines based on a combination of complementary split ring resonators (CSRRs) and series gaps. To illustrate the potentiality and accuracy of the approach, we report in this work the synthesis of a CRLH line. This work represents a clear progress on the synthesis of CSRR-based artificial lines, since we are now able to synthesize structures with a combination of CSRRs and series capacitances. These structures exhibit a CRLH behavior of great interest in microwave engineering to implement enhanced bandwidth components, multi-band devices, or compact filters, among others. Therefore, the reported approach will have a direct impact in reducing the efforts for designing these metamaterial based planar components.

Index Terms — Space mapping, Broyden matrix, composite right/left handed lines, complementary split ring resonators.

I. INTRODUCTION

Metamaterial transmission lines are artificial lines consisting on a host line loaded with reactive elements. Typically, these lines have smaller size than conventional lines and, due to the presence of the loading elements, their dispersion behavior and characteristic impedance can be further controlled, as compared to conventional lines. For these reasons, metamaterial transmission lines have been a subject of interest in recent years, and many applications of these structures in microwave engineering have been highlighted [1]-[3]. There are basically two approaches for the implementation of metamaterial transmission lines: one consists on loading a host line with series capacitances and shunt inductances (CL-loaded approach) [4]-[6]; the other approach, known as resonant-type, combines resonant sub-wavelength particles, such as split ring resonators (SRRs) or

their complementary counterparts (i.e. the CSRRs), with shunt inductances or series capacitances [7],[8]. In both cases, the lines exhibit left handed (backward) wave propagation at low frequencies and right handed (forward) wave propagation at high frequencies. Thus, these lines are usually identified as composite right/left handed (CRLH) lines (a term firstly introduced in [9]), and the enhanced functionalities or performances of microwave circuits based on these artificial lines are mainly (although not exclusively) based on this composite behavior.

The implementation of metamaterial transmission lines in a fully planar technology reduces fabrication costs. Thus, these artificial lines have been mostly implemented by loading them with semi-lumped components, i.e., electrically small planar elements, such as inductive stubs, gaps and interdigital capacitances, as well as split rings, among others. However, the main drawback of these fully planar artificial lines is related to their synthesis. There are models that link the geometry to the electrical parameters of the reactive elements (even including parasitic effects), but these models are valid under conditions that sometimes are not fulfilled. Therefore, optimization is necessary.

This work is focused on the resonant-type metamaterial transmission line approach, in particular on the synthesis of CRLH microstrip lines loaded with CSRRs and series gaps. These lines have been demonstrated to be useful in many applications that require the tailoring of their dispersion diagram (dispersion engineering), such as dual-band components or enhanced bandwidth components, among others. Therefore, the present paper constitutes an important progress on the application of aggressive space mapping (ASM) to the automated optimization of microstrip lines loaded with CSRRs. The presence of the gap in such lines enhances the complexity of the ASM algorithm, but the novel algorithm opens the path to the automated layout generation

of fully planar CSRR-based CRLH transmission lines and related circuits. The aim of this work is thus to apply the so called aggressive space mapping (ASM) [10] to accelerate and simplify the synthesis process of CSRR-loaded CRLH microstrip lines. The topology and circuit model of such lines are explained in Section II. The particularities of the ASM algorithm, derived from the presence of the series gap, are discussed in section III. Finally, we report the application of this algorithm to the automated design of a specific structure in section IV. To the best of our knowledge, this is the first time that ASM is applied to the synthesis of CRLH lines based on complementary split rings. Owing to the potentiality of these artificial lines, the ASM algorithm presented in this work will contribute to the penetration of these lines into the market by alleviating their main drawback, that is, their automated synthesis.

II. CRLH MICROSTRIP LINES BASED ON CSRRs AND SERIES GAPS: TOPOLOGY AND CIRCUIT MODEL

The unit cell of a CRLH microstrip line loaded with CSRRs and series gaps is depicted in Fig. 1(a), with the relevant dimensions describing the structure. As long as the electrical size of the CSRRs (diameter) is much smaller than the guided wavelength at their resonance frequency, the structure can be accurately described by means of the lumped model of Fig. 1(b), which was first introduced in [11]. In this model, L and C_L are, respectively, the line inductance and capacitance, L_c and C_c are the inductance and capacitance of the CSRR, and the gap is modeled by its series capacitance C_s and the fringing capacitance C_f .

