An Experimental Investigation of Electrically Thick Rectangular Microstrip Antennas

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Abstract—**The electromagnetic properties of electrically thick rectangular microstrip antennas were investigated experimentally. Antennas were fabricated with different patch sizes and with electrical thicknesses ranging from 0.03 to 0.23 wavelengths in the dielectric substrate. The resonant frequencies were measured and compared to existing formulas. The bandwidth was calculated as a function of electrical thickness and the antenna radiation patterns were measured.**

I. Introduction

DURING THE PAST TEN YEARS, microstrip antennas experienced a great gain in popularity and have become a major research topic in both theoretical and applied electromagnetics. They are well known for their highly desirable physical characteristics such as low profile, light weight, low cost, ruggedness, and conformability. Numerous researchers have investigated their basic characteristics and recently extensive efforts have also been devoted to the design of "frequency agile," "polarization agile," or dual-band microstrip antennas [1], [2], [3]. Most of the previous theoretical and experimental work has been carried out only with electrically thin microstrip antennas. Recent interest has developed in radiators etched on electrically thick substrates. This interest is primarily due to two major reasons. First, as these antennas are used for applications with increasingly higher operating frequencies, and consequently shorter wavelengths, even antennas with physically thin substrates become thick when compared to a wavelength. Second, microstrip antennas have inherently narrow bandwidths and are normally not suitable for broad bandwidth applications. Increasing the bandwidth is possible, but the methods used [4], [5], [6] invariably increase the volume of the antenna by either extending the radiating surface or by increasing the overall antenna thickness. To aid in the design of broader band microstrip antennas, a careful experimental study of the resonant frequency, bandwidth, and radiation patterns of rectangular microstrip antennas as a function of electrical thickness of the substrate was undertaken. The measured resonant frequencies were compared to formulas previously developed for predicting the resonant frequency of electrically thin rectangular microstrip antennas.

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II. Experimental Procedures

The microstrip antennas investigated are rectangular patches with geometry as illustrated in Fig. 1. They are fabricated on 3M CuClad 233 and on Rogers RT/duroid 5870 microwave substrates. The CuClad material is made of a polytetrafluoroethylene (PTFE) woven glass laminate material while the RT/duroid material is made of a glass microfiber reinforced PTFE composite. Both substrates have a nominal dielectric constant (ϵ_r) of 2.33, and all the antennas are fed using an SMA coaxial feed. In this investigation the feed is located at the midpoint of the longer side ($x' = a/2$) and at a distance from the edge ($y' = 0.15$ cm). In each case the dimension "a" has been chosen to be approximately one and one-half times the dimension "b" with a 10 cm × 10 cm ground plane. Two sets of regular microstrip antennas have been fabricated. The ones in the first set have the same substrate thickness "h" but have nine different patch sizes; the ones in the second set have the same patch size but have three different substrate thicknesses. In addition a so-called "air-dielectric" model radiator has been fabricated to allow an even more detailed study of the resonant frequency. Its geometry models that of the regular rectangular microstrip antenna shown in Fig. 1 but with a substrate whose height can be changed by placing sheets of styrofoam ($\epsilon_r \approx 1.05$) with varying thicknesses between the ground plane and the radiating patch. The 1.78 cm × 2.67 cm aluminum radiating patch has a thickness of $t = 0.16$ cm and is coaxial fed over a 14 cm × 14 cm aluminum ground plane. This fixture allows the resonant frequency to be measured for a wide range of electrical thicknesses using exactly the same rectangular radiating patch.

The resonant frequency, impedance and radiation pattern measurements were all performed at the University of Houston-University Park, Applied Electromagnetics Laboratories, using an automated network analyzer system and dedicated computer programs. Accuracy enhancement techniques [7] have been used to partially correct for effective directivity, effective source match, and frequency tracking errors when taking impedance measurements. The radiation pattern measurements were taken with the antenna under test placed inside an anechoic chamber and mounted on a one meter diameter circular aluminum ground plane.

