

Chapter 13: Introduction to Switched-Capacitor Circuits

13.1 General Considerations

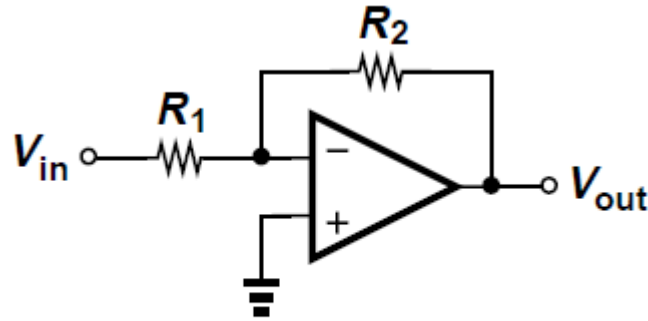
13.2 Sampling Switches

13.3 Switched-Capacitor Amplifiers

13.4 Switched-Capacitor Integrator

13.5 Switched-Capacitor Common-Mode Feedback

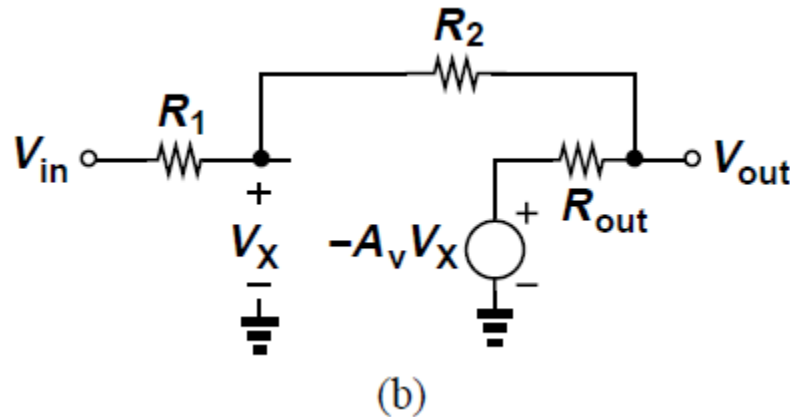
General Considerations



(a)

- For continuous-time amplifier [Fig. (a)], $V_{out}/V_{in} = -R_2/R_1$ ideally
- Difficult to implement in CMOS technology
- Typically, open-loop output resistance of CMOS op-amps is maximized to maximize A_v
- R_2 heavily drops open-loop gain, affecting precision

General Considerations



- In equivalent circuit of Fig. (b), we can write

$$-A_v \left(\frac{V_{out} - V_{in}}{R_1 + R_2} R_1 + V_{in} \right) - R_{out} \frac{V_{out} - V_{in}}{R_1 + R_2} = V_{out}$$

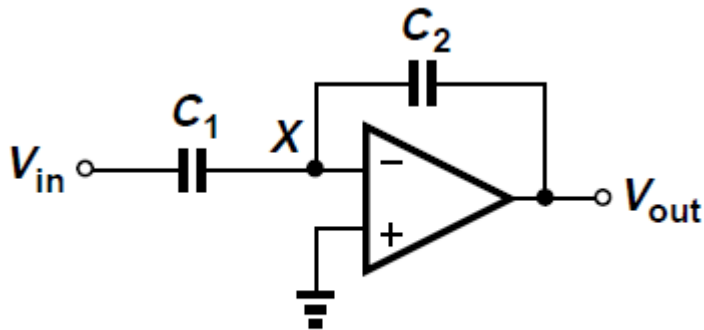
- Hence,

$$\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1} \cdot \frac{A_v - \frac{R_{out}}{R_2}}{1 + \frac{R_{out}}{R_1} + A_v + \frac{R_2}{R_1}}$$

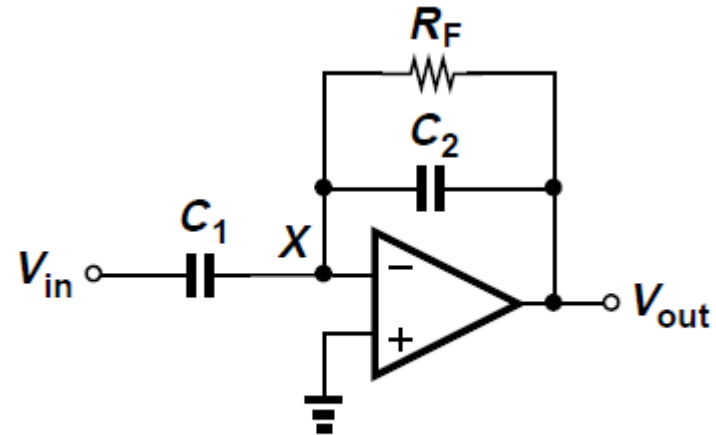
- Closed-loop gain is inaccurate compared to when $R_{out} = 0$

General Considerations

- To reduce open-loop gain, resistors can be replaced by capacitors [Fig. (a)]
- Gain of this circuit is ideally $-C_1/C_2$
- To set bias voltage at node X, large feedback resistor can be added [Fig. (b)]



(a)



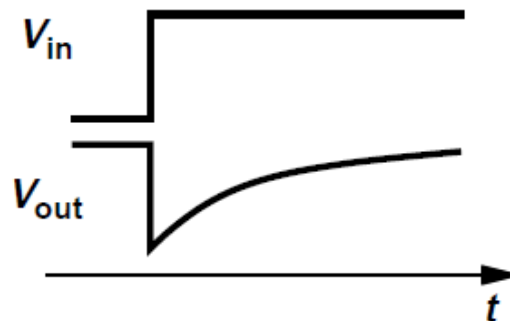
(b)

General Considerations

- Feedback resistor is not suited to amplify wideband signals
- Charge on C_2 is lost through R_F resulting in “tail”
- Circuit exhibits high-pass transfer function given by

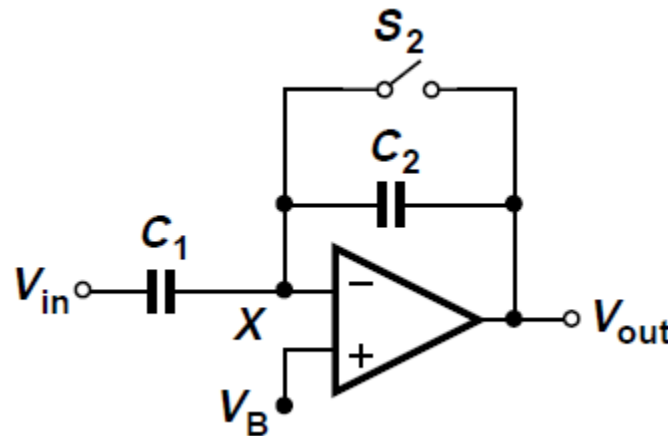
$$\begin{aligned}\frac{V_{out}}{V_{in}}(s) &\approx -\frac{R_F \frac{1}{C_2 s}}{R_F + \frac{1}{C_2 s}} \div \frac{1}{C_1 s} \\ &= -\frac{R_F C_1 s}{R_F C_2 s + 1},\end{aligned}$$

- $V_{out}/V_{in} \approx -C_1/C_2$ only if $\omega \gg (R_F C_2)^{-1}$.

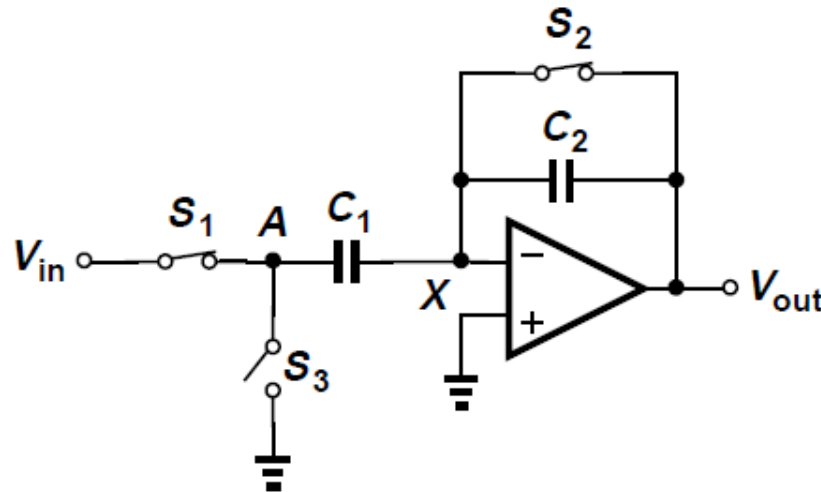


General Considerations

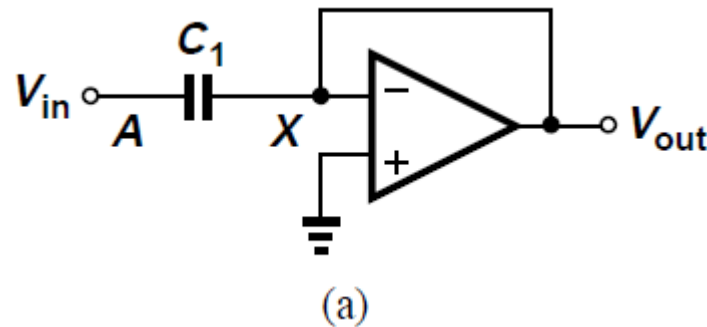
- R_F can be replaced by a switch
- S_2 is turned on to place op amp in unity gain feedback to force V_X equal to V_B , a suitable common-mode value
- When S_2 turns off, node X retains the voltage allowing amplification
- When S_2 is on, circuit does not amplify V_{in}



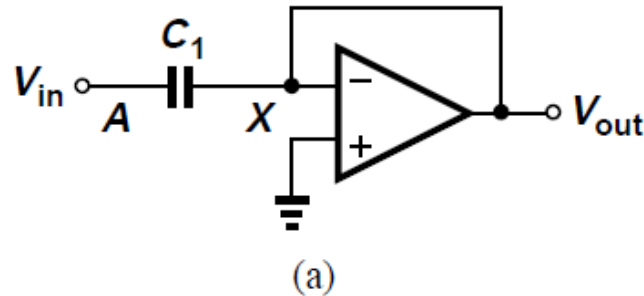
General Considerations



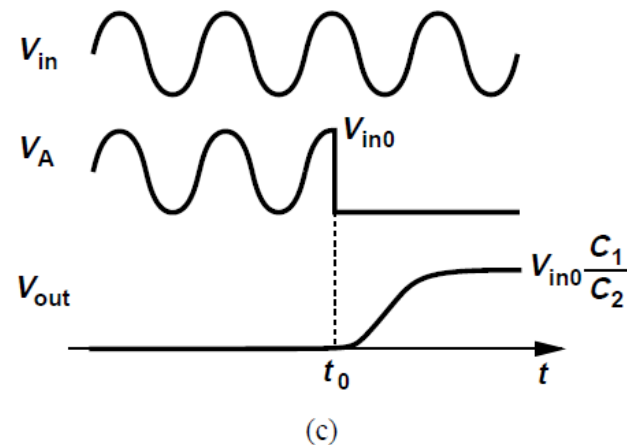
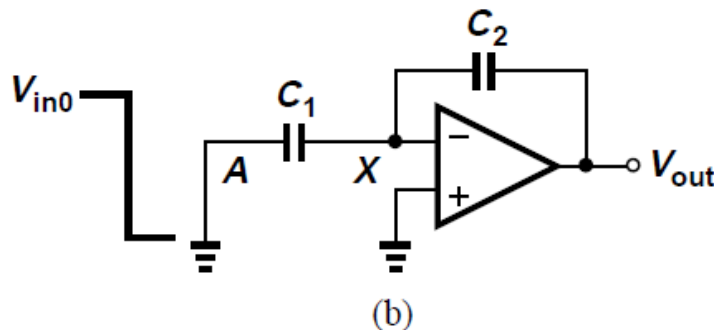
- In above circuit, S_1 and S_3 connect left plate of C_1 to V_{in} and ground, S_2 for unity-gain feedback
- Assume large open-loop gain of op amp
- First phase: S_1 and S_2 on, S_3 off [Fig. (a)]