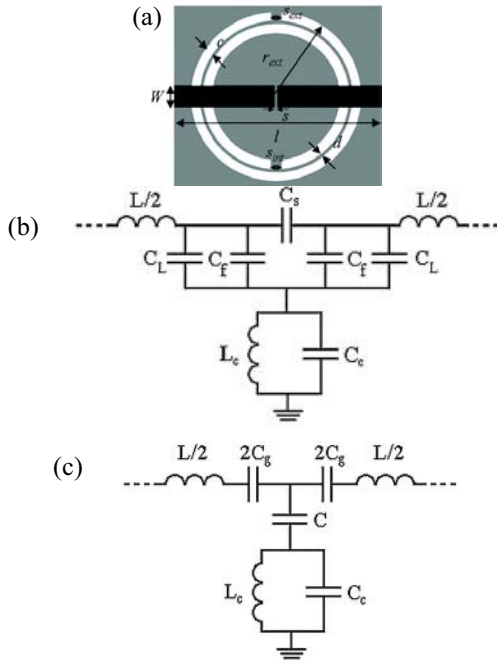


Fig. 1 Typical topology of a CRLH line unit cell based on the combination of CSRRs and series gaps (a), and lumped element equivalent circuit model (b). The CSRRs are etched in the ground plane, depicted in grey. The transformed T-model is depicted in (c).

The circuit model of Fig. 1(b) can be transformed to that of Fig. 1(c) (formerly reported in [12]), which is the useful model for design purposes. The transformation equations are [11]:

$$2C_g = 2C_s + C_{par} \quad (1)$$

$$C = \frac{C_{par}(2C_s + C_{par})}{C_s} \quad (2)$$

with $C_{par} = C_f + C_L$. As it will be latter discussed, the proposed ASM algorithm includes a parameter extractor module, where the element parameters of the circuit of Fig. 1(c) are obtained from the electromagnetic simulation of the CRLH structures, following a method similar to the one reported in [13].

As it was mentioned before, although there are models that link the geometry of the structure to the circuit parameters, the line, the CSRRs and the gaps are not isolated but coupled, so that such simple models cannot be straightforwardly used to find the cell topology. This justifies the present work, where ASM is applied to the automated determination of the layout reproducing the response of a given circuit (target response).

III. THE ASM IMPLEMENTATION

Aggressive space mapping (ASM) is a widely recognized technique for the efficient optimization of microwave devices. Each strategy based on ASM assumes that the structure under consideration can be simulated using two models: a fine model which is complex and computationally demanding but accurate, and a coarse model that is CPU efficient but less precise [10]. In this work, the efficient optimization of the fine model (i.e., a full-wave electromagnetic (EM) analysis tool) is achieved through the iterative optimization of the coarse model (the equivalent circuit described before). The ASM algorithm tries to find a mapping (P), relating the parameters of the fine (x_{em}) and coarse (x_c) models:

$$x_c = P(x_{em}) \quad (3)$$

In our case, the five electric parameters of the circuit of Fig. 1(c), L , C , C_g , L_c and C_c , are the elements defining the coarse model. In order to simplify the synthesis technique, not all the geometry parameters of the EM-model will be considered. The chosen set of design variables in x_{em} are the width of the microstrip line W , the external CSRR radius r_{ext} , the width of the slotted rings c , the distance between them d , and the gap distance, s (see Fig. 1a). The length of the line l_{strip} is fixed to the diameter of the external ring of the CSRR.

We have followed a constrained Broyden-based input space mapping algorithm [14], which makes use of a quasi-Newton type iteration for predicting the next solution of the EM-model

$$x_{em}^{(j+1)} = x_{em}^{(j)} + h \quad (4)$$

where h is the step size in the quasi-Newton direction. Using the constrained approach, h will be decreased in the same quasi-Newton direction by a factor δ when the new vector x_{em} is not within the acceptable established limits, thus letting the algorithm continue with normal evolution flow once it is adjusted. We have found empirically that the shrinking factor leading to a good convergence of the algorithm lies between

$\delta=0.7$ and $\delta=0.8$. The ASM algorithm converges when the root of the system of non linear equations given by

$$f(x_{em}) = P(x_{em}) - x_c^* \quad (5)$$

is found, which implies that the EM model response should approximate the optimal coarse model response x_c^* .