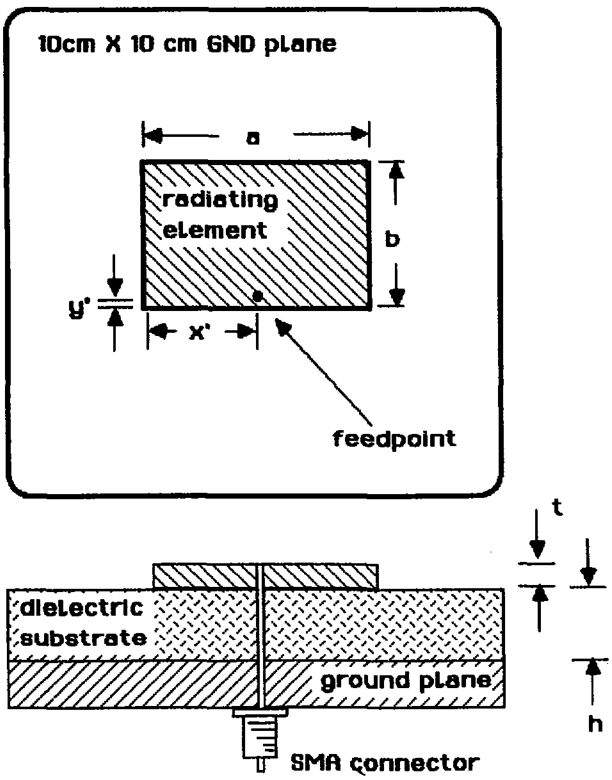

Fig. 1. Rectangular microstrip antenna geometry.

III. Experimental Results

A. Impedance

During the course of this research, the input impedances ($Z = R + jX$) and the radiation patterns of each of the antennas have been measured. Before the impedance data were used to determine the resonant frequencies (f_r) and the bandwidths (BW), they were smoothed in order to take out any residual ripples or oscillations that are due to reflections internal to the measurement equipment and have not been corrected by the accuracy enhancement routines. The values of admittance, $Y = G + jB$, as a function of frequency (f) were computed from the measured impedance versus frequency data through the relation $Y = Z^{-1}$. Depending on the values of admittance, either a cubic or quadratic least squares regression polynomial is fitted through the values of the admittance versus frequency curve. From this fitted polynomial, the smoothed admittance at each measured value of frequency is computed and then the reciprocal is taken to obtain the smoothed impedance. The actual smoothing operation is carried out with the admittance data since both the real and imaginary parts of the admittance are monotonic functions in the neighborhood of resonance and thus result in a better polynomial fit. The smoothed curves follow the general form of the measured traces very closely and allow the true peak of the resistance curve to be determined more accurately for resonant frequency measurements. Fig. 2 shows a comparison of typical smoothed and measured impedance versus frequency curves with data points taken every 10 MHz for an antenna with $h/\lambda_d = 0.110$.

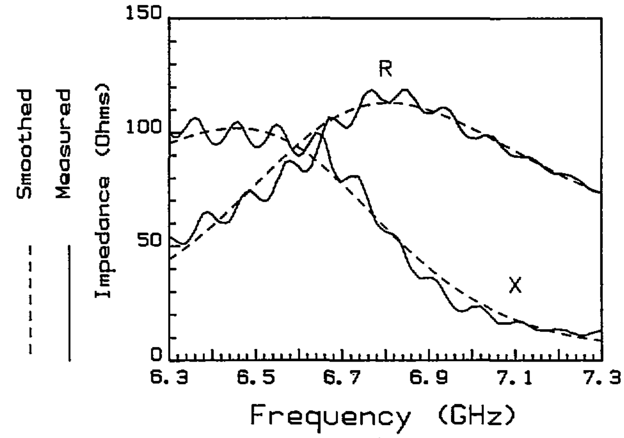

Fig. 2. Comparison of measured and smoothed impedance (1.1 cm × 1.7 cm radiating patch, 0.3175 cm substrate, $\epsilon_r = 2.33$).

B. Resonant Frequency

Generally, the resonant frequency of a microstrip antenna is defined as the frequency at which the reactance is equal to zero. For electrically thin antennas, this point is also very close to the frequency where the resistance reaches a maximum. However, in this investigation many of the reactance curves exhibit an inductive shift due to the coaxial feed passing through the electrically thick substrate [8], [9]. In fact, for the thicker antennas, the reactance curve never passes through zero at all (see Fig. 2). For this reason, the resonant frequency has been redefined as the point at which the resistance reaches a maximum, independent of the value of reactance. Furthermore, since the bandwidth of an electrically thin microstrip antenna is commonly defined in terms of the impedance (and thus is dependent on the reactance), an alternate definition of bandwidth that is not affected by the inductive shift of the reactance is used to obtain the antenna bandwidths of this paper. This last point will be discussed in detail in a later section.