General Considerations

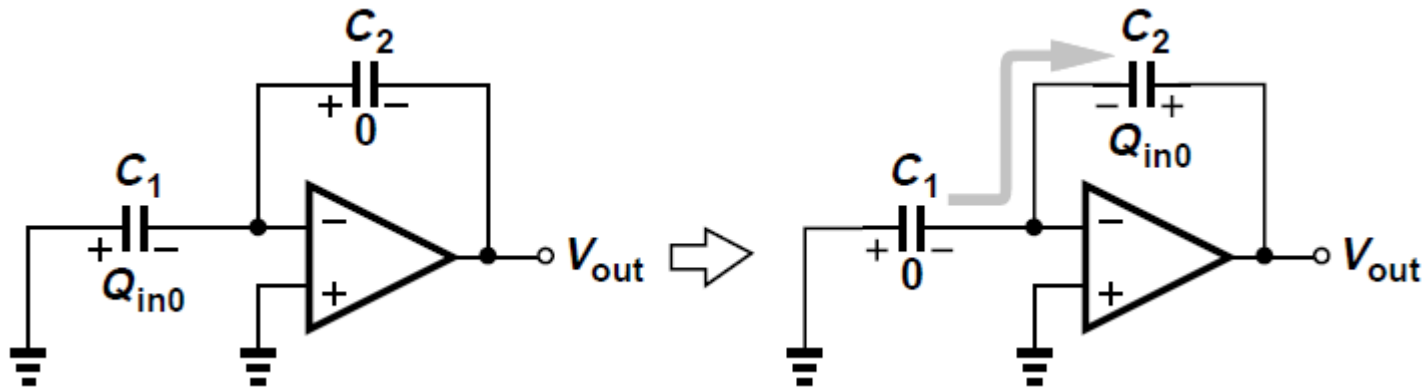


- Here, $V_B = V_{out} \approx 0$ and C_1 samples the input V_{in}
- Second phase: At $t = t_0$, S_1 and S_2 turn off and S_3 turns on, pulling node A to ground [Fig. (b)]
- V_A changes from V_{in} to 0, therefore V_{out} must change from zero to $V_{in0} C_1 / C_2$ [Fig. (c)]



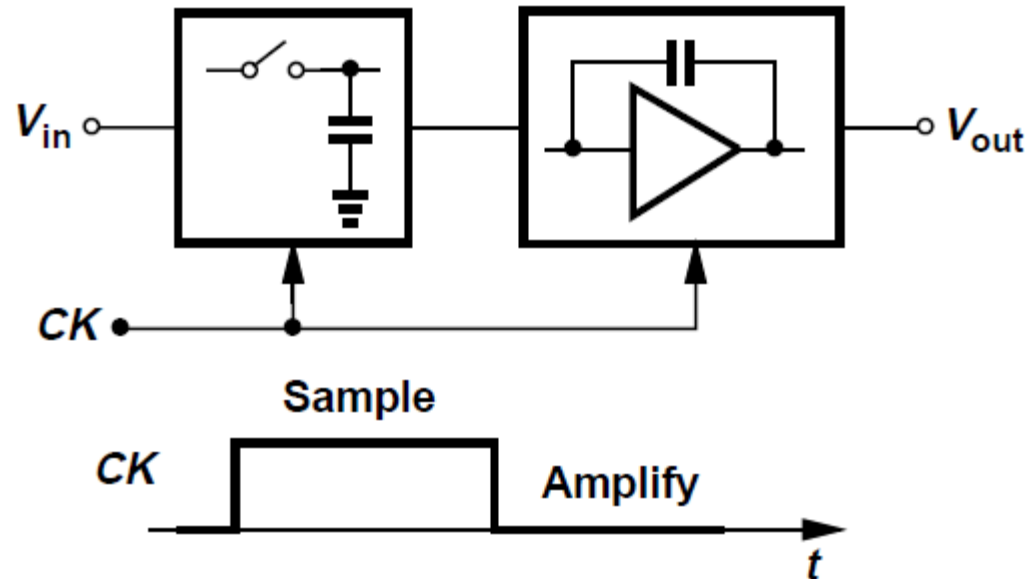
General Considerations

- Circuit devotes some time to sample input, setting output to zero and providing no amplification
- After sampling, for $t > t_0$, circuit ignores input voltage, amplifies sampled voltage



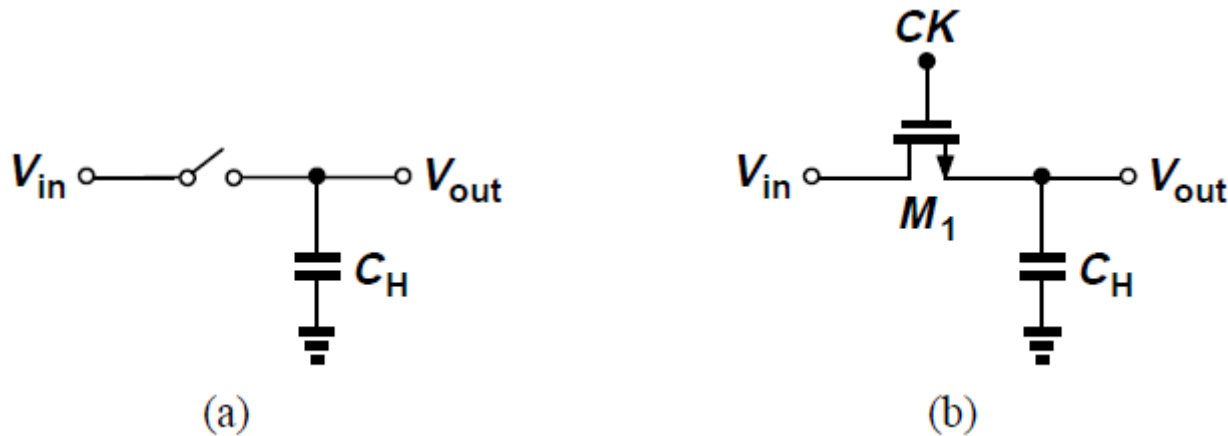
General Considerations

- Switched-capacitor amplifiers operate in two phases: Sampling and Amplification
- Clock needed in addition to analog input V_{in}

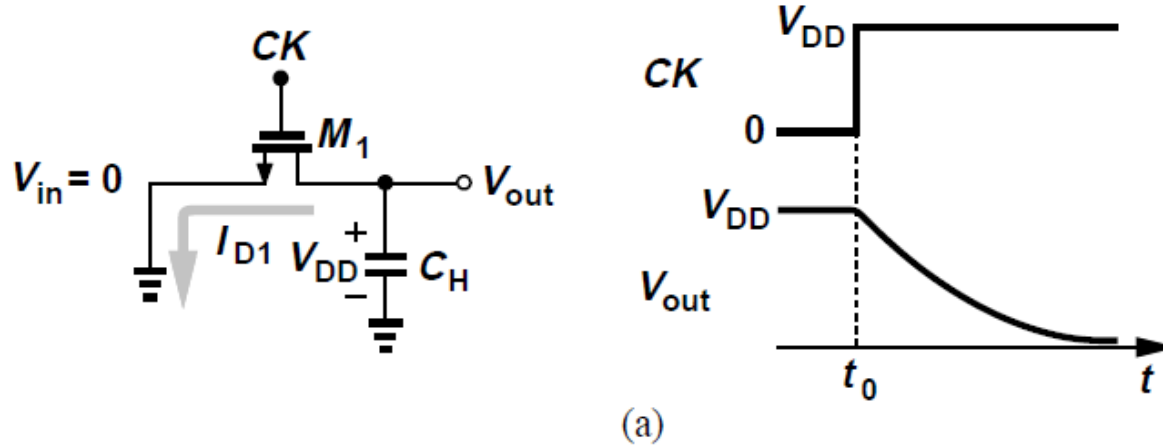


MOSFETS as Switches

- Sampling circuit consists of a switch and a capacitor [Fig. (a)]
- MOS transistor can function as switch [Fig. (b)] since it can be on while carrying zero current

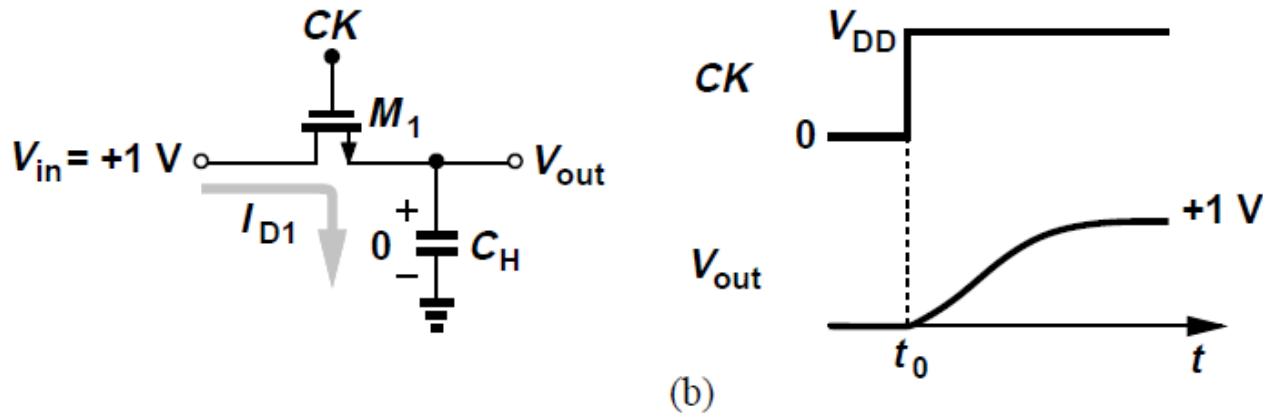


MOSFETS as Switches



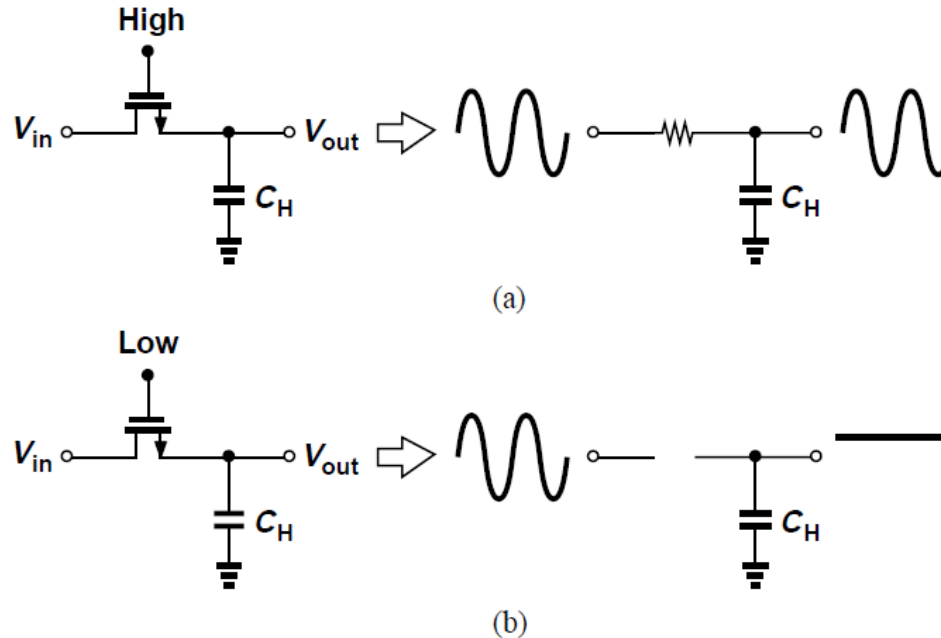
- CK goes high at $t = t_0$
- Assume $V_{in} = 0$ and capacitor has initial voltage V_{DD}
- At $t = t_0$, M_1 is in saturation and draws current
- As V_{out} falls, at some point M_1 goes into triode region
- C_H is discharged until V_{out} reaches zero
- For $V_{out} \ll 2(V_{DD} - V_{TH})$, transistor is an equivalent resistor

