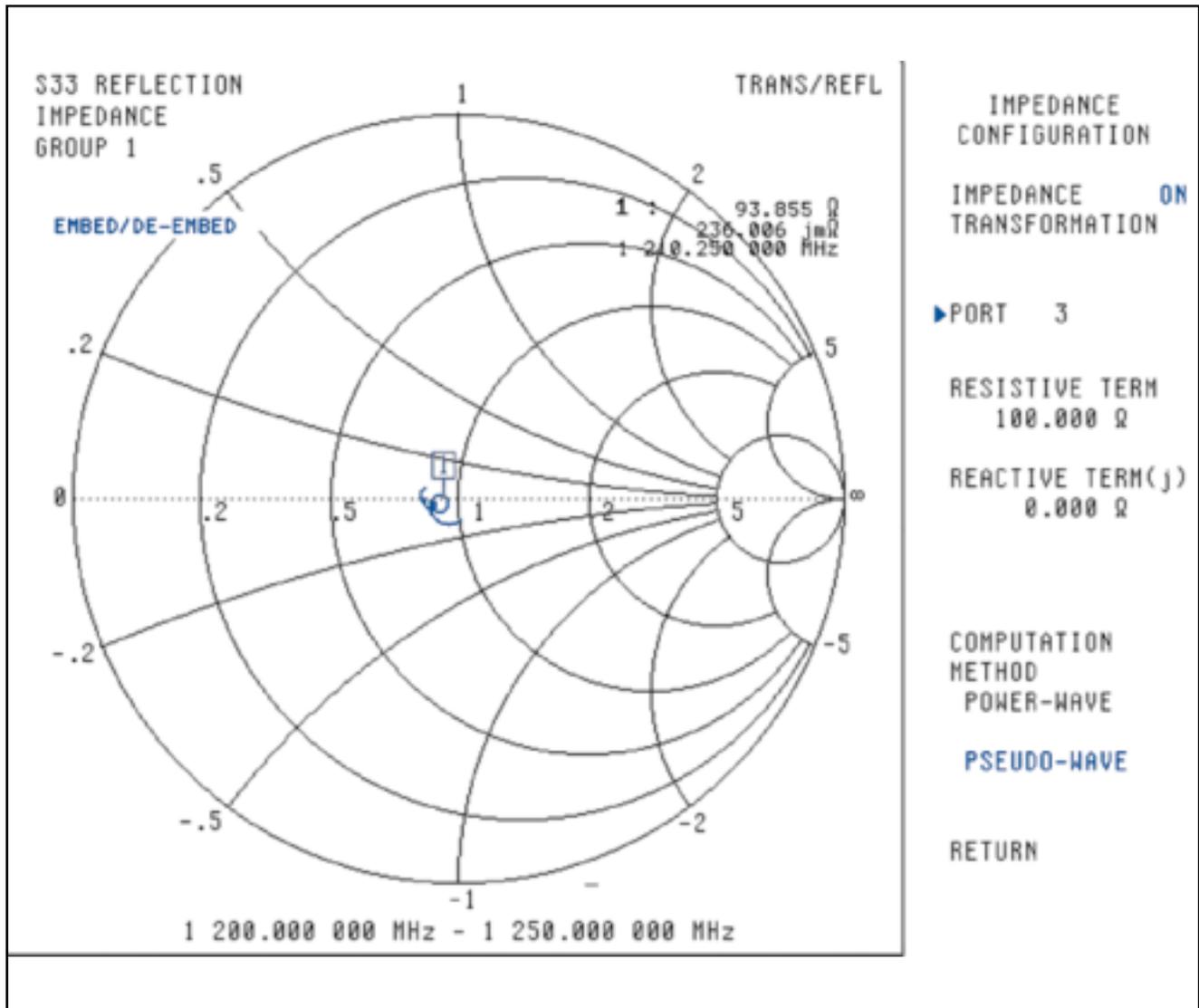


Application Note Arbitrary Impedance

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Introduction

S-parameters are dependent on the impedance to which they are referenced and many measurements are now performed in a non-50 ohm world. Matching and other tasks are now often referenced to something other than 50 ohms but calibrations in the native unusual impedance may be difficult or untraceable. Arbitrary impedance transformations allow measurements to be presented as if performed in the desired impedance.

In an earlier era of rf/microwave testing, the measurement environment could almost always be assumed to be 50 ohms (or perhaps 75 ohms in the video-related world). Matching was performed in a 50 ohm environment and everyone expected the S-parameters to be expressed relative to a 50 ohm world. In recent years, this has started to change, particularly in balanced environments now used in many consumer receivers. The reasons for the shift are many-fold but basically boil down to extracting additional performance while reducing cost and size:

- Practical A/D input impedances are not 50 ohms (often closer to 1 kohm). As the A/D converter moves closer and closer to the front-end, matching to it becomes increasingly important. Even with a transformer, the impedance to be matched to will probably still be a couple of hundred ohms.
- Mixer impedances are quite often not 50 ohms and their matching is always critical (more so as baluns start to disappear)
- Power amplifier output impedances have, in a raw sense, generally are not very close to 50 ohms and they can often be more efficiently used in a different impedance environment.