Since the main concern in the measurement of the resonant frequency is the effect of the changing electrical thickness of the substrate, a normalized resonant frequency is defined where $f_{\text{norm}} = f_r/f_{r0}$, and f_{r0} is the zeroth-order prediction of the resonant frequency. This approximation for f_{r0} assumes that the antenna thickness is infinitesimally thin and that b is equal to $\lambda_d/2$. Then knowing that $\lambda_d = c/(f_{r0}\sqrt{\epsilon_r}) = 2b$, f_{r0} can be computed. For the units of c in m/s and those of b in cm,

$$f_{r0} = 15/(b\sqrt{\epsilon_r}) \text{ GHz.} \tag{1}$$

Table I shows the measured resonant frequency, zeroth order prediction, physical dimensions, and electrical thickness of each antenna. Fig. 3 shows a plot of the normalized resonant frequency plotted as a function of electrical thickness for the nine antennas etched on the same thickness of substrate and for the three identically sized antennas on different substrate thicknesses.

Table II shows the measured resonant frequencies and the

TABLE I
MEASURED AND PREDICTED ANTENNA RESONANT FREQUENCIES

a (cm)	b (cm)	h (cm)	Meas'd (GHz)	James (GHz)	Hammerstad (GHz)	h/λ_d
5.7	3.8	0.3175	2.31	2.30	2.38	0.037
4.55	3.05	0.3175	2.89	2.79	2.90	0.047
2.95	1.95	0.3175	4.24	4.11	4.34	0.068
1.95	1.3	0.3175	5.84	5.70	6.12	0.094
1.7	1.1	0.3175	6.80	6.47	7.01	0.110*
1.4	0.9	0.3175	7.70	7.46	8.19	0.125
1.2	0.8	0.3175	8.27	8.13	9.01	0.141
1.05	0.7	0.3175	9.14	8.89	9.97	0.148
0.9	0.6	0.3175	10.25	9.82	11.18	0.166
1.7	1.1	0.1524	7.87	7.46	7.84	0.061
1.7	1.1	0.3175	6.80	6.47	7.01	0.110*
1.7	1.1	0.9525	4.73	4.32	5.27	0.229

*These two are the same antennas.

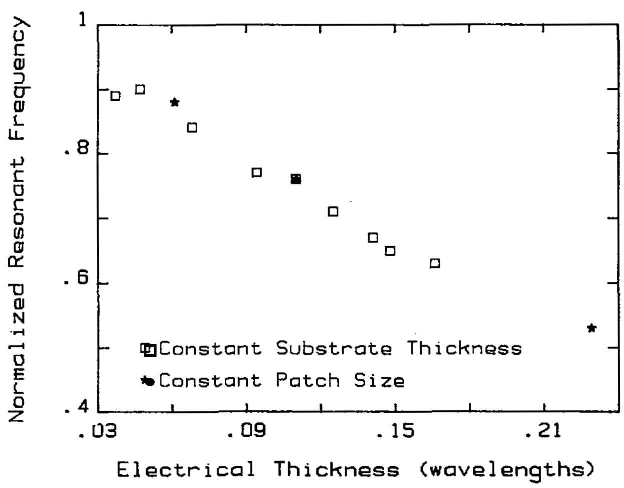

Fig. 3. Normalized antenna resonant frequency versus electrical thickness.

TABLE II
RESONANT FREQUENCY VERSUS h FOR AIR-DIELECTRIC FIXTURE (1.78 cm × 2.67 cm RADIATING PATCH)

h (cm)	Meas'd (GHz)	James (GHz)	Hammerstad (GHz)	h/λ_d
0.64	5.14	4.54	5.75	0.112
0.79	5.12	4.18	5.42	0.138
0.99	4.33	3.78	5.07	0.146
1.19	4.27	3.46	4.77	0.174
1.44	3.32	3.13	4.46	0.163
1.64	3.06	2.91	4.25	0.171
2.04	2.56	2.55	3.91	0.178
2.34	2.29	2.34	3.70	0.183

corresponding electrical thicknesses of the air dielectric fixture, while the circles in Fig. 4 represent the same data in graphical form. It is evident from Figs. 3 and 4 that the resonant frequency indeed decreases as the antennas become electrically thicker as has been shown in previously published results [10], [11]. It is perhaps unexpected, however, that this trend continues even to thicknesses approaching one quarter wavelength.