MOSFETS as Switches



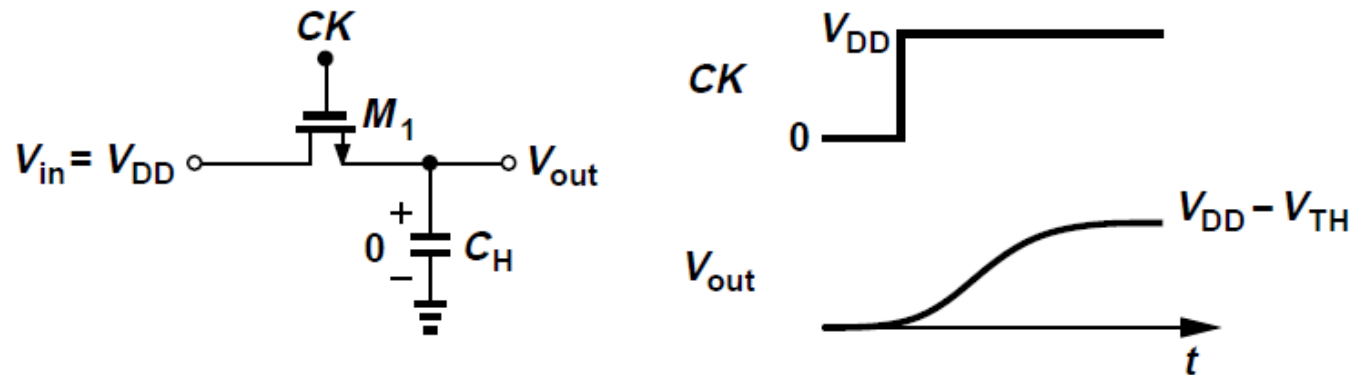
- If $V_{in} = +1\text{ V}$, $V_{out}(t = t_0) = +0\text{ V}$ and $V_{DD} = +3\text{ V}$
- Terminal of M_1 connected to C_H acts as source, and the transistor turns on with $V_{GS} = +3\text{ V}$ but $V_{DS} = +1\text{ V}$
- M_1 operates in triode region and charges C_H until V_{out} approaches $+1\text{ V}$
- For $V_{in} = +1\text{ V}$, M_1 exhibits an on-resistance of $R_{on} = [\mu_n C_{ox} (W/L) (V_{DD} - V_{in} - V_{TH})]^{-1}$

MOSFETS as Switches



- When switch is on [Fig. (a)], V_{out} follows V_{in}
- When switch is off [Fig. (b)], V_{out} remains constant
- Circuit “tracks” signal when CK is high and “freezes” instantaneous value of V_{in} across C_H when CK goes low

MOSFETS as Switches



- Suppose $V_{in} = V_0$ instead of +1 V
- M_1 is saturated and we have:

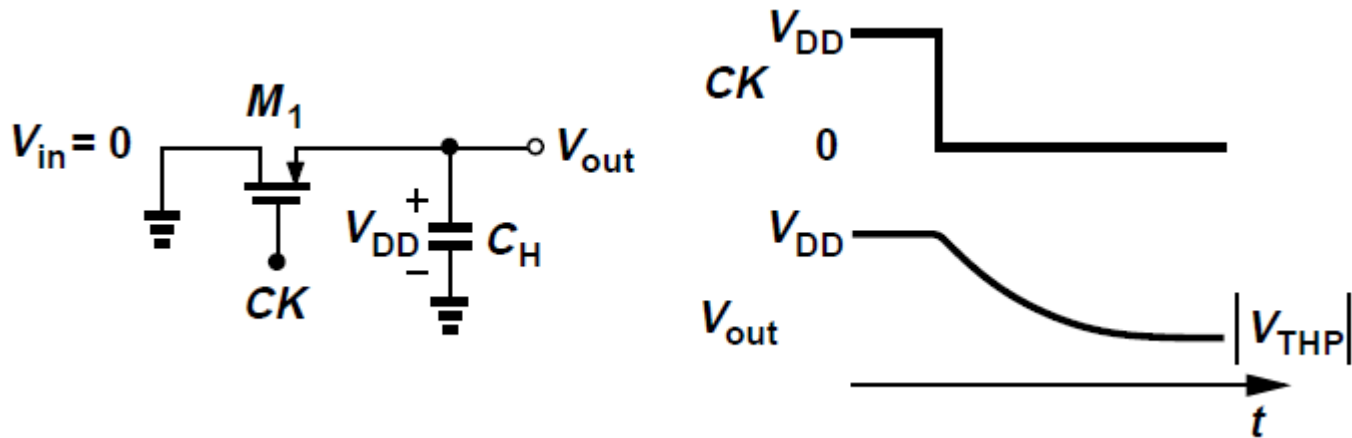
$$\begin{aligned} C_H \frac{dV_{out}}{dt} &= I_{D1} \\ &= \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{DD} - V_{out} - V_{TH})^2 \end{aligned}$$

- Solving,

$$V_{out} = V_{DD} - V_{TH} - \frac{1}{\frac{1}{2} \mu_n \frac{C_{ox}}{C_H} \frac{W}{L} t + \frac{1}{V_{DD} - V_{TH}}}$$

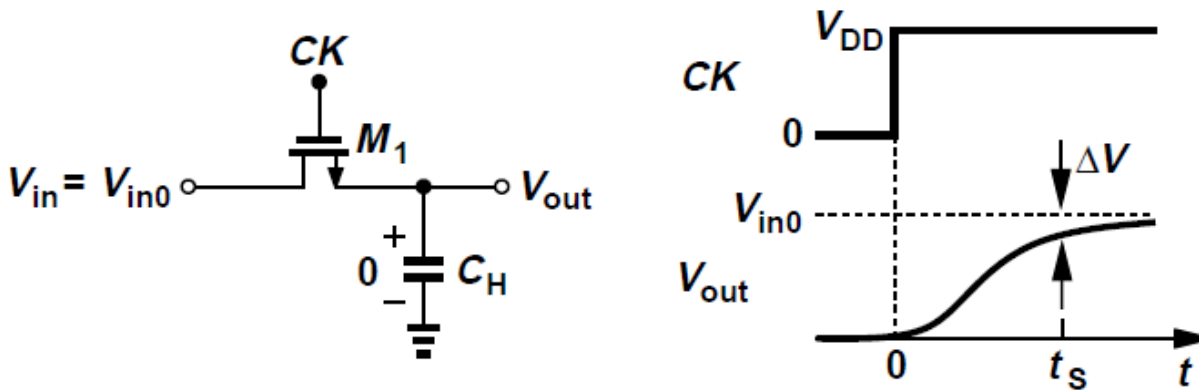
- As $t \rightarrow \infty$, $V_{out} \rightarrow V_{DD} - V_{TH}$ so NMOS cannot pull up to V_{DD}

MOSFETS as Switches



- Similarly, PMOS transistor fails to operate as a switch if gate is grounded and drain senses an input voltage of $|V_{THP}|$ or less
- On resistance rises rapidly as input and output levels fall to $|V_{THP}|$ above ground

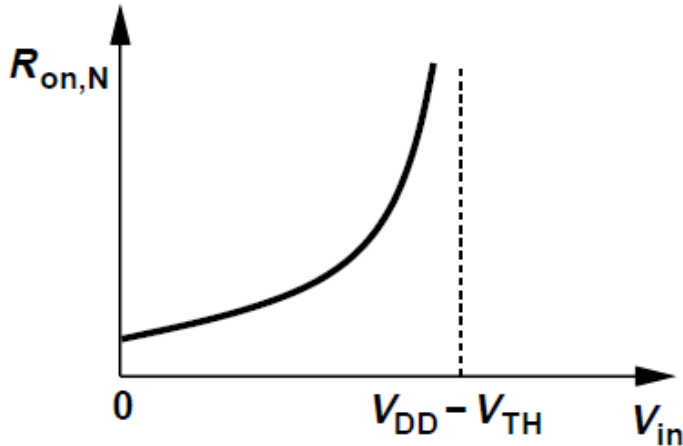
MOSFETs as Switches: Speed Considerations



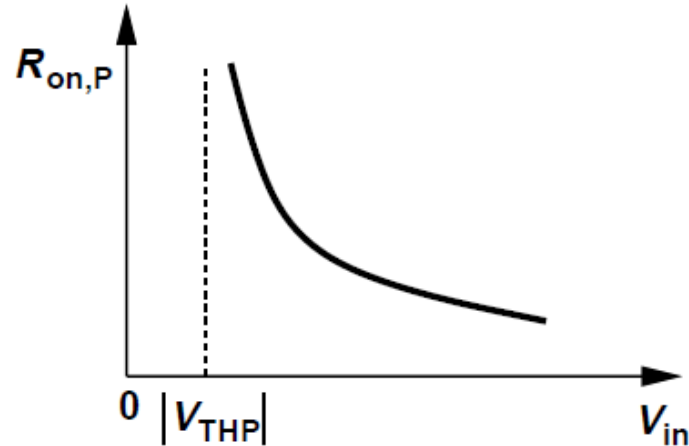
- Measure of speed is the time required for output to go from zero to the maximum input level after switch turns on
- Consider output settled within a certain “error band” ΔV around final value
- If output settles to 0.1% accuracy after t_s seconds, then $\Delta V/V_{in0} = 0.1\%$
- After $t = t_s$, consider source and drain voltages to be approximately equal

MOSFETs as Switches: Speed Considerations

- Sampling speed is given by two factors: switch on-resistance and sampling capacitance
- For higher speed, large aspect ratio and small capacitance are needed
- On-resistance also depends on input level for both NMOS and PMOS



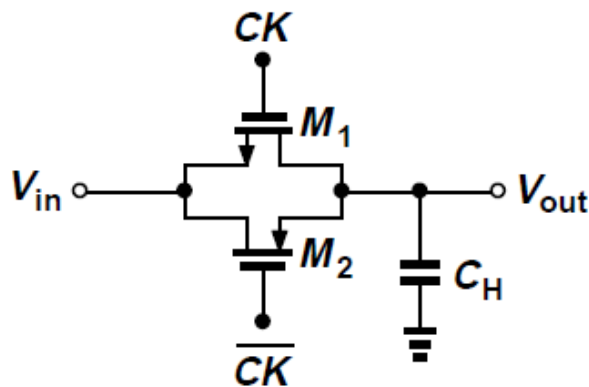
(a)



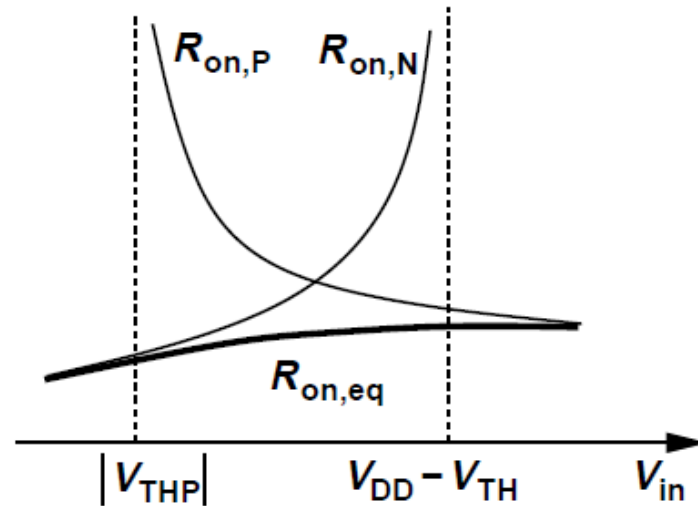
(b)

MOSFETs as Switches: Speed Considerations

- To allow greater input swings, we can use “complementary” switches, requiring complementary clocks [Fig. (a)]
- Equivalent on-resistance shows following behavior [Fig. (b)], revealing much less variation



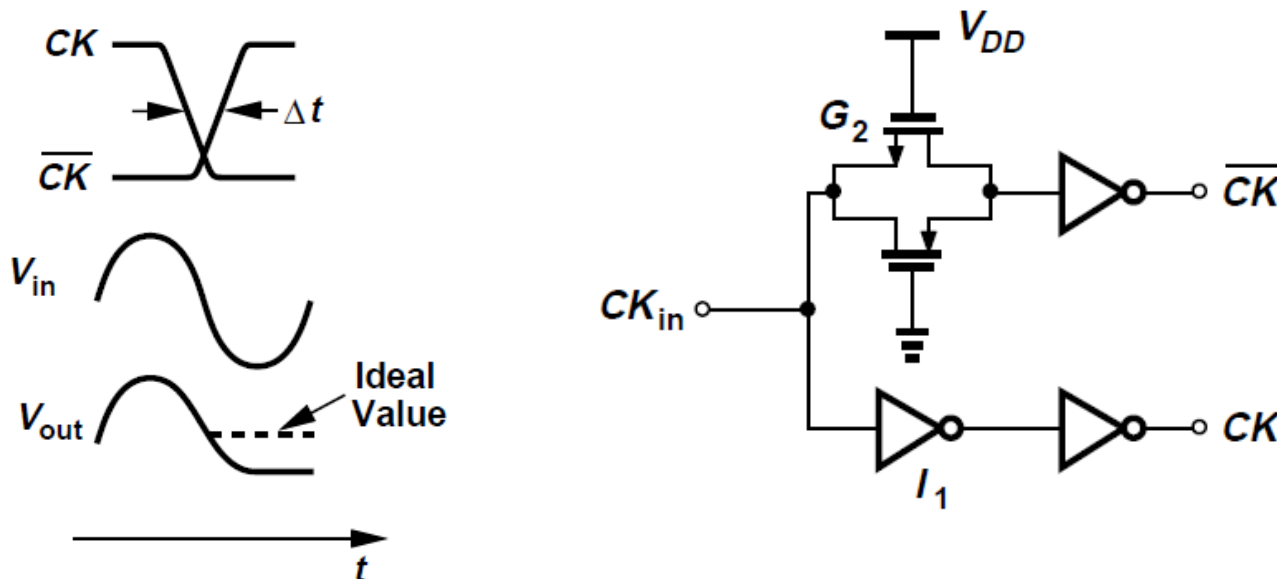
(a)