For these and other reasons, entire receiver chains may be more conveniently referenced to an environment other than 50 ohms. Figure 1 shows two example receiver architectures that may contain few non-50 ohm environments or many.

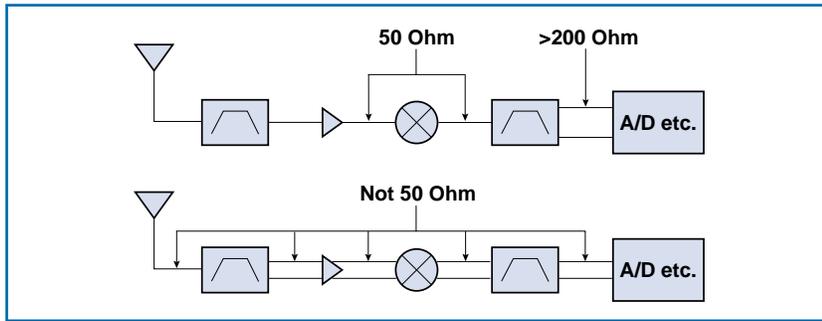


Figure 1. Two example receiver architectures are shown here. The first, somewhat more conventional, is largely referenced to 50 ohms. The second, somewhat more contemporary, structure makes much greater use of balanced and non-50 ohm interfaces for improved performance and efficiency.

A key concept is that S-parameters are dependent on the reference impedances thus the representations must change as impedances shift. This is particularly obvious when thinking of S11 as a reflection coefficient (see Fig. 2) given by:

$$S_{11}^{1-port} = \frac{Z - Z_{ref}}{Z + Z_{ref}}$$

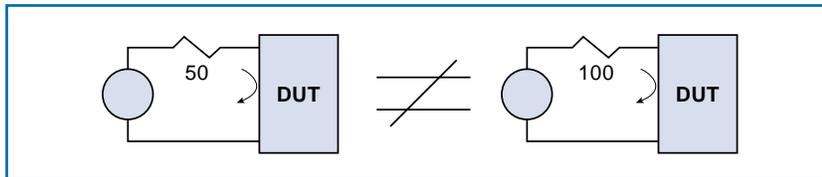


Figure 2. A simple diagram is shown here that quickly illustrates that S11 of a one port (reflection coefficient) is a function of the reference impedance. Because all terminating impedances on other ports also change, all S-parameters on a multiport device can be affected.

One way of establishing the reference impedance for a measurement is through the calibration (use a Z ohm cal kit for example). The easiest way to conceptualize this is that the cal will use Z ohm loads and Z ohm transmission lines. These will be defined, for the instrument, as being reflectionless. This procedure, including creating calibration kits for the different environments, will create severe measurement issues, the problems would be:

- a) The variety of impedances being used
- b) Traceability issues
- c) And for many, these cal environments would be on circuit boards, causing accuracy issues associated with board-based standards.

Since it is usually much easier to perform matching, etc. in the impedance environment in which it will be used, the S-parameters of the DUT should be expressed in terms of that impedance. Thus it may be a useful concept to be able to perform the calibration in a 50 ohm world (using conventional calibration kits) and then transform the resulting measurement data to what it would look like if calibrated in some other arbitrary impedance environment. The environment need not be the same at every port and it need not even be real (although the real part is required to be positive). Generally this transformation is straightforward and well-behaved; the two exceptions are when the impedance environment being transformed to is extreme ($|Z| < 1$ or $|Z| > 1000$ as a coarse estimate) or when the environment is complex.

This note will examine the basics of the impedance transformation, how the transformations work, and how they can be used in practice. The appendix briefly covers some of the more specialized details and subtleties associated with complex reference impedances.

Basics of the Transformation

Implicit in the definition of the S-parameter is knowledge of the impedance environment in which it is being examined. If for no other reason, this occurs because undriven ports must be terminated in their reference impedance for the given S-parameter to be measured. On a more practical level, the impedances of the transmission lines and loads used during the calibration procedure establish the starting reference impedance for the measurement. In all cases, however, the data is the same since it is considered to be a linear system; the reference impedance represents a point-of-view for looking at the data.

Conceptually it is fairly easy to understand that looking at the S-parameter data in a different environment means that the undriven ports would be terminated in a different impedance, the driving source impedance would be different, and any reference plane extensions would be done using transmission lines of a different impedance. Reflection coefficients obviously change since they are now relative to a different impedance. Transmission coefficients now also change since the reference impedance shift has shifted reflections at the ports and hence the net wave functions ending up at the ports.