A. Optimal Coarse Solution x_c^*

The initial step in any ASM-based synthesis methodology requires to find an optimal solution of the coarse model x_c^* , i.e. the electrical parameter values that define the target frequency response to synthesize. These values either are known in advance or must be calculated in order to satisfy the desired specifications. In the second case, it involves the solution of simple equations that result, for instance, by forcing the image impedance and phase shift of the cell to certain values at two frequencies (as is the case of dual-band components, where the phase must be set to -90° and $+90^\circ$ at the lower and upper frequencies, respectively, and the image impedance to the necessary value for impedance matching). In enhanced bandwidth components, the criterion to chose the element values of the coarse model is to achieve the required phase slope of the unit cell so that the operative bandwidth of the devices using these artificial lines can be improved, as compared to conventional implementations. In this work, rather than pursuing specific performance or functionality for the considered CRLH lines, the main aim is to demonstrate the potentiality of the ASM algorithm; therefore, we have directly considered an arbitrary set of coarse model parameters, whose electrical response is our target response. These parameters are indicated in table I, and are those extracted from the structure reported in [13]. Then, we can check the capability of the ASM algorithm to synthesize that structure, and hence the accuracy of the approach.

TABLE I
COARSE MODEL PARAMETERS

	L [nH]	C [pF]	L_c [nH]	C_c [pF]	C_g [pF]
x_c^*	4.92	35.87	3.41	3.85	1.05

B. Estimation of $x_{em}^{(1)}$

The next step consists on finding an initial geometry of the cell, i.e. $x_{em}^{(1)}$. From the values of C and C_g , and inverting equations (1) and (2), we can obtain C_s and C_{par} . Typically, C_{par} is dominated by the line capacitance C_L , therefore, we can neglect C_f in a first order approximation, and consider $C_L = C_{par}$. With this value and the value of the line inductance, L , we can estimate the characteristic impedance of the host line and, using standard formulas [15], we can infer the line width, W , corresponding to the initial layout. With this value, the substrate height h (which is not a variable), and the value of C_s , we can determine the gap separation, s . Regarding the initial geometry of the CSRR, we use the model equations that appear in [12], but fixing one of the geometry parameters in order to find a deterministic solution. Specifically, c is fixed to 0.25mm, since smaller values than 0.15 mm cannot be easily manufactured, and much bigger ones would involve too big

dimensions for the CSRR. Then, with the electrical parameters of the CSRR and c , the external radius, r_{ext} , and rings separation, d , are obtained. Finally, the length of the line is set to twice the external radius of the CSRR. In Table II, the estimation of $x_{em}^{(1)}$ for the considered example is summarized.

TABLE II
FIRST ESTIMATION OF PHYSICAL DIMENSIONS

	r_{ext} [mm]	c [mm]	d [mm]	W [mm]	s [mm]
$x_{em}^{(1)}$	5.46	0.25	0.33	1.14	0.11

C. Parameter Extraction Algorithm

The equations needed to extract the circuit parameters of Fig. 1(c) from the electromagnetic simulation are:

$$f_o = \frac{1}{2\pi\sqrt{L_c C_c}} \quad (6)$$

$$f_s = \frac{1}{2\pi\sqrt{L C_g}} \quad (7)$$

$$f_z = \frac{1}{2\pi\sqrt{L_c(C + C_c)}} \quad (8)$$

$$Z_s(j\omega_{-\pi/2}) = -Z_p(j\omega_{-\pi/2}) \quad (9)$$

where f_o , f_s and f_z are, respectively, the frequency at the intercept point of S_{11} with the unit resistance circle in the Smith chart, the frequency at the intercept point of S_{11} with the unit conductance circle in the Smith chart, and the transmission zero frequency. On the other hand, $Z_s(j\omega)$ and $Z_p(j\omega)$ are the series and shunt impedances of the equivalent T-circuit model shown in Fig. 1, and $\omega_{-\pi/2}$ is the frequency where the phase shift of the unit cell is $-\pi/2$. In spite that we have 4 independent equations, from f_o (expression 6) we can infer the resonance frequency of the CSRR and the reactance of the series branch at CSRR resonance. Therefore, we have the required 5 independent conditions to univocally infer the 5 parameters of the circuit model. Hence, the corresponding x_c vector is straightforwardly derived from the EM simulation results. For the full-wave simulations we have used the Agilent Momentum® commercial software.