(b)

MOSFETs as Switches: Speed Considerations

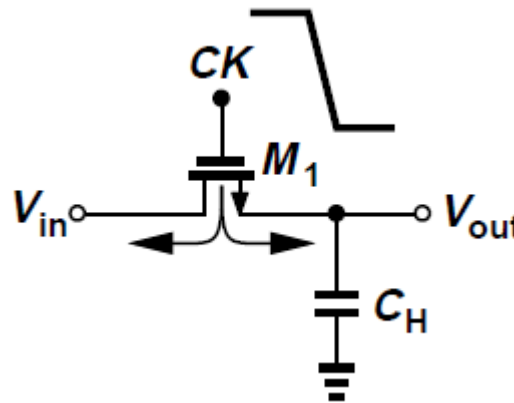
- For high speed signals, NMOS and PMOS switches must turn off simultaneously to avoid ambiguity in sampled value
- If NMOS turns off Δt seconds before PMOS, output tends to track input for the remaining Δt seconds, causing distortion
- For moderate precision, circuit below is used to provide complementary clocks



MOSFETs as Switches: Precision Considerations

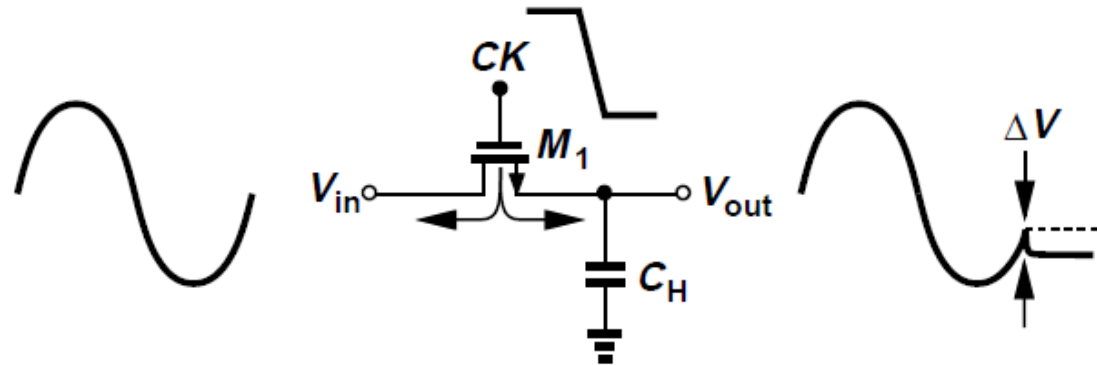
- Speed trades with precision
- Channel Charge Injection:
- For MOSFET to be on, a channel must exist at the oxide-silicon interface
- Assuming $V_{in} \approx V_{out}$, total charge in the inversion layer is

$$Q_{ch} = WLC_{ox}(V_{DD} - V_{in} - V_{TH})$$



- When switch turns off, Q_{ch} exits through the source and drain terminals (“channel charge injection”)

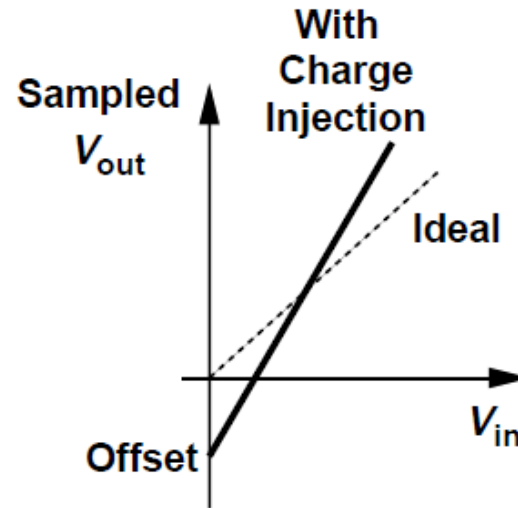
MOSFETs as Switches: Precision Considerations



- Charge injected to the left is absorbed by input source, creating no error
- Charge injected to the right deposited on C_H , introducing error in voltage stored on capacitor
- For half of Q_{ch} injected onto C_H , error (negative pedestal) equals

$$\Delta V = \frac{WLC_{ox}(V_{DD} - V_{in} - V_{TH})}{2C_H}$$

MOSFETs as Switches: Precision Considerations



- If all of the charge is deposited on C_H ,

$$V_{out} \approx V_{in} - \frac{WLC_{ox}(V_{DD} - V_{in} - V_{TH})}{C_H}$$

$$V_{out} = V_{in} \left(1 + \frac{WLC_{ox}}{C_H} \right) - \frac{WLC_{ox}}{C_H}(V_{DD} - V_{TH})$$

- Since we assume Q_{ch} is a linear function of V_{in} , circuit exhibits only gain error and dc offset

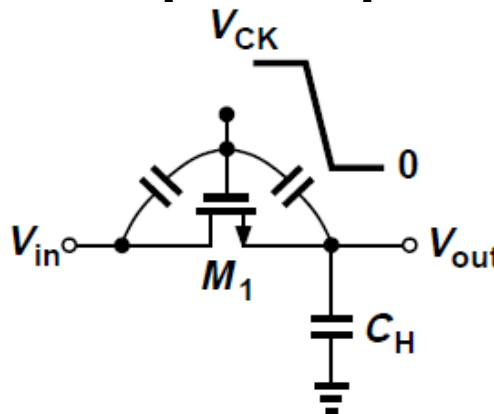
MOSFETs as Switches: Precision Considerations

- Clock Feedthrough:
- MOS switch couples clock transitions through C_{GD} or C_{GS}

- Sampled output voltage has error due to this give by

$$\Delta V = V_{CK} \frac{WC_{ov}}{WC_{ov} + C_H}$$

- C_{ov} is the overlap capacitance per unit width
- Error ΔV is independent of input level, manifests as constant offset in the input/output characteristic

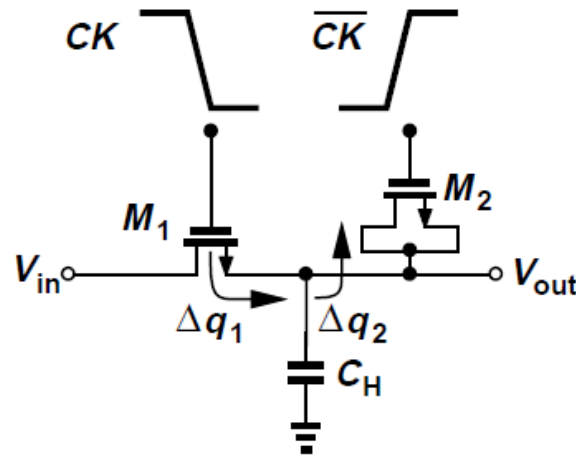


MOSFETs as Switches: Precision Considerations

- kT/C Noise:
- Resistor charging a capacitor gives a total RMS noise voltage of $\sqrt{kT/C}$.
- On resistance of switch introduces thermal noise at output which is stored on the capacitor when switch turns off
- RMS voltage of sampled noise is still approximately equal to $\sqrt{kT/C}$.



Charge Injection Cancellation

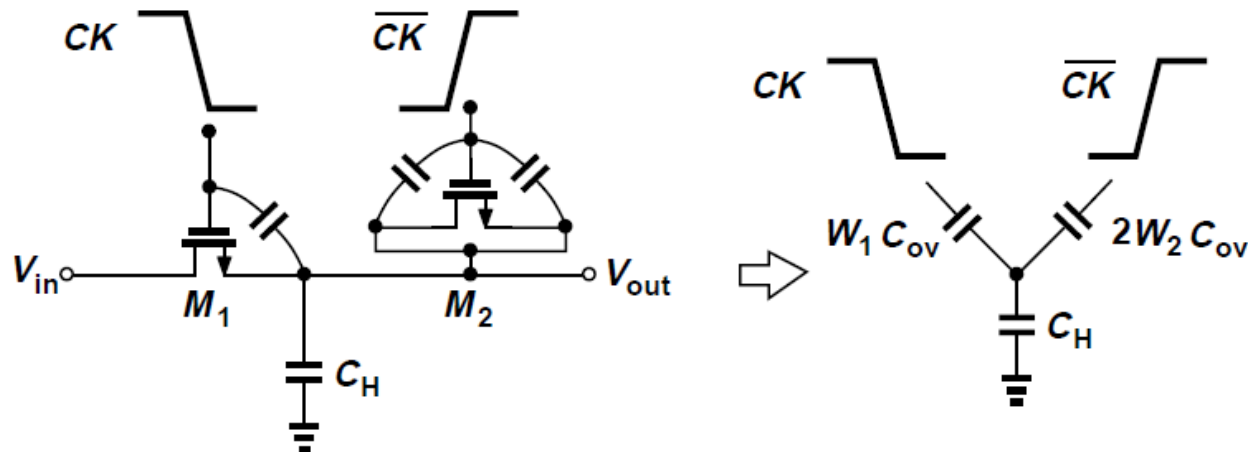


- Charge injected by main transistor removed by a dummy transistor M_2
- M_2 is driven by \overline{CK} so that after M_1 turns off and M_2 turns on, channel charge deposited by M_1 on C_H is absorbed by M_2 to create a channel
- If $W_2 = 0.5W_1$, then charge injected by M_1 , Δq_1 is equal to that absorbed by M_2

$$\Delta q_1 = \frac{W_1 L_1 C_{ox}}{2} (V_{CK} - V_{in} - V_{TH1})$$

$$\Delta q_2 = W_2 L_2 C_{ox} (V_{CK} - V_{in} - V_{TH2})$$

Charge Injection Cancellation

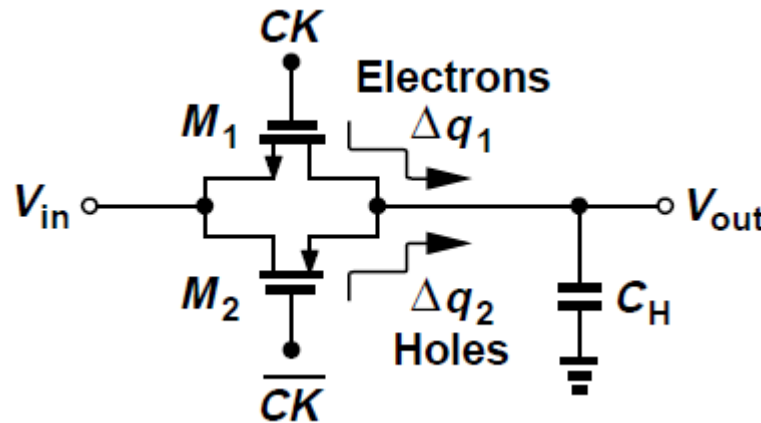


- If $W_2 = 0.5W_1$ and $L_2 = L_1$, effect of clock feedthrough is suppressed
- Total change in V_{out} is zero because

$$-V_{CK} \frac{W_1 C_{ov}}{W_1 C_{ov} + C_H + 2W_2 C_{ov}} + V_{CK} \frac{2W_2 C_{ov}}{W_1 C_{ov} + C_H + 2W_2 C_{ov}} = 0$$

Charge Injection Cancellation

- Incorporate both PMOS and NMOS devices so that opposite charge packets injected cancel each other



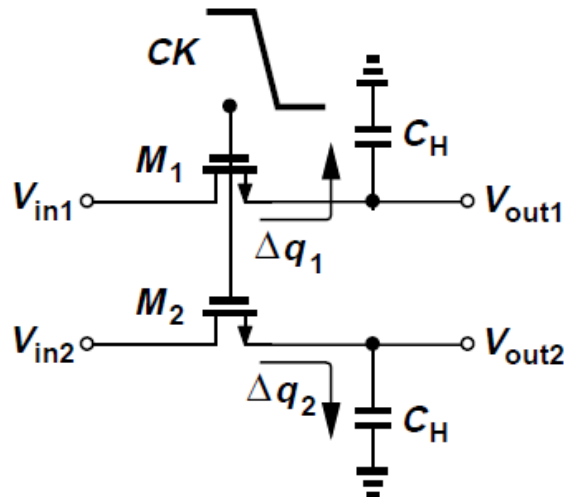
- For Δq_1 to cancel Δq_2 , we must have

$$W_1 L_1 C_{ox} (V_{CK} - V_{in} - V_{THN}) = W_2 L_2 C_{ox} (V_{in} - |V_{THP}|)$$