The goal then is to find a transformation from one impedance realm to another. To perform this transformation, one can use the fact that the Impedance matrix (Z-parameters in the older literature) of a network is independent of the impedance environment since it is purely defined by voltages and currents. Since many textbooks describe how to get between Z and S-parameters and the Z matrix for the two sets of reference impedances must be the same, it is straightforward (although algebraically complicated) to express one set of S-parameters in terms of the others. Again the concept is that the different reference impedances merely represent a different way of viewing the same data. We will not present the derivation here but merely state the result (essentials in [1]-[5] among other places)

$$S' = P^{-1}(S - \gamma)(I - \gamma S)^{-1}P \quad \text{Eq. (1)}$$

Where S' is the new S-parameter matrix, S is the old S-parameter matrix and γ and P are matrices dependent solely on the new and old impedance environments. This transformation is processed on the entire set of S-parameters at a time (e.g., S11, S12, S21, and S22 for a two port) and the MS462x system will use the type of calibration in place to decide how many ports are involved. As might be expected, the S-parameters interact heavily with each other and the different reference impedances in this transformation. As an example, when looking at a reflection coefficient, not only is there a different source impedance but the other load impedances will be involved and modulated by the DUT transmission parameters (e.g., S21 and S12). More so than many other measurements, this calculation makes less sense uncalibrated. More details on this equation and the meaning of the coefficient matrices can be found in the appendix. The important points are the transformation is only dependent on the initial S-parameters and the reference impedances.

While not explicitly stated above, the different ports do not have to be at the same reference impedance and the impedances need not be real. The system will require that the real part be positive, however. One may wonder when a complex reference impedance would be of value. Since complex impedances are rarely frequency invariant, it would seem less useful to establish that as the reference. As an example, one application is related to transceiver filters where the primary value for a different reference impedance is in setting up matching problems. Since these matching issues need only be resolved in the passband of the filter, it may be narrow enough that a complex impedance environment can be considered frequency-invariant. Since the impedances to which the filter must be matched (A/D converters, mixers, etc.) are often not real (particularly on the RF side), it may be reasonable to set the reference as being complex and proceed with the matching problem in that environment.

Some complications can occur when using complex reference impedances in that different interpretations are possible depending on the simulator or textbook that is being used for comparison/analysis. This issue is discussed to a certain level in the appendix and in the references [1]-[5]. The system defaults into the most physically consistent computation method (called pseudo-wave; it is derived directly from travelling wave circuit theory) although when comparing data to that generated by certain simulators, the other method may be more appropriate (called power wave, more of a visualization domain derived from voltages and currents). The distinctions are discussed in the appendix but *these methods are identical when the reference impedances are real.*

How To Use The Transformations

The impedance transformation is one of the last steps performed (internally and by the user) since the basic measurement must be acquired correctly first. The calibration, and any embedding or de-embedding steps, are all done first. Once all of that is prepared, the reference impedance for each of the ports is simply entered and the impedance transformation is turned on. The computation method, discussed further in the appendix, can be left in the default state and only need be considered if any of the reference impedances entered are complex.

The starting impedance environment is generally set by the calibration and will usually be 50 ohms (except for waveguide calibrations or when using a 75 ohm system). Once the impedance transformation is turned on, all displayed S-parameters will be viewed from the vantage point of the new reference impedances.

Examples

An example is shown in Fig. 3 where a straight thru line (in 50 ohms) is transformed to an environment with port 2 at 150 ohms. As would be expected, both reflection and transmission coefficients are affected. A common unequal impedance scenario is with balanced to unbalanced filters where the unbalanced side may be used in a 50 ohm environment where the balanced side may be used in a 150-250 ohm environment. In this case, the 150-250 ohm value is a differential impedance and one would normally enter half that value for each of the balanced ports. The default computation method is used here but since the reference impedances are all real, there would be no difference.

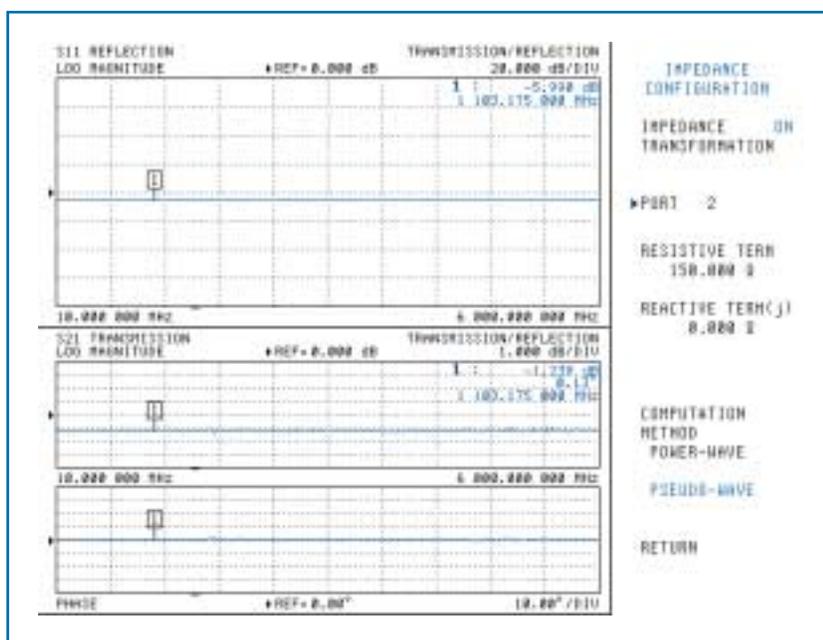


Figure 3. A 50 ohm thru line when viewed with a port 2 reference impedance of 150 ohms is shown here. Both reflection and transmission parameters are affected.

