Complex Power

Note Title 4/14/2011

Suppose we have V & I as follows:

$$V = V_0 \cos(\omega t + \alpha) = Re\left[V_0 e^{j\alpha} e^{j\omega t}\right] \Longrightarrow V_p = V_0 e^{j\alpha}$$
$$I = I_0 \cos(\omega t + \beta) = Re\left[I_0 e^{j\beta} e^{j\omega t}\right] \Longrightarrow I_p = I_0 e^{j\beta}$$

$$P = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} V_0 I_0 \cos(\omega t + \alpha) \cos(\omega t + \beta)\, dt$$

$$= \frac{\omega}{2\pi} \frac{V_0 I_0}{2} \int_0^{2\pi/\omega} \left[\cos(2\omega t + \alpha + \beta) + \cos(\alpha - \beta)\right] dt$$

$$= \frac{\omega}{2\pi} \frac{V_0 I_0}{2} \left[\sin\left(2\omega \frac{2\pi}{\omega} + \alpha + \beta\right) - \sin(\alpha + \beta) + \left(\frac{2\pi}{\omega} - 0\right) \cos(\alpha - \beta)\right]$$

$$= \frac{\omega}{2\pi} \frac{V_0 I_0}{2} \frac{2\pi}{\omega} \cos(\alpha - \beta)$$

$$P = \frac{V_0 I_0}{2} \cos(\alpha - \beta)$$

To get the same thing in in phasor arithmetic

Say we do $Re[V_p I_p] = V_0 I_0 \cos(\alpha + \beta)$

But that is not what we need. We need α & β to have opposite signs $\therefore$ we need to take the conjugate of 1 phasor

Hence $P = \frac{1}{2} Re\left[V_p I_p^*\right]$