- Cancellation occurs for only one input level
- Clock feedthrough is not completely suppressed since C_{GD} of NFETs is not equal to that PFETs

Charge Injection Cancellation

- Charge injection appears as a common-mode disturbance, may be countered by differential operation

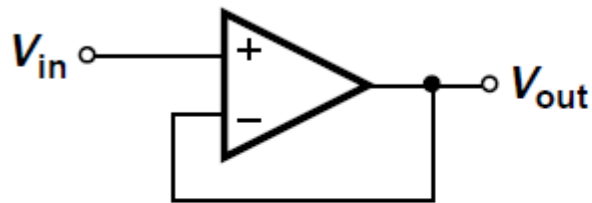


$\forall \Delta q_1 = \Delta q_2$ only if $V_{in1} = V_{in2}$, thus overall error is not suppressed for differential signals

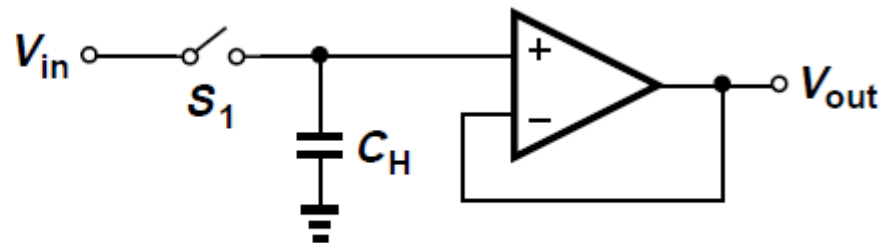
- Removes constant offset and nonlinear component

$$\begin{aligned}\Delta q_1 - \Delta q_2 &= WLC_{ox}[(V_{in2} - V_{in1}) + (V_{TH2} - V_{TH1})] \\ &= WLC_{ox} \left[V_{in2} - V_{in1} + \gamma \left(\sqrt{2\phi_F + V_{in2}} - \sqrt{2\phi_F + V_{in1}} \right) \right]\end{aligned}$$

Unity-Gain Sampler/ Buffer



(a)

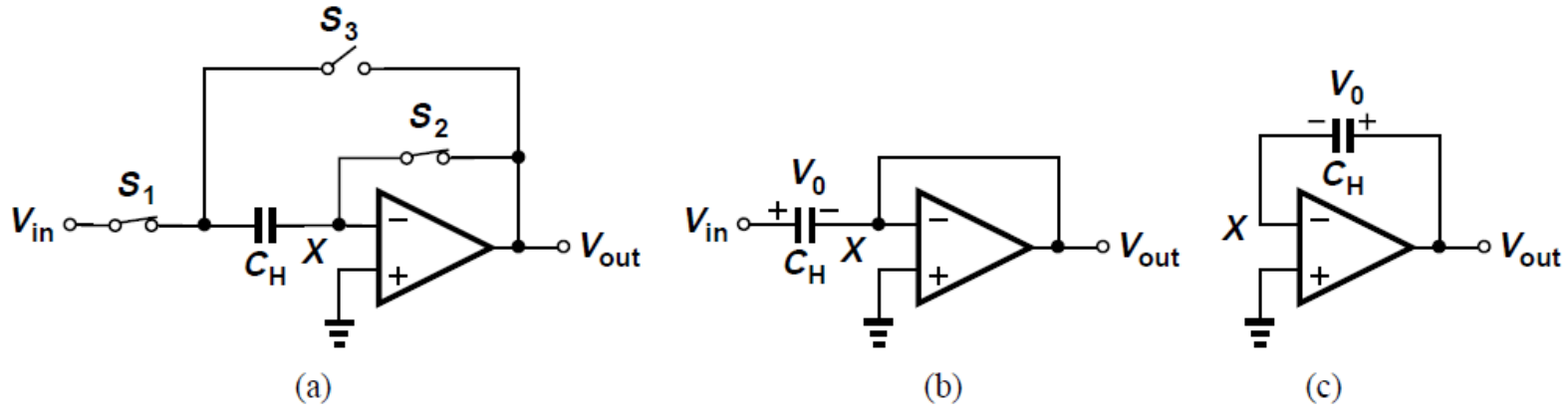


(b)

- For discrete-time applications, unity-gain amplifier [Fig. (a)] requires a sampling circuit [Fig. (b)]
- Accuracy limited by input-dependent charge injected by S_1 onto C_H

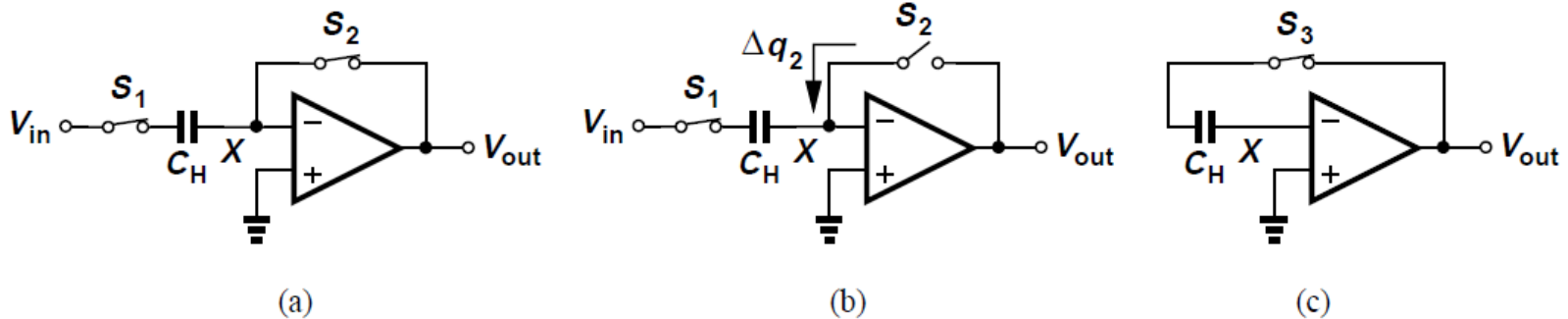
Unity-Gain Sampler/ Buffer

- Consider the topology shown in Fig. (a)

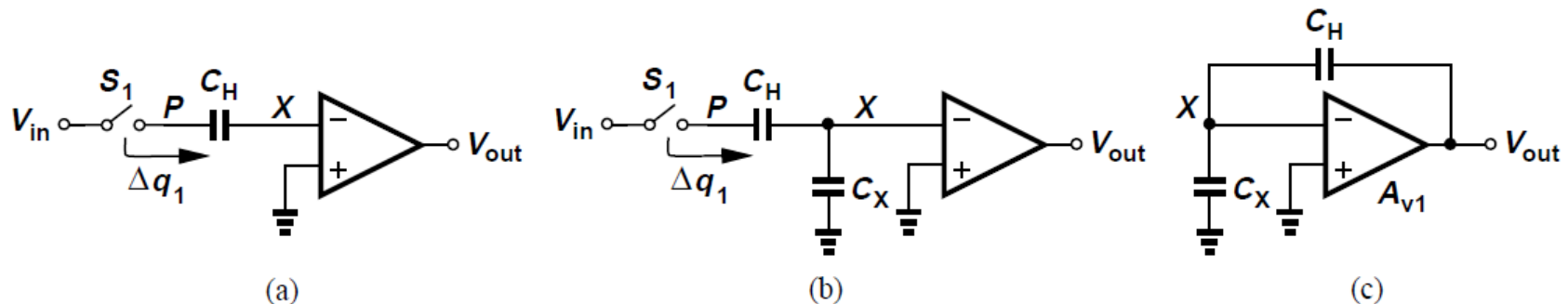


- In sampling mode, S_1 and S_2 are on, S_3 is off yielding circuit in Fig. (b)
- Thus, $V_{out} = V_X \approx 0$, and the voltage across C_H tracks V_{in}
- At $t = t_0$, when $V_{in} = V_0$, S_1 and S_2 turn off and S_3 turns on, yielding circuit of Fig. (c) [amplification mode]
- Op amp requires node X is still a virtual ground, V_{out} rises to approximately $V_0 \rightarrow$ “frozen” for processing by subsequent stages

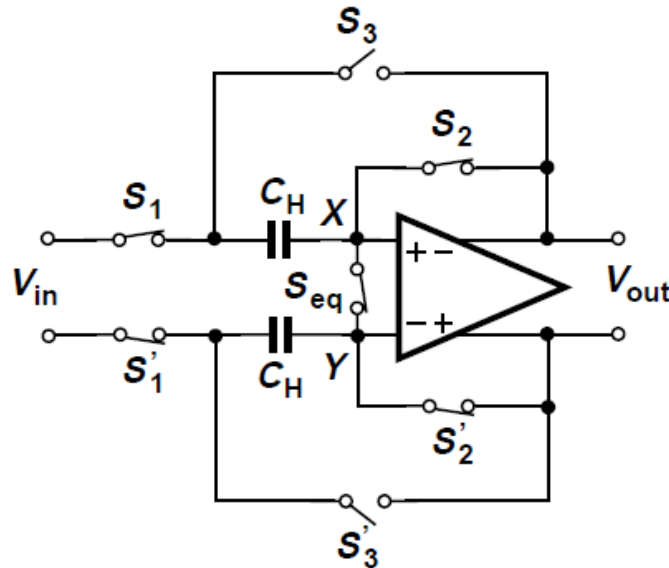
Unity-Gain Sampler/ Buffer



- S_2 turns off slightly before S_1 during transition from sampling mode to amplification mode
- Charge injected by S_2 , Δq_2 is input-independent and constant, producing only an offset
- After S_2 turns off, total charge at node X stays constant and charge injected by S_1 does not affect output voltage



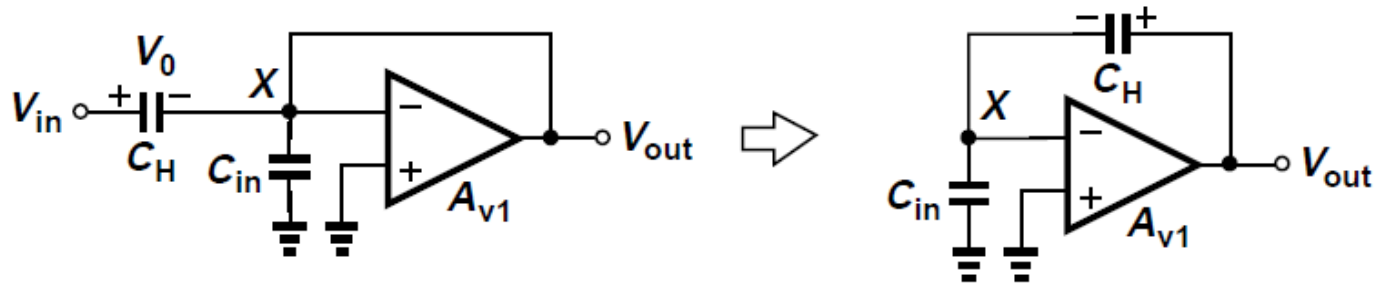
Unity-Gain Sampler/ Buffer



- Input-independent charge injected by S_2 can be cancelled by differential operation as shown
- Charge injected by S_2 and S_2' appears as common-mode disturbance at nodes X and Y
- Charge injection mismatch between S_2 and S_2' resolved by adding another switch S_{eq} that turns off slightly after S_2 and S_2' , equalizing the charge at nodes X and Y

Unity-Gain Sampler/ Buffer

- Precision Considerations:
- Assume op-amp has a finite input capacitance C_{in} and calculate output voltage when circuit goes from sampling to amplification mode