Another example, this time the measurement of a filter, is shown in Fig. 4. One trace is the measurement in the filter's designed impedance environment (lower trace in S11, flat trace in S21) while the other trace shows the filter response in a 100 ohm environment (both ports 1 and 2). As might be expected, the passband ripple and mismatch increase dramatically when seen outside of the filter's designed environment. This is a characteristic behavior when the reference impedances are not set correctly for a particular DUT.

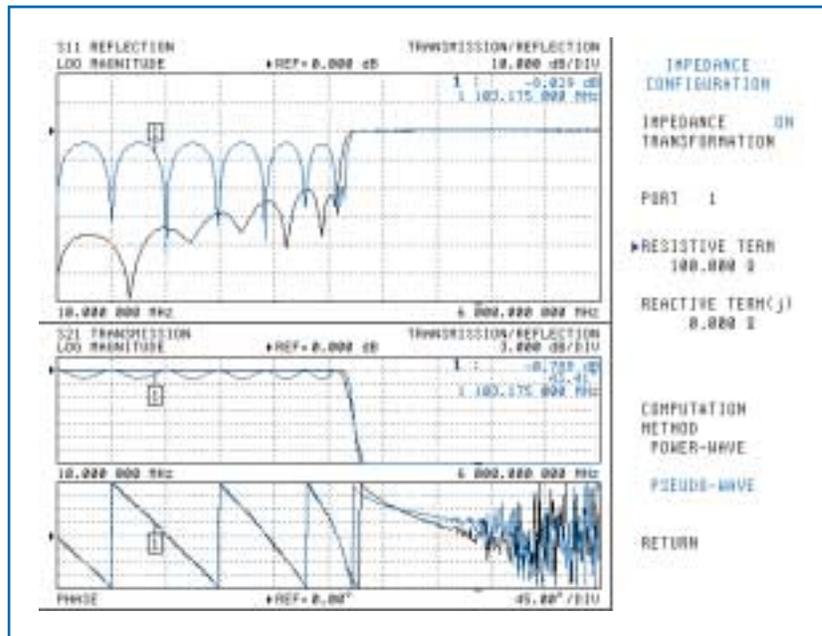


Figure 4. A filter measurement is shown here when viewed in the reference impedance environment that it was designed for (lower trace in S11, flatter magnitude trace in S21) and with 100 ohm reference impedances (ports 1 and 2). As might be expected, the passband match and flatness are much better in the design environment.

Since balanced devices are an important class of parts that use non-standard impedances, one was chosen for the next example. The mixed-mode S-parameters for a balun are shown in Fig. 5 (for the reader needing more information on mixed-mode S-parameters, consult references such as [6]-[9]). This particular balun was designed to be connected to 50 ohm single-ended ports for a differential impedance of 100 ohms. Thus the output differential match S_{DD} appears good even with the impedance transformation turned off. If one thought that the devices wanted to see 50 ohms differential (25 ohms single ended) and performed the transformation, a poorer match results. The general rule is that the differential reference impedance will be the sum of the two single-ended reference impedances and the common-mode reference impedance will be the parallel combination of the single-ended reference impedances.

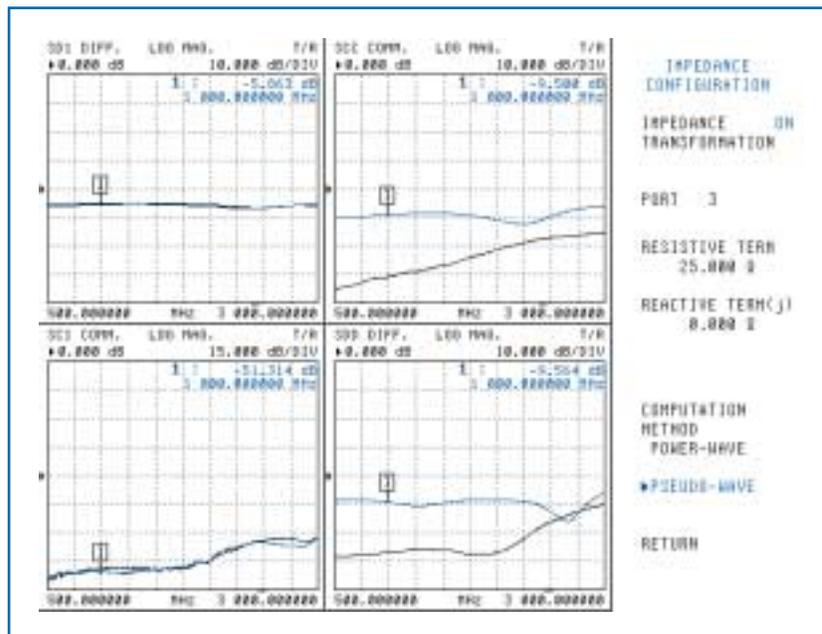


Figure 5. A balun measurement is shown here with 50 ohm single ended impedances on all ports (100 ohm differential on the balanced side, lower traces on S_{CC} and S_{DD}) and with 25 ohm single ended impedances on the balanced side (upper traces on S_{CC} and S_{DD}). Since this device was designed for a 100 ohm differential impedance, the output match appears better in the former case. There is little effect on the transmission parameters.