C. Bandwidth

The percent bandwidth of the antennas was determined from the impedance data. For ease of notation the term bandwidth refers to percent bandwidth unless otherwise specified. Bandwidth is normally defined as

$$\text{percent BW} = [(f_{r2} - f_{r1})/f_r]\ 100 \text{ percent} \tag{2}$$

where f_r is the resonant frequency, while f_{r2} and f_{r1} are the frequencies between which the magnitude of the reflection coefficient of the antenna is less than or equal to 1/3 (which corresponds to a voltage standing-wave ratio (VSWR) $\leqslant 2.0$). However, this definition is found not to be directly applicable

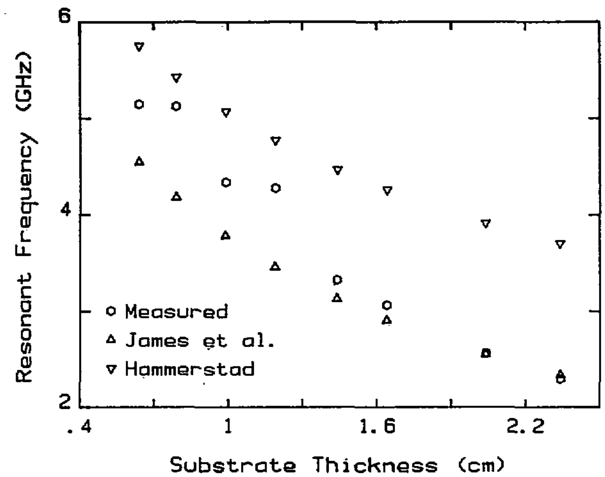

Fig. 4. Air-dielectric fixture resonant frequencies versus substrate thickness (1.78 cm × 2.67 cm radiating patch, $\epsilon_r \approx 1.05$).

to the experimental data because of the inductive shift. Thus, an alternate definition is found in order to determine the bandwidth of the test antennas.

The case where the impedance at resonance is purely resistive ($Z_{res} = R_{max}$) may be represented by a parallel RLC circuit, and an analytical expression for the input impedance in terms of the antenna Q-factor, R_{max} and f_r may be written as

$$Z(f) = \frac{\frac{1}{R_{max}} + j\left[\frac{Qf_r}{R_{max}f} - \frac{fQ}{R_{max}f_r}\right]}{\frac{1}{R_{max}^2} + \left[\frac{Qf}{R_{max}f_r} - \frac{Qf_r}{R_{max}f}\right]^2}. \quad (3)$$

Using (3), it was found that R and the magnitude of Z had a definite relationship to R_{max} that is independent of the other parameters. Namely,

$$|Z(f_{r1})| = |Z(f_{r2})| = 0.818R_{max}, \quad (4)$$

and

$$R(f_{r1}) = R(f_{r2}) = 0.670R_{max}. \quad (5)$$

For the special case of a prescribed VSWR = 2.0, this method is equivalent to the previously derived analytical expression [12]

$$BW = (VSWR - 1)/(Q\sqrt{VSWR}). \quad (6)$$

For electrically thick radiators with the associated large inductive shift in the impedance, the VSWR may not be below 2.0 for any range of frequencies. Using only the resistance data a projected bandwidth can be calculated, however, following this resonant circuit model by locating the frequencies where the resistance is equal to 0.670 times the value of R_{max}. Table III shows the bandwidths so obtained arranged in order of increasing electrical thickness along with the corresponding f_{r1} and f_{r2}, while Fig. 5 shows the same bandwidth data in graphical form. It is seen that values of bandwidth on the order of 20 percent may be achieved using electrically thick antennas.

It should be noted, however, that no attempt has been made in this investigation to actually match the antennas to a standard 50 Ω transmission line by a technique such as moving the feed point away from the edge. Thus the bandwidths reported in Table III and Fig. 5 are, in effect, projected ones that might be obtainable under the more usual definition of bandwidth. These values are most useful for comparison purposes to characterize the dependence of the bandwidth on the various antenna parameters.

D. Radiation Pattern

The radiation pattern for each element was measured at its resonant frequency in both the E-plane and the H-plane. The experimental results show that the radiation patterns of electrically thick antennas are very similar to those of electrically thin antennas. The H-plane patterns remain virtually unchanged while the E-plane ones begin to show some asymmetries only for the larger substrate thicknesses. Fig. 6 shows the radiation patterns of a representative thicker antenna ($h/\lambda_d = 0.110$, taken at 6.8 GHz in 0.5° steps) and it is typical of the remaining antennas.