- It can be shown from the above fig. that

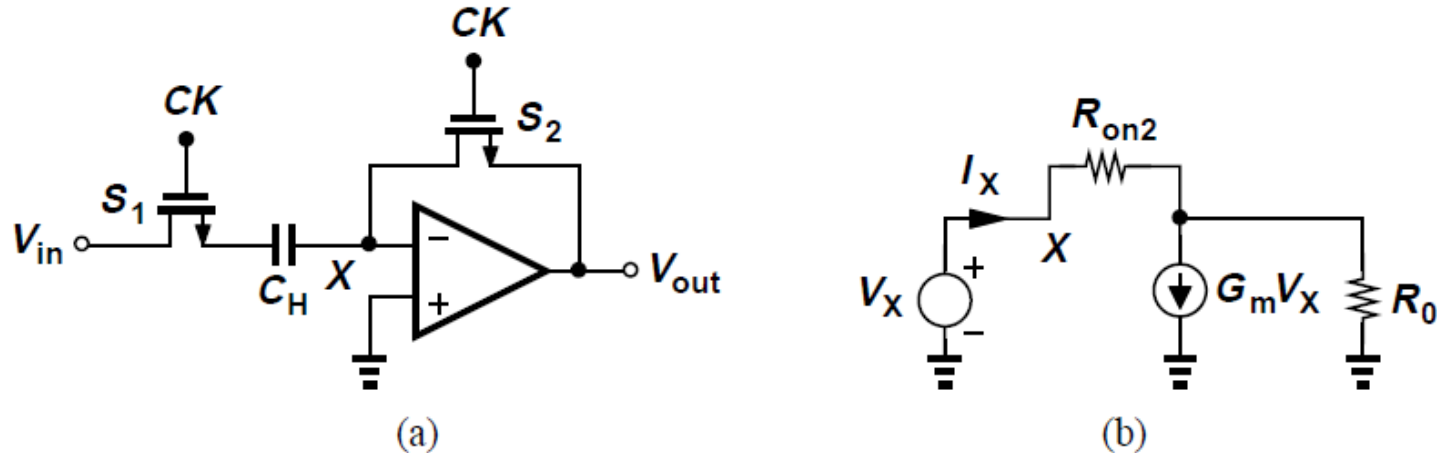
$$V_{out} = \frac{V_0}{1 + \frac{1}{A_{v1}} \left(\frac{C_{in}}{C_H} + 1 \right)}$$

$$\approx V_0 \left[1 - \frac{1}{A_{v1}} \left(\frac{C_{in}}{C_H} + 1 \right) \right]$$

- Circuit suffers from gain error of approximately $-(C_{in}/C_H + 1)/A_{v1}$

Unity-Gain Sampler/ Buffer

- Speed Considerations:
- In sampling mode, circuit appears as in Fig. (a)



- Use equivalent circuit of Fig. (b) to find time constant in sampling mode
- Total resistance in series with C_H is R_{on1} and the resistance between X and ground, R_X

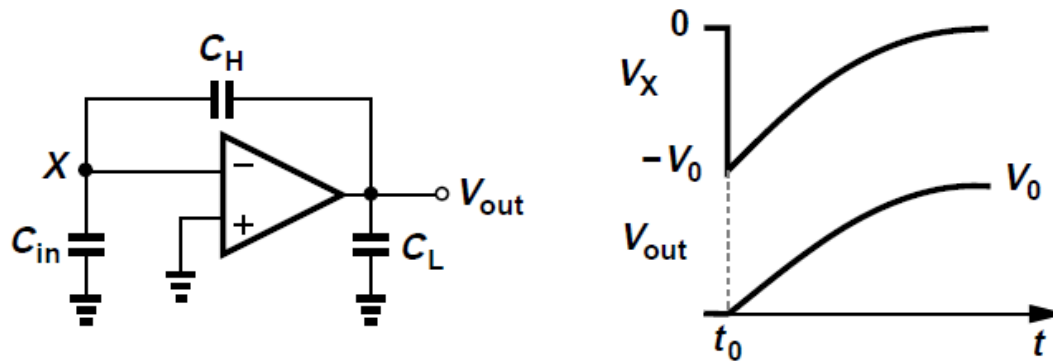
$$R_X = \frac{R_0 + R_{on2}}{1 + G_m R_0}$$

Unity-Gain Sampler/ Buffer

- Since typically $R_{on2} \ll R_0$ and $G_m R_0 \gg 1$, $R_X \approx 1/G_m$
- Time constant in sampling mode is thus

$$\tau_{sam} = \left(R_{on1} + \frac{1}{G_m} \right) C_H$$

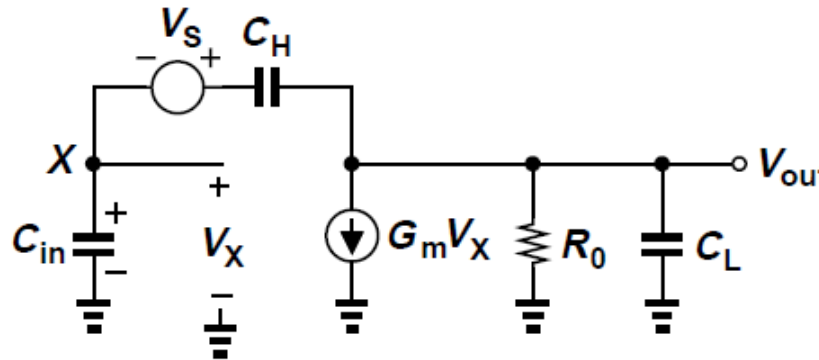
- Consider circuit as it enters amplification mode



- Circuit must begin with $V_{out} \approx 0$ and eventually produce $V_{out} \approx V_0$
- For relatively small C_{in} , voltages across C_L and C_H do not change instantaneously so that $V_X = -V_0$ at the beginning of amplification

Unity-Gain Sampler/ Buffer

- Represent charge on C_H by a voltage source V_S that goes from zero to V_0 at $t = t_0$, while C_H carries no charge itself



- The transfer function $V_{out}(s)/V_{in}(s)$ can be obtained as

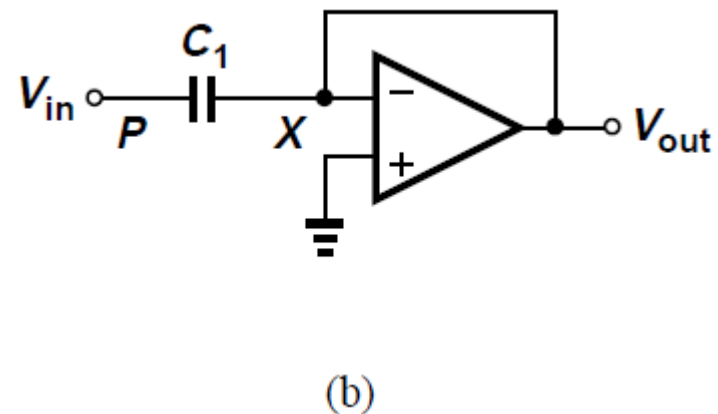
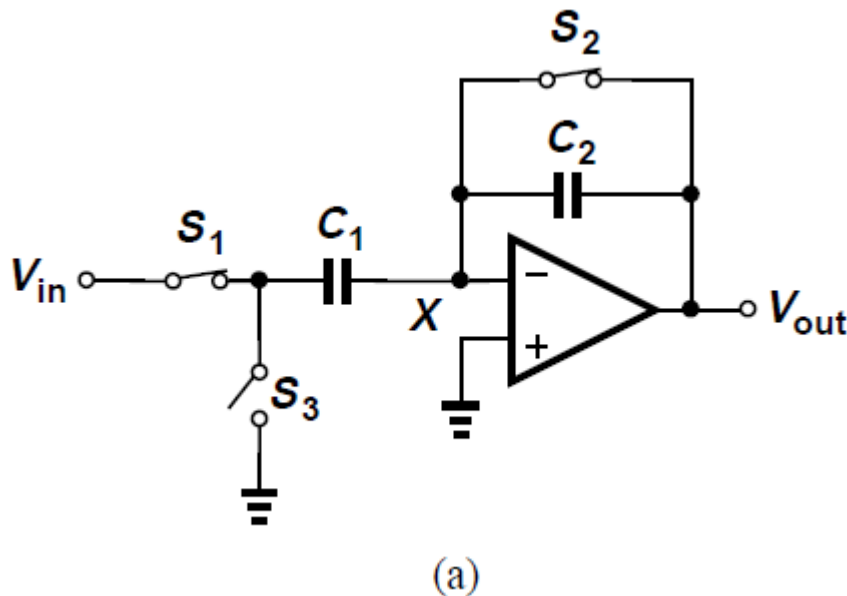
$$\frac{V_{out}}{V_S}(s) = \frac{(G_m + C_{in}s)C_H}{(C_L C_{in} + C_{in} C_H + C_H C_L)s + G_m C_H}$$

- This response is characterized by a time constant independent of op-amp output resistance

$$\begin{aligned} \tau_{amp} &= \frac{C_L C_{in} + C_{in} C_H + C_H C_L}{G_m C_H} \\ &= \frac{1}{G_m} \left[C_{in} + \left(1 + \frac{C_{in}}{C_H} \right) C_L \right] \end{aligned}$$

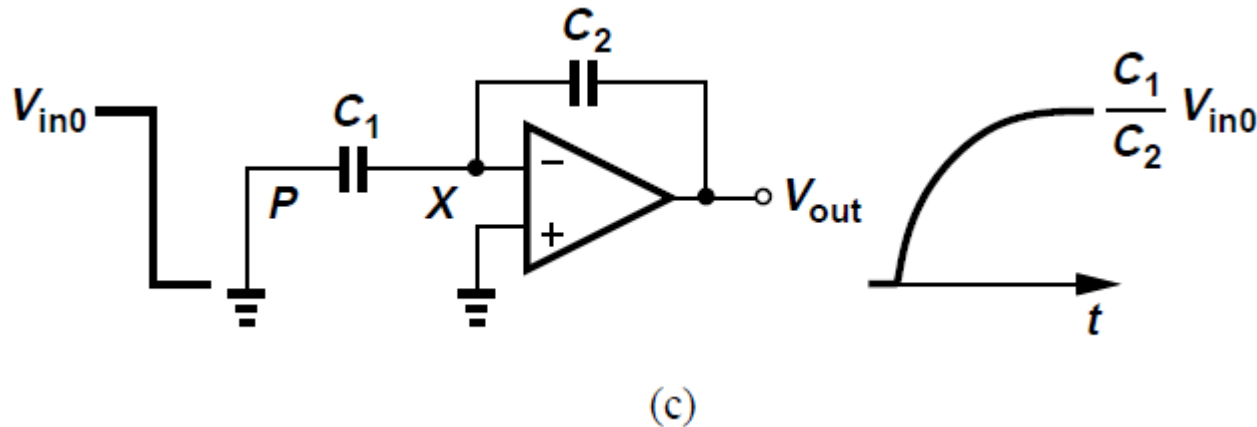
Noninverting Amplifier

- In non-inverting amplifier of Fig. (a), in sampling mode, S_1 and S_2 are on while S_3 is off, creating a virtual ground at X and allowing voltage across C_1 to track V_{in} [Fig. (b)]



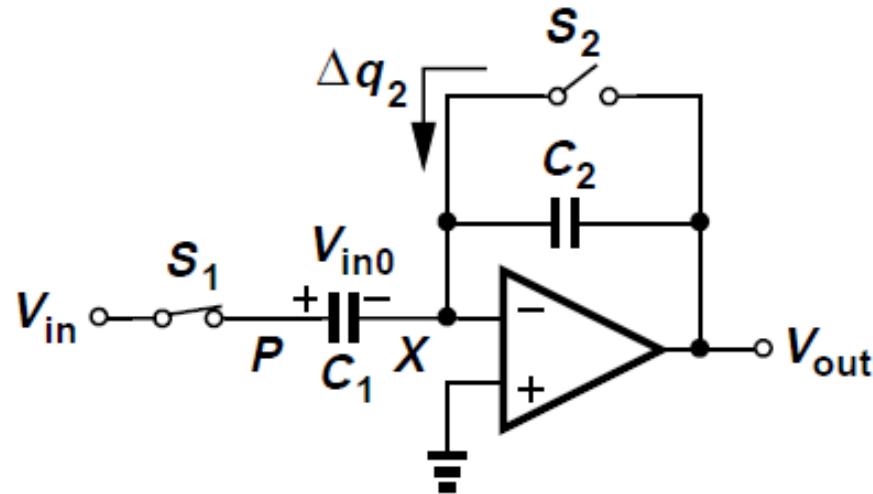
Noninverting Amplifier

- At the end of sampling mode, S_2 turns off first, injecting a constant charge Δq_2 onto node X , after which S_1 turns off and S_3 turns on [Fig. (c)]
- Since V_p goes from V_{in0} to 0, output voltage changes from 0 to approximately $V_{in0}(C_1/C_2)$, providing a gain of C_1/C_2
- Called a “noninverting amplifier” since output polarity is the same as V_{in0} and the gain can be greater than unit