Limitations of the Transformation

Since the receiver still has a physical characteristic impedance of 50 ohms (or perhaps 75), there are practical limits to what can be transformed just based on signal-to-noise ratio. As an example, it is difficult for the physical system to differentiate between a 10000 ohm load and an open or between a 0.1 ohm load and a short. Thus expecting the transformation to reliably convert data to reference impedances of 0.1 or 10kohms is perhaps not realistic. One measure of the practicality of the measurement is the difference in reflection coefficient (in the native sense) between the desired reference impedance and the closest pure reactance. As this difference approaches the measurement uncertainty, the measurement will increasingly be of dubious quality. A plot of this difference vs. reference impedance is shown in Fig. 6.

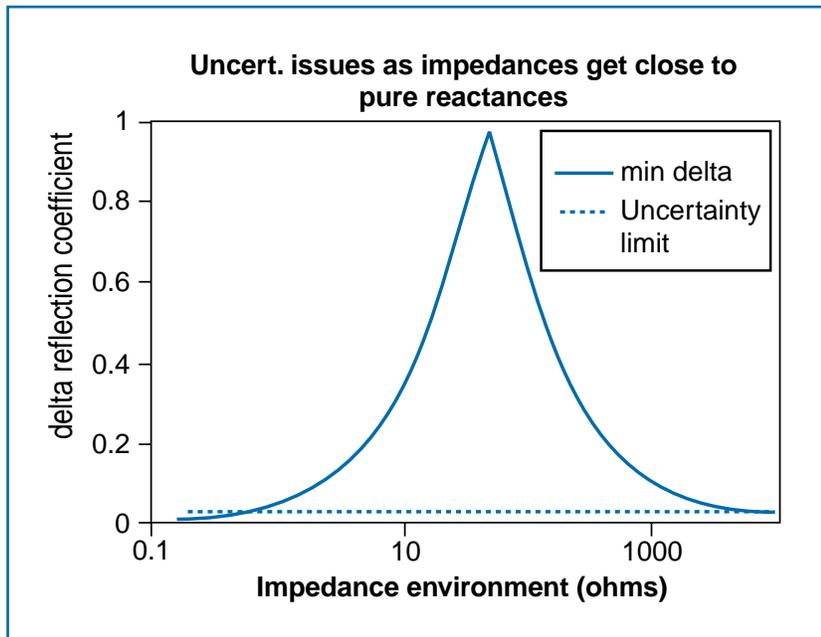


Figure 6. A plot of the difference in reflection coefficient that a 50 ohm instrument sees between a Z ohm load and the nearest pure reactance is shown here. As this difference approaches the measurement uncertainty, the system will be less accurately able to measure devices relative to the Z ohm reference impedance. This is primarily a signal to noise limit and can be affected by cal quality, IFBW, etc.

From the graph, it is clear that transforming to reference impedances less than an ohm or more than about 2000 ohms will become difficult. Depending on the IF bandwidth used and other measurement stability issues (e.g., mechanically unstable test fixture), the practical range may be somewhat smaller.

Conclusions

This note has described the impedance transformation function basics and implementation. A few examples have shown its practical use and the limitations on allowed impedances have been discussed. Some more details on the transform and some subtleties associated with complex impedance environments are discussed in the appendix.

Appendix

This section contains some more advanced topics related to the impedance transformations that some users may be interested in. These issues primarily revolve around the computation method selection item and its implications.

Some Terminology:

A careful exposition of some of the terminology used is important for some of the subtleties to be discussed. A more detailed description is available in a treatise by Marks and Williams [1].

Characteristic impedance: The physical impedance of a transmission line or waveguide. This is of particular importance in the LRL family of calibrations (and indirectly in other calibrations) in that it establishes the impedance environment of the cal. The magnitude of the characteristic impedance is in some sense arbitrary (and the choice is often less than obvious in some waveguide systems). The phase is a fixed characteristic of the mode.

Travelling waves: Refers to the unidirectional, single moded waves that form the basis for scattering matrix analysis. Circuit theory based on travelling waves is physical and has led to the standard methodology of handling rf/microwave problems: the usual S-matrix.

Pseudo waves: Linear combinations of travelling waves that are sometimes convenient (for reasons of measurement for example) to use in lieu of travelling waves. These can also be referenced to an arbitrary port reference impedance for use in applications. VNA-measured parameters are basically derived from pseudo-waves. These are less physical than travelling waves but are a practical abstraction for measurement and other purposes.

Reference impedance: A port impedance to which pseudo-wave parameters can be referenced. It is completely arbitrary and forms the basis for the transformations to be discussed here. Inherent in the definition of S_{ij} is that all ports except j are terminated in their reference impedance and the source impedance of port j is equal to its reference impedance. A starting reference impedance is established by the calibration (line impedance = characteristic impedance in LRL, load impedance is SOLT) but can be later transformed. There are a number of reasons why this may be chosen over the characteristic impedance: the characteristic impedance is not known or is a function of frequency, the environment in which the subcircuit will be embedded is of a different characteristic impedance, etc.