IV. Comparison of Measured and Predicted Resonant Frequencies

The measured resonant frequencies can be compared to the predicted resonant frequencies as a mutual check of the experimental data and of the validity of the theories. The equations for the resonant frequency proposed by Hammerstad [13] and by James, Hall, and Wood [11] are used for these comparisons. Both methods share the concept of an effective dielectric constant (ϵ_{eff}) given by [14]

$$\epsilon_{eff}(w) = \frac{(\epsilon_r + 1)}{2} + \frac{(\epsilon_r - 1)(1 + 10h/w)^{-1/2}}{2} \quad (7)$$

where w is a variable and can be either the patch dimension "a" or "b." Hammerstad gives a predicted

$$f_r = \frac{c}{2(b + 2\Delta b)\sqrt{\epsilon_r}}, \quad (8)$$

$$\Delta b = \frac{0.412h(\epsilon_{eff}(a) + 0.3)(a/h + 0.264)}{(\epsilon_{eff}(a) - 0.258)(a/h + 0.8)}. \quad (9)$$

James *et al.* give a predicted

$$f_r = \frac{f_{r0}\epsilon_r}{\sqrt{\epsilon_{eff}(a)\epsilon_{eff}(b)}(1 + \delta)}, \quad (10)$$

where

$$\delta = (h/b)0.882 + \left[\frac{0.164(\epsilon_r - 1)}{\epsilon_r^2}\right] + \left[\frac{(\epsilon_r + 1)[0.758 + \ln(b/h + 1.88)]}{\pi\epsilon_r}\right]. \quad (11)$$

The predicted resonant frequencies obtained using these two methods are shown in Tables I and II for the test antennas and for the air-dielectric fixture.

TABLE III
PROJECTED VALUES OF BANDWIDTH AND CORRESPONDING F_{r1} AND F_{r2}

h/λ_d	f_{r1} (GHz)	f_{r2} (GHz)	BW (%)
0.037	2.280	2.352	3.117
0.047	2.834	2.952	4.083
0.061	7.632	8.152	6.607
0.068	4.120	4.396	6.509
0.094	5.632	6.140	8.699
0.110	6.494	7.272	11.441
0.125	7.314	8.468	14.987
0.141	7.848	9.024	14.220
0.148	8.380	10.560	23.850
0.166	9.442	11.560	20.660
0.229	4.320	5.180	18.180

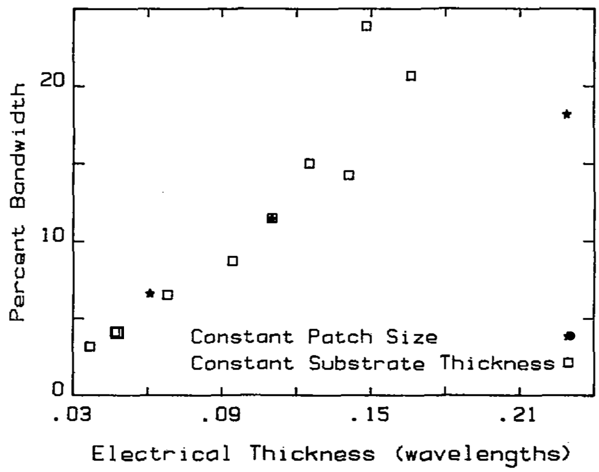

Fig. 5. Projected antenna bandwidth versus electrical thickness.

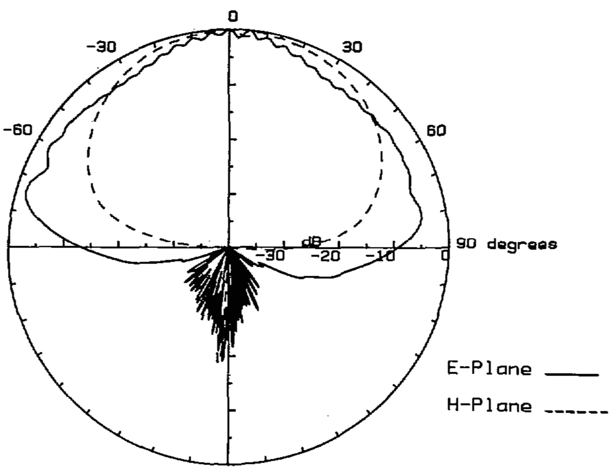

Fig. 6. Antenna radiation pattern (1.1 cm × 1.7 cm radiating patch, 0.3175 cm substrate, ϵ_r = 2.33).