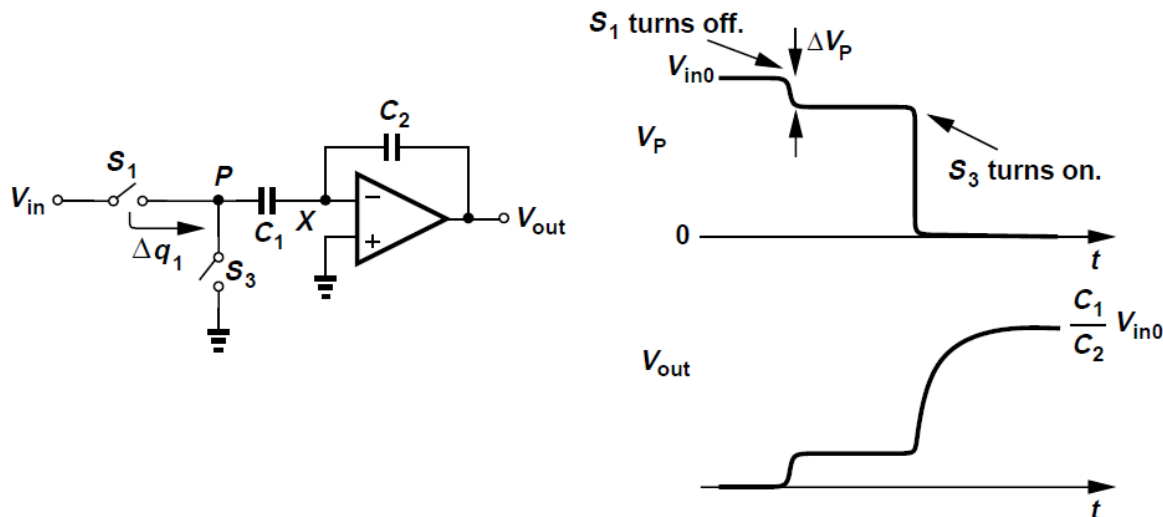
Noninverting Amplifier

- Noninverting amplifier avoids input-dependent charge injection by turning off S_2 before S_1
- After S_2 is off, total charge at node X remains constant, making the circuit insensitive to charge injection of S_1 or charge “absorption” of S_3



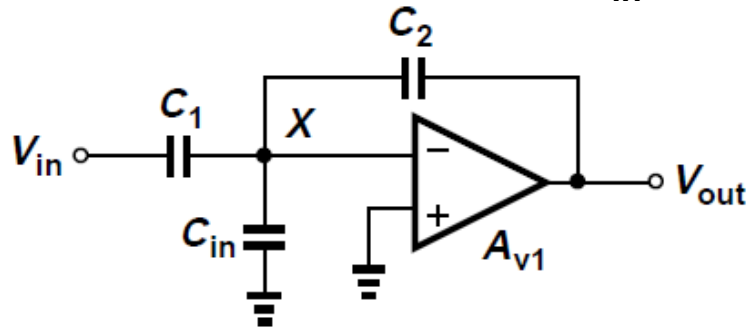
Noninverting Amplifier

- Charge injected by S_1 , Δq_1 changes voltage at node P by $\Delta V_P = \Delta q_1 / C_1$ and output voltage by $-\Delta q_1 C_1 / C_2$
- After S_3 turns on, V_P becomes zero so overall change in V_P is $0 - V_{in0} = -V_{in0}$, producing overall change in output of $-V_{in0}(-C_1 / C_2) = V_{in0} C_1 / C_2$
- V_P goes from V_0 to 0 with a perturbation due to S_1
- Since output is measure after node P is connected to ground, charge injected by S_1 does not affect final output



Noninverting Amplifier

- Precision Considerations:
- Calculate actual gain if op amp has finite open-loop gain of A_{v1} and input capacitance C_{in}



$$\left| \frac{V_{out}}{V_{in}} \right| \approx \frac{C_1}{C_2} \left(1 - \frac{C_2 + C_1 + C_{in}}{C_2} \cdot \frac{1}{A_{v1}} \right)$$

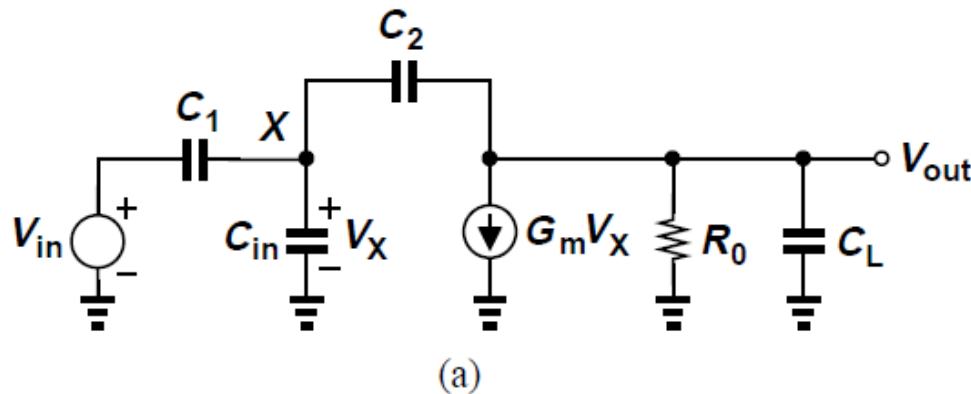
- It can be shown that

$$(C_2 + C_1 + C_{in}) / (C_2 A_{v1})$$

- Amplifier suffers from a gain error of

Noninverting Amplifier

- Speed Considerations:
- Consider equivalent circuit in amplification mode [Fig. (a)]



- It can be shown for a large $G_m R_0$ that

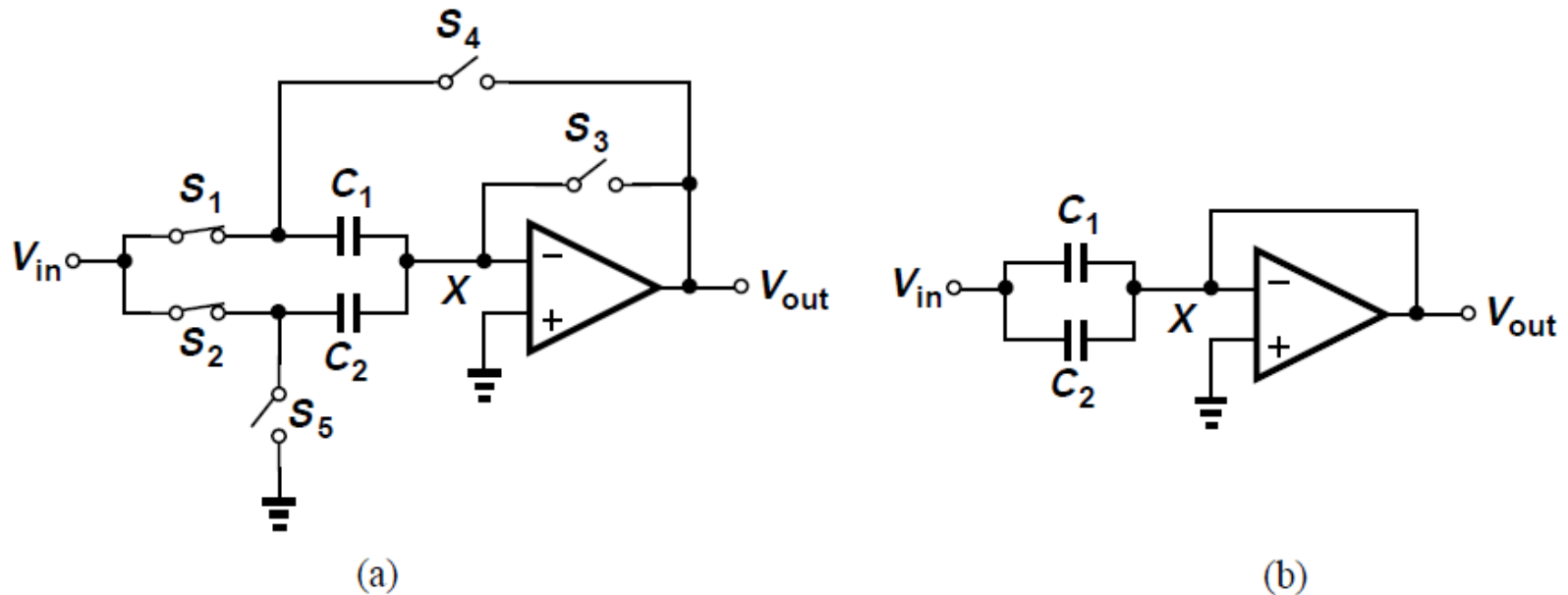
$$\frac{V_{out}}{V_{in}}(s) \approx \frac{-C_{eq} \frac{C_1}{C_1 + C_{in}} (G_m - C_2 s) R_0}{R_0 (C_L C_{eq} + C_L C_2 + C_{eq} C_2) s + G_m R_0 C_2}$$

- This gives a time constant of

$$\tau_{amp} = \frac{C_L C_{eq} + C_L C_2 + C_{eq} C_2}{G_m C_2}$$

Precision Multiply-by-Two Circuit

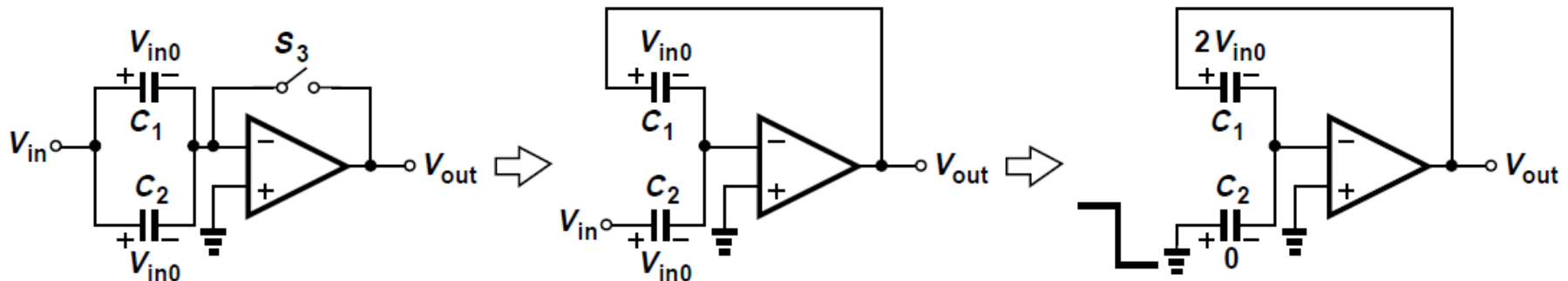
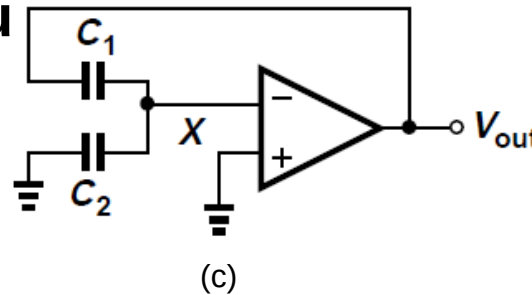
- Topology shown in Fig. (a) provides a nominal gain of two while achieving higher speed and lower gain error



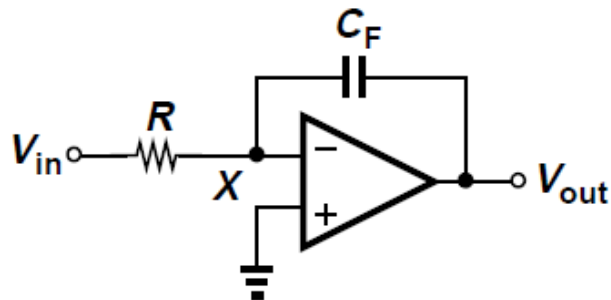
- Incorporates two equal capacitors $C_1 = C_2 = C$
- In sampling mode [Fig. (b)], node X is a virtual ground, allowing voltage across C_1 and C_2 to track V_{in}

Precision Multiply-by-Two Circuit

- During transition to amplification mode [Fig. (c)], S_3 turns off first, placing C_1 around op-amp and left plate of C_2 is grounded
- At the moment S_3 turns off, total charge on C_1 and C_2 equals $2V_{in0}C$ and since voltage across C_2 approaches zero in amplification mode, final voltage across C_1 and hence output $2V_{in0}$