Power waves: Different from both pseudo-waves and travelling waves, this is a wave-like construct to allow analysis of waveguide and transmission-line circuits. These are not at all physical but are derived as if the voltages and currents associated with the involved waves were already known. As such, the power wave analysis is more of a simulation or visualization tool (an ancillary computational construct). A port reference impedance can be associated with power waves as well but the implications are different from when used on pseudo-waves. We will use ‘reference impedance’ to denote the impedance environment assigned to either power or pseudo waves but the reader must be aware of the context.

Impedance transformation: When the user desires to express the parameters in a different set of reference impedances from the starting set (or from the starting characteristic impedance depending on the cal), a transformation can be employed. This transformation will be slightly different if the representation is assumed to be pseudo-waves (with travelling waves considered to be a limiting case of pseudo-waves) as opposed to power waves.

The two rather different S-parameter definitions (one based on pseudo-waves, the abstraction from travelling wave analysis; the other based on the power wave computational construct) both have been in the literature for years and the definitions have an impact on how this transformation is done. Both are used in simulators and other applications so both computation options are provided. The power wave construct (see e.g. [5]), expresses the incident and reflected/transmitted “wave-like” variables as:

$$\hat{a} = \frac{v + iZ}{2\sqrt{\text{Re}(Z)}} \quad \text{Eq. (2)}$$
$$\hat{b} = \frac{v - iZ^*}{2\sqrt{\text{Re}(Z)}}$$

Where Z is the impedance environment (reference impedance as discussed above) of the port in question. One reason behind this formulation is that it forces delivered power to be:

$$|\hat{b}|^2 - |\hat{a}|^2 \quad \text{Eq. (3)}$$

This structure does help in visualizing the behavior of a set of S-parameters (in terms of power flow) but it does have some unusual implications that will be discussed below.

The pseudo-wave method (derived from the physical travelling wave analysis) has the wave variables expressed as follows (requiring the real part of $Z_{ref} > 0$)

$$\begin{aligned} a &= K \left[\frac{\sqrt{\text{Re}(Z_{ref})}}{2|Z_{ref}|} \right] (v + iZ_{ref}) \\ b &= K \left[\frac{\sqrt{\text{Re}(Z_{ref})}}{2|Z_{ref}|} \right] (v - iZ_{ref}) \end{aligned} \quad \text{Eq. (4)}$$

Where K represents a normalizing term dependent on how voltage is measured on the given transmission structure [1] (e.g., there is an obvious choice for TEM modes, just measure between points on each of the active conductors). Since we are primarily concerned with S-parameters, this normalizing term will cancel. The important difference from equation 2 is in the absence of conjugation on the reference impedance term in b .

When Z is real, the two definitions are computationally identical. Since this covers the majority of practical situations, most users are generally not aware of the differences. Since both formulations are prevalent in the literature and in commercial simulators, however, the MS462X allows both definitions to be invoked when performing the impedance transformation. It is sometimes important then to understand the differences between them.

When the impedance realm becomes complex, some major differences start to appear.

- Power transfer
- Smith chart behavior
- Transfer matrix behavior
- Conjugate Matching

As mentioned earlier, the power wave formulation has a particularly simple power transfer relation. For a pseudo-wave computation, the result is a bit more involved if Z_{ref} is not real [1]

$$|a|^2 \left[1 - |\Gamma|^2 - 2 \text{Im}(\Gamma) \frac{\text{Im}(Z_{ref})}{\text{Re}(Z_{ref})} \right] \quad \text{Eq. (5)}$$

This has another side effect in that $|\Gamma|$ can be greater than 1 for a passive device if the reference impedance is not real. The reason is that the third term in Eq. 5 allows this to happen without violating conservation of energy. Again, this is much more physical than the power-wave formulation in that it is at least based on a linear combination of traveling waves but it can make intuitive understanding a bit more complicated.

In the pseudo-wave formulation, the Smith chart behaves in much the way one would expect even if Z is complex. Such cannot be said for the power wave formulation [1]. While an open will still be represented by a reflection coefficient of 1, a short would be represented by:

$$\Gamma_{short} = \frac{-Z^*}{Z} = e^{j(\pi - 2\text{arg}(Z))} \quad \text{Eq. (6)}$$

This concept is displayed in Fig. 7. A calibration was initially done in 50 ohms (the characteristic impedance in this case) and a transformation to the reference port impedance of $20 + j20$ is desired while a short is being measured. The Smith chart corresponding to the pseudo-wave formulation places the short at the expected $\Gamma = -1$ position. The Smith chart corresponding to the power wave formulation, however, places the short at the $\Gamma = j$ position.