D. Initialization and Update of Broyden Matrix

The initialization of the Broyden matrix follows essentially a finite differences scheme and the update of the matrix responds to the classical approach [12]:

$$B^{(j+1)} = B^{(j)} + \frac{f^{(j+1)} h^{(j)T}}{h^{(j)T} h^{(j)}} \quad (10)$$

This Broyden matrix is used during the ASM algorithm to obtain the new step h , needed to calculate the next x_{em} in (4).

IV. RESULTS

A validation example is reported in this section in order to illustrate the efficiency and robustness of the proposed synthesis technique. In the example, the optimal coarse solution is chosen equal to the one provided in [13] for the CRLH structure (see table I). The considered substrate is the

commercial Rogers RO3010 with thickness $h=1.27\text{mm}$ and dielectric constant $\epsilon_r=10.2$. The ASM algorithm has been applied, and convergence has been achieved after 25 iterations (90 minutes). Our convergence criterion is defined by obtaining a norm lower than 0.04, where the norm is given by:

$$\|f(x_f)\| = \sqrt{\left(1 - \frac{L}{L^*}\right)^2 + \left(1 - \frac{C_g}{C_g^*}\right)^2 + \left(1 - \frac{C}{C^*}\right)^2 + \left(1 - \frac{C_c}{C_c^*}\right)^2 + \left(1 - \frac{L_c}{L_c^*}\right)^2} \quad (11)$$

In Fig. 2, we can see that the simulation results for the optimal coarse solution x_c^* and the final layout x_{em}^* exhibit a very good agreement. It must be noticed that the dimensions of the final solution (see Table III) are very close to the ones published in [13].

TABLE III
LAYOUT FOUND FOR THE OPTIMIZED SOLUTION

	$r_{ext}[\text{mm}]$	$c[\text{mm}]$	$d[\text{mm}]$	$W[\text{mm}]$	$s[\text{mm}]$
x_{em}^*	5.62	0.31	0.19	3.90	0.31

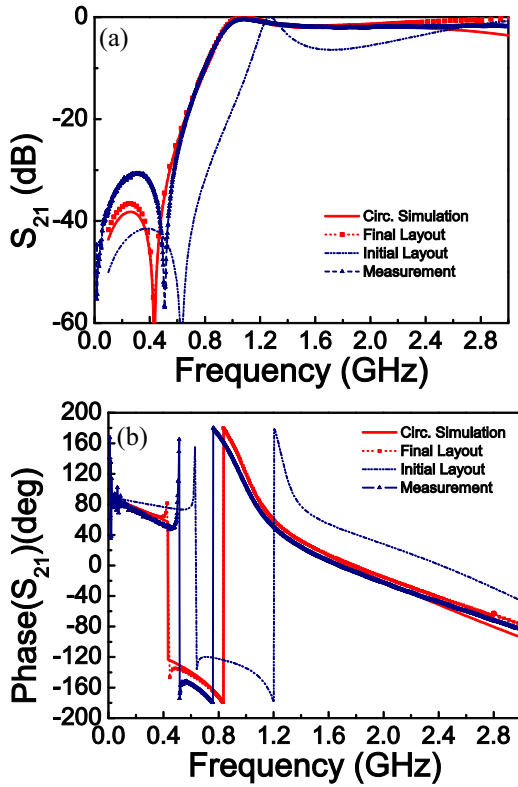


Fig. 2 Magnitude (a) and phase (b) of the transmission coefficient at initial solution $x_{em}^{(l)}$, final solution x_{em}^* , target response x_c^* , and measurement. The slight discrepancy with measurement is due to tolerances in the fabrication process.

V. CONCLUSION

It has been demonstrated that ASM is useful for the synthesis of CRLH microstrip lines based on a combination of CSRRs and series gaps. An excellent agreement between the target response and the final solution has been obtained in the reported example, thus validating the proposed approach. This work means a significant progress on the synthesis of artificial

lines based on split rings, since for the first time we have demonstrated the possibility to automatically generate the layout of composite right/left handed transmission lines.

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