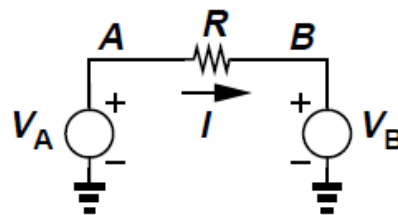


Switched-Capacitor Integrator

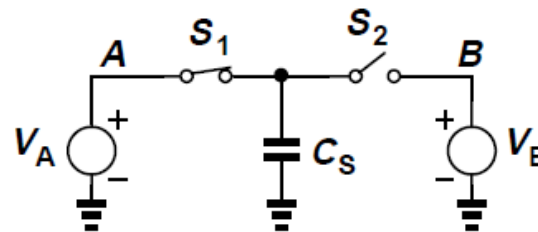


- Output of a continuous-time integrator can be expressed as

$$V_{out} = -\frac{1}{RC_F} \int V_{in} dt$$



(a)

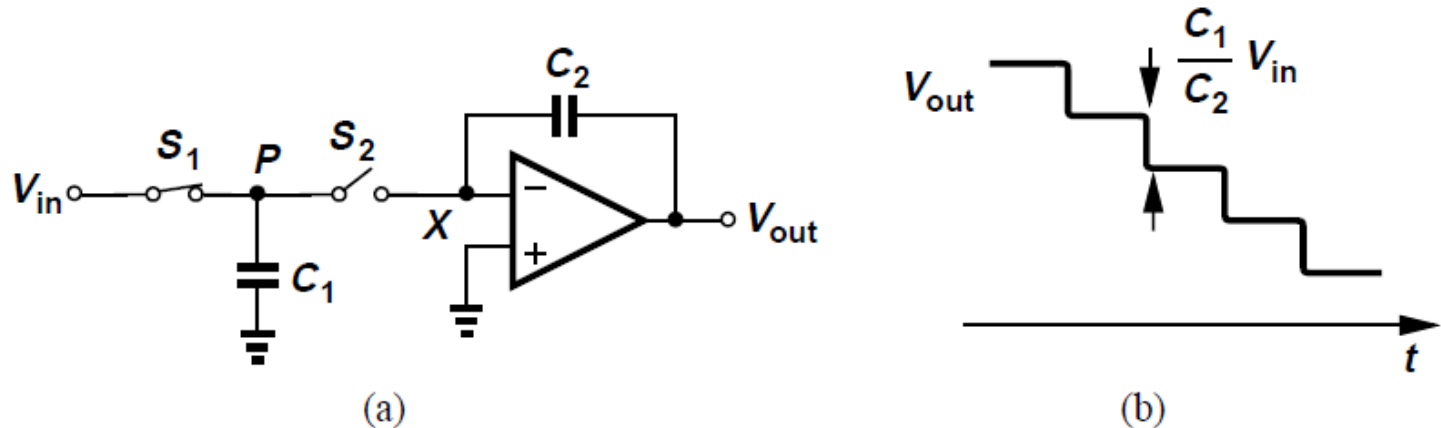


(b)

- In Fig. (a), resistor R carries a current of $(V_A - V_B)/R$
- In circuit of Fig. (b), C_S is alternately connected to nodes A and B at a clock rate f_{CK}
- Average current flowing from A to B is the charge moved in one clock period
- Can be viewed as a resistor of value

$$\begin{aligned} \overline{I_{AB}} &= \frac{C_S(V_A - V_B)}{f_{CK}^{-1}} \\ &= C_S f_{CK} (V_A - V_B) \end{aligned}$$

Switched-Capacitor Integrator

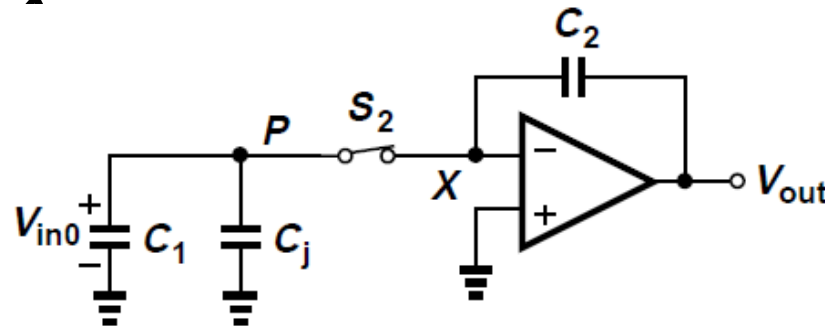


- Fig. (a) shows discrete-time integrator
- In every clock cycle, C_1 absorbs a charge equal to $C_1 V_{in}$ when S_1 is on and deposits it on C_2 when S_2 is on
- If V_{in} is constant, output changes by $V_{in} C_1 / C_2$ every clock cycle [Fig. (b)]
- Final value of V_{out} after clock cycle can be written as

$$V_{out}(kT_{CK}) = V_{out}[(k-1)T_{CK}] - V_{in}[(k-1)T_{CK}] \cdot \frac{C_1}{C_2}$$

Switched-Capacitor Integrator

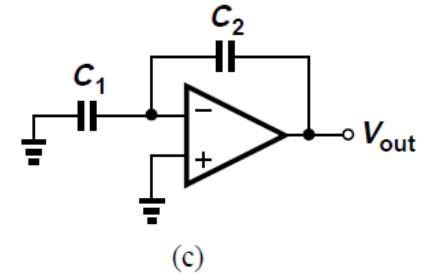
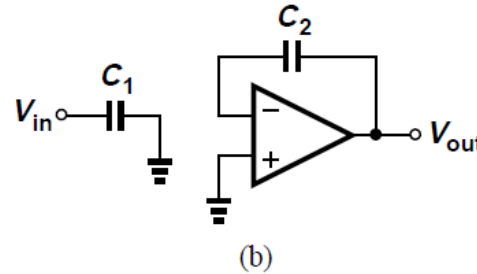
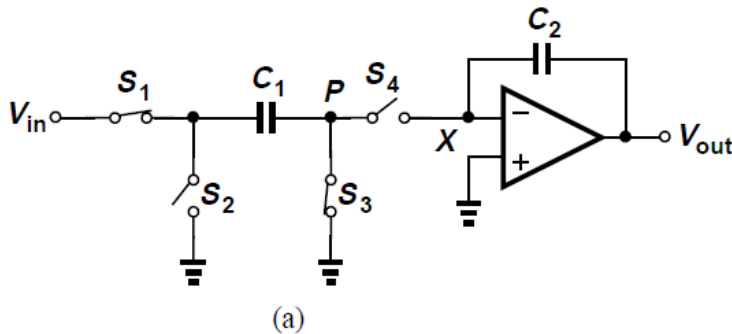
- Input-dependent charge injection of S_1 introduces nonlinearity in output voltage
- Nonlinear capacitance at node P resulting from source/drain junctions of S_1 and S_2 leads to a nonlinear charge-to-voltage conversion when C_1 is switched to Ψ



- Charge stored on the total junction capacitance, C_j is not equal to $V_{in0}C_j$, but rather equal to

$$q_{cj} = \int_0^{V_{in0}} C_j dV.$$

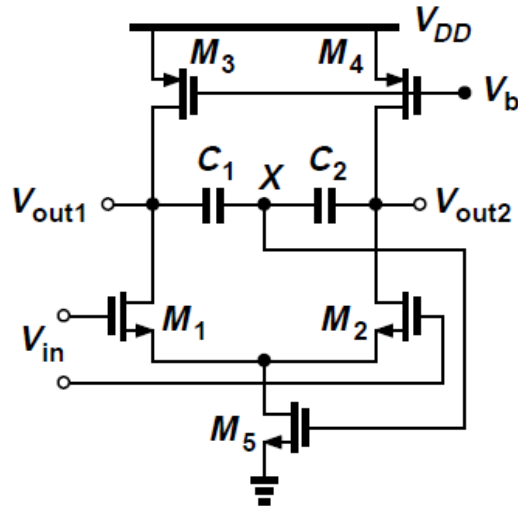
Switched-Capacitor Integrator



- Circuit of Fig. (a) resolves the issues in the simple integrator
- In sampling mode [Fig. (b)], S_1 and S_3 are on, S_2 and S_4 are off, allowing voltage across C_1 to track V_{in} while op amp and C_2 hold previous value
- In the transition to integration mode, S_3 turns off first, injecting a constant charge onto C_1 , S_1 turns off next, and subsequently S_2 and S_4 turn on
- Charge stored on C_1 is transferred to C_2 through the virtual ground node

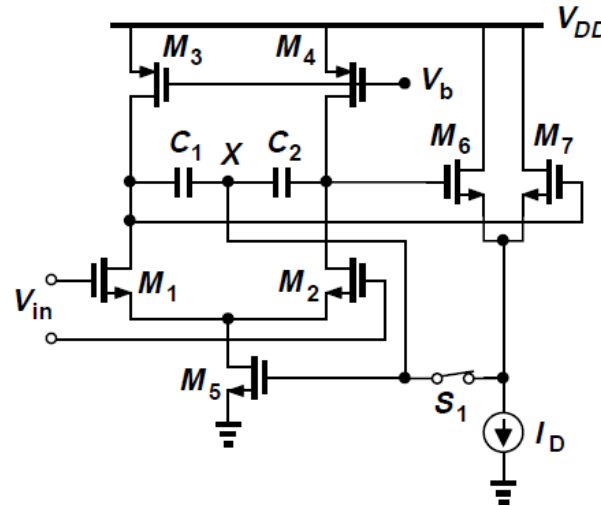
Switched-Capacitor Common-Mode Feedback

- In switched-capacitor common-mode feedback, outputs are sensed by capacitors rather than resistors



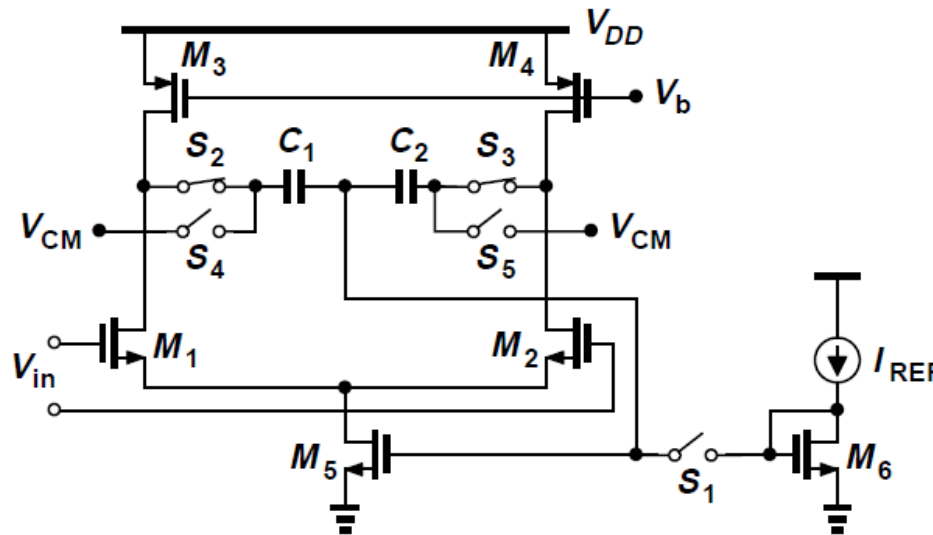
- In circuit above, equal capacitors C_1 and C_2 reproduce at node X the average of the changes in each output voltage
- If V_{out1} and V_{out2} experience a positive CM change, then V_X and I_{D5} increase, pulling V_{out1} and V_{out2} down
- Output CM is V_{GS2} plus voltage across C_1 and C_2

Switched-Capacitor Common-Mode Feedback



- Voltage across C_1 and C_2 defined as shown above
- During CM level definition, amplifier differential input is zero and S_1 is on
- M_6 and M_7 act as a linear sense circuit since their gate voltages are nominally equal
- Circuit settles such that output CM level is equal to $V_{GS6,7} + V_{GS5}$
- At the end of this mode, S_1 turns off, leaving a voltage equal to $V_{GS6,7}$ across C_1 and C_2

Switched-Capacitor Common-Mode Feedback



- For more accuracy in CM level definition, above circuit may be used
- In the reset mode, one plate of C_1 and C_2 is switched to V_{CM} while the other is connected to the gate of M_6
- Each capacitor sustains a voltage of $V_{CM} - V_{GS6}$
- In the amplification mode, S_2 and S_3 are on and the other switches are off, yielding an output CM level of $V_{CM} - V_{GS6} + V_{GS5}$, which is equal to V_{CM} if I_{D3} and I_{D4} are copied properly from I_{REF} so that $V_{GS5} = V_{GS6}$