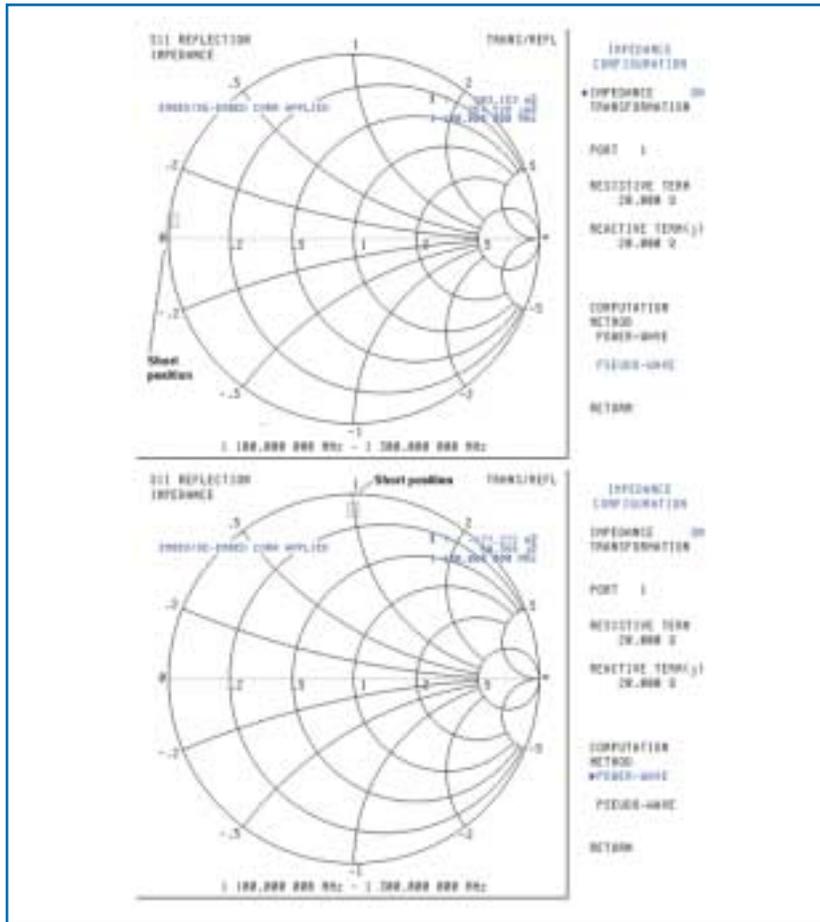


Figure 7. The Smith chart representation of a short for a reference impedance of $20+j20$ ohms is shown here using both the pseudo-wave (top) and power-wave (bottom) computation methods. The position on the Smith chart when using the power-wave method is dependent on the phase of the reference impedance.

This obviously causes some complications for calibration routines such as SOLT, which tend to make assumptions about short reflection coefficients that do not include this effect. It also leads to difficulties in performing tuning using the Smith chart in that trajectories will appear very different depending on the value of the reference impedance. Generally speaking if one wants a graphical interpretation similar to the Smith chart for power wave formulations, one must redo it for every phase of Z .

Transfer matrices are an important computational tool used and they behave quite differently in the two formulations. The process of multiplying two such matrices works in the pseudo-wave formulation assuming the output port reference impedance of block 1 is equal to that of the input port of block 2. This is usually a reasonable assumption (and a converting block can be inserted between two mismatched blocks if necessary). In the power wave construct, one must be the conjugate of the other. This can lead to some complications when moving ‘black boxes’ of data into and out of simulators since they may be combinable in some but not in other contexts. This power wave result is physically much less plausible and leads to some difficulties with interpretation. This difficulty extends into matching for maximum power transfer in that conjugate matching no longer works. These complications point to some of the limitations of a more computational construct.

The interpretation of $|\Gamma| > 1$ also requires some care particularly when the reference impedance is complex. A passive network can have a magnitude of reflection coefficient, under the pseudo-wave formulation, greater than unity when Z_{ref} is not real. This is not true in the power wave computation: S_{ij} will have a maximum magnitude of unity assuming the device is passive. Again this fits into the role of power waves as helping visualize power flow (the ‘reflected power’ interpretation of $|b|^2$ will still make some sense in a complex reference impedance environment).

One similarity between the two formulations is worth note because it is not particularly intuitive. If Z_{ref} is not real, the S matrix need not be symmetric even if the network in question is reciprocal [1]. This can lead to confusion if the asymmetric result is thought to be symptomatic of an error.

The pseudo-wave and power-wave transformation equations are shown here for completeness. The matrix equation takes a pseudo scattering matrix S with reference impedances A_{ii} to a new matrix S' with reference impedances B_{ii} .

$$S' = P^{-1}(S - \gamma)(I - \gamma S)^{-1}P \quad \text{pseudo-wave(7)}$$

where

$$P_{ii} = \sqrt{\frac{\text{Re}(A_{ii})}{\text{Re}(B_{ii})}} \frac{B_{ii}}{A_{ii}} \frac{2A_{ii}}{A_{ii} + B_{ii}} \quad \text{pseudo-wave(8)}$$

$$\gamma_{ii} = \frac{B_{ii} - A_{ii}}{B_{ii} + A_{ii}} \quad \text{pseudo-wave(9)}$$

$$S' = Q^{-1}(S - \Gamma^+)(I - \Gamma S)^{-1}Q^+ \quad \text{power-wave(10)}$$

where

$$Q_{ii} = \sqrt{\frac{\text{Re}(B_{ii})}{\text{Re}(A_{ii})}} \frac{A_{ii} + A_{ii}^*}{B_{ii}^* + A_{ii}} \quad \text{power-wave(11)}$$

$$\Gamma_{ii} = \frac{B_{ii} - A_{ii}}{B_{ii} + A_{ii}^*} \quad \text{power-wave(12)}$$

An important thing to note about equations 7 and 10 is that the relationship is not necessarily invariant to changes in S. Therefore the order of post-processing operations one performs on S data is important (get reference planes in the proper position before performing the impedance transformation since one will be moving around reflected wave interactions).