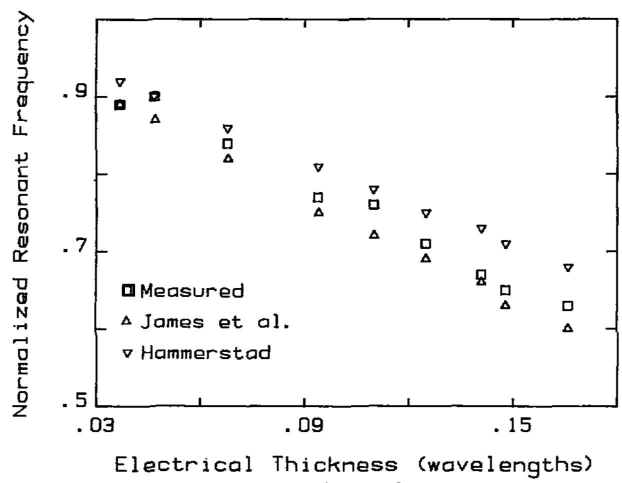

Fig. 7. Comparison of normalized antenna resonant frequencies versus electrical thickness (h = 0.3175 cm, ϵ_r = 2.33).

The predicted resonant frequencies are normalized to the zeroth order predictions and are compared to the measured, normalized resonant frequencies. Fig. 7 shows this comparison for the nine different patches on the substrates with identical physical thickness, while Fig. 4 shows the predicted values of resonant frequency for the air-dielectric fixture compared to the actual measured values. It should be noted that the theoretical data in Fig. 7 are presented as discrete points rather than a continuous curve so that the theory can correspond to the exact cases of the experimental cases, some of which have slightly varying values of a/b ratio and of dielectric constant. This use of individual points causes the data to no longer be aligned along smooth curves. In both cases it is clear that the theories follow the trend of the experimental data quite well even for electrically thick substrates. In fact, based on the information presented here and on additional research data [15], some general observations may be made concerning these two methods. Both predict the resonant frequency very closely for electrically thin rectangular microstrip antennas, but as h becomes greater than 0.1 λ_d, James *et al.* give consistently better predictions than the method by Hammerstad. Specifically, James *et al.* usually predict values approximately 4 percent lower than the measured resonant frequency while Hammerstad predicts values around 8 percent higher than the measured resonant frequency. It should be noted, however, that these two theories are not intended for use with electrically thick substrates. For $h \leqslant 0.1\ \lambda_d$, the measured resonant frequency is very nearly the mean of the predicted resonant frequencies from the two different formulas. A third algebraic formula for the resonant frequency has been proposed by Sengupta [16], but as the author states, it only applies to electrically thin structures and does not predict the proper behavior for the thicker substrates measured in this investigation.

V. CONCLUSION

The effect of varying the electrical thickness for rectangular microstrip antennas has been investigated experimentally during the course of this research. In addition, an air-dielectric model radiator has been fabricated with a single patch size and a variable substrate thickness. The resonant frequency, bandwidth, and radiation pattern have been measured over a range of substrate thickness from 0.03 to 0.23 of a wavelength in the dielectric.

The resonant frequency of a rectangular microstrip antenna was found to decrease as a function of electrical thickness. The validity of this finding is confirmed by previously published results and by the existing theories of Hammerstad and of James *et al.* The predicted resonant frequencies are all very close to the measured resonant frequency for electrically thin substrates. As the electrical thickness is increased, the theories due to Hammerstad and to James *et al.* generally predict values that are approximately 8 percent high and 4 percent low respectively when compared to the actual values. The impedance of the thicker antennas is characterized by an inductive shift in the reactance away from zero at resonance.

The projected bandwidth of the unmatched antennas was calculated to determine the effect of substrate thickness on this characteristic as well. The resulting data show that bandwidths as high as 20 percent could be achieved by simply using electrically thick substrates. Finally, the radiation patterns of electrically thick antennas were seen to be very similar to those of the more usual thin ones. Overall it has been shown that electrically thick rectangular microstrip antennas retain most of the desirable electrical characteristics of thinner ones and may be utilized for broad-band applications assuming a reactive network is used for impedance matching.

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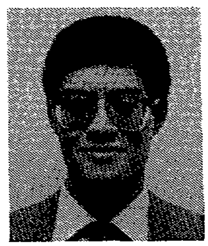

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William F. Richards (S'69–M'70–M'80), for a photograph and biography please see page 561 of the May 1985 issue of this TRANSACTIONS.