Figure 8 shows the effect of mixing order of operations. One trace shows the result of changing reference planes before the impedance transformation is performed while the other shows the result of moving the reference planes after the impedance transformation. A difference may be expected since the entire standing wave pattern is different in these two scenarios. The MS462X will automatically ensure that these operations are done in the correct order (embedding and reference plane shifts first) but it is important to remember this effect if moving back and forth between the instrument and simulators while changing reference impedances.

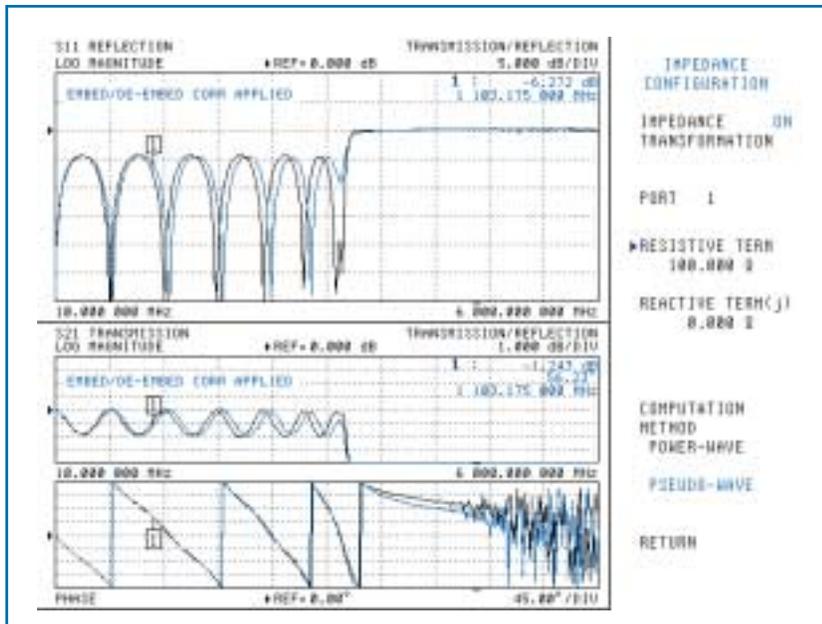


Figure 8. The effect of doing a reference plane shift before and after a reference plane extension is shown here. The results are different since the assumed transmission line impedance will be different for the two techniques. The MS462X forces all plane extensions and embedding/de-embedding to occur before the impedance transformation so that the plane can be uniquely defined.

As a final example in this section, consider a filter to be referenced to a $30+j30$ environment (on both ports). Both the pseudo-wave and power wave representations are shown in Fig. 9 (upper curves being pseudo-wave). As discussed previously, there are now substantial differences between the formulations since Z_{ref} is complex. Also note that the magnitudes of both S_{11} and S_{21} are greater than 1 for this passive device under the pseudo-wave formulation.

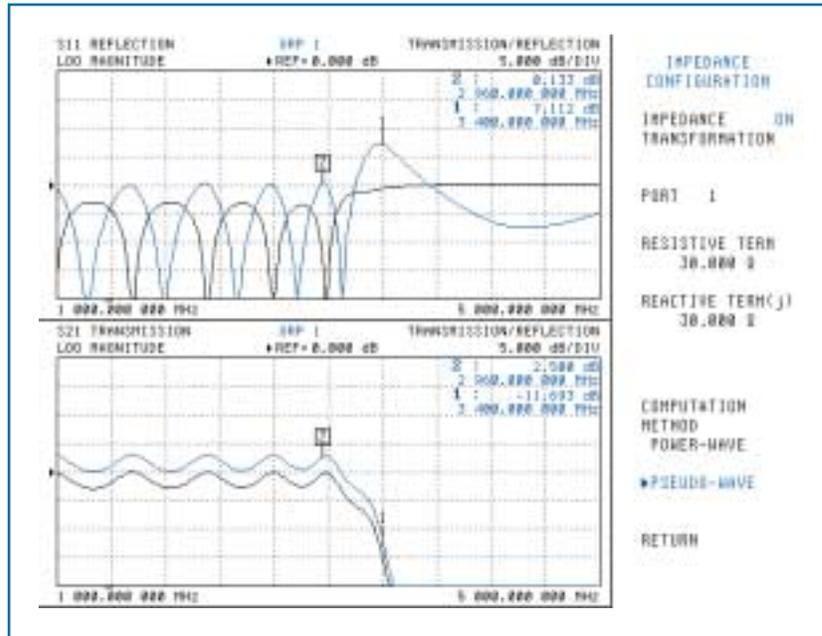


Figure 9. The S-parameters of a filter in a $30+j30$ ohm reference impedance environment are shown here using the two different computation methods (upper trace-pseudo-wave, lower trace-power wave). $|S_{ij}|$ can be greater than unity for a passive device using the pseudo-wave computation method.

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