

ANALYSIS OF VIVALDI ANTENNAS

Huang Jingxi and Fan Zhibo

Wuhan University, People's Republic of China

INTRODUCTION

The tapered slot-line antenna is a new mm-wave integrated circuit antenna which can be easily integrated with other MIC components. It is fabricated in a thin metal deposited on one side of a dielectric substrate with the tapered slot, i.e., exponentially tapered (Vivaldi), Linearly tapered (LTSA) and constant width (CWSA). A Vivaldi antenna is illustrated in Fig.1.

These antennas are light, compact, easily reproducible, thin and flat in profile, so they have widely been applied in mm-wave single beam as well as in imaging systems, and have experimentally been investigated. In the other paper (2), we have derived characteristic equations for guide wavelength of the tapered slot-line antenna by using Galerkin's method in the Fourier domain. This paper presents different hybrid-mode solution for the guide wavelength. The solution to the hybrid-mode equations is obtained by applying the transverse equivalent transmissionline concept in the spectral domain in conjunction with a simple coordinate transformation rule.

Since the slot do not have a uniform, the propagation constant is not constant. It will be assumed that all the variations in the propagation constant are sufficiently gradual so that the propagation constant at a point along a nonuniform traveling wave structure is the same as that for infinitely long uniform structure of the same cross section as the nonuniform structure at the point that is being considered.

METHOD OF ANALYSIS

Fig.2. shows the cross sectional view of a Vivaldi antenna

As we know, in the absence of any metallization on the surfaces of the slot line substrate the modal spectrum consists of TM to y and TE to y modes only, the modes of the structure being studied are superimposed from

these fields. The Fourier transform for the magnetic-component along the y-direction and its corresponding inverse transform are respectively, given by

$$\hat{H}_y(a, y) = \int_{-\infty}^{+\infty} H_y(x, y) e^{j\alpha x} dx$$

$$H_y(x, y) e^{-j\beta z} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \hat{H}_y(a, y) e^{-j(\alpha x + \beta z)} da$$

The electric-field component along the y-direction has similar expression. It can be seen that all the field components are a superposition of inhomogeneous plane waves propagating in the direction of θ from the z axis where $\theta = \cos^{-1}(\beta/\sqrt{\alpha^2 + \beta^2})$. Now the plane waves in the v-direction are decomposed into TM to y ($\hat{E}_y, \hat{E}_v, \hat{H}_u$) and TE to y ($\hat{H}_y, \hat{H}_v, \hat{E}_u$) where the coordinates v and u are shown as in Fig.3. The current densities J_v and J_u that generate the fields of TM to y and TE to y waves respectively are introduced. Hence, we can draw equivalent circuits for the TM and TE fields as shown in Fig.4. The characteristic admittances in each region are

$$Y_{TMi} = \frac{j\omega\epsilon_0\epsilon_{ri}}{Y_i}$$

$$Y_{TEi} = \frac{Y_i}{j\omega\mu_0}$$

where $Y_i = \sqrt{\alpha^2 + \beta^2 - \epsilon_{ri}k_0^2}$ is the propagation constant in the i-th region. ϵ_0, μ_0 are the free-space permittivity and permeability, respectively, k_0 is the free space wavenumber.

According to the transmission-line theory, we can easily derive the following relations between the "voltage" and "current"

$$\hat{J}_v = Y_e \hat{E}_v \quad (1a)$$

$$\hat{J}_u = Y_h \hat{E}_u \quad (1b)$$

where

$$Y_e = Y_{TM1} + Y_{TM2} \frac{Y_{TM1} + Y_{TM2} \tanh(Y_2 d)}{Y_{TM2} + Y_{TM1} \tanh(Y_2 d)} \quad (2a)$$

$$Y_h = Y_{TE1} + Y_{TE2} \frac{Y_{TE1} + Y_{TE2} \tanh(Y_2 d)}{Y_{TE2} + Y_{TE1} \tanh(Y_2 d)} \quad (2b)$$

When Y_2 is imaginary, $Y_2 = jY_2'$, $\tanh(Y_2 d)$ in (2) are replaced by $\tanh(Y_2' d)$. By complex algebraic manipulation, we obtain

$$Y_e = jY_e^0 \quad (3a)$$

$$Y_h = jY_h^0 \quad (3b)$$

where Y_e^0 and Y_h^0 are real functions.

According to the coordinate transform rule in Fig. 3, the following relations can be obtained.

$$\begin{bmatrix} \hat{J}_x \\ \hat{J}_z \end{bmatrix} = \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix} \begin{bmatrix} \hat{J}_v \\ \hat{J}_u \end{bmatrix} \quad (4a)$$

$$\begin{bmatrix} \hat{E}_v \\ \hat{E}_u \end{bmatrix} = \begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix} \begin{bmatrix} \hat{E}_x \\ \hat{E}_z \end{bmatrix} \quad (4b)$$

From (1), (3), (4), we obtain

$$\begin{aligned} \begin{bmatrix} \hat{J}_x \\ \hat{J}_z \end{bmatrix} &= \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix} \begin{bmatrix} jY_e^0 & 0 \\ 0 & jY_h^0 \end{bmatrix} \begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix} \begin{bmatrix} \hat{E}_x \\ \hat{E}_z \end{bmatrix} \\ &= j \begin{bmatrix} Y_e^0 \sin^2\theta + Y_h^0 \cos^2\theta & (Y_e^0 - Y_h^0) \sin\theta \cos\theta \\ (Y_e^0 - Y_h^0) \sin\theta \cos\theta & Y_e^0 \cos^2\theta + Y_h^0 \sin^2\theta \end{bmatrix} \begin{bmatrix} \hat{E}_x \\ \hat{E}_z \end{bmatrix} \\ &= j[Y^0] \begin{bmatrix} \hat{E}_x \\ \hat{E}_z \end{bmatrix} \quad (5) \end{aligned}$$

where $[Y^0]$ is the Fourier transforms of dyadic Green's functions which are real functions.

In order to obtain the numerical values of eigenvalues efficiently and with a high computational velocity and convergence, it is important to use well-suited expansion functions for the series expansion of the slot fields. Therefore, considering "singularities" near the edges, we may choose the following functions to expand the slot electric fields.

$$E_x(X) = \sum_{l=0}^{l_1} \frac{C_l \cos(Y_l X)}{\sqrt{1 - (X/W)^2}} \quad |X| < W$$

$$E_z(X) = \sum_{l=1}^{l_2} \frac{D_l \sin(Y_l X)}{\sqrt{1 - (X/W)^2}} \quad |X| < W$$

where $Y_l = \frac{l\pi}{W}$, coefficients C_l , D_l are unknown. The Fourier transforms of $E_x(X)$, $E_z(X)$ are derived

$$\hat{E}_x(a) = \sum_{l=0}^{l_1} C_l U_l(a) \quad (6a)$$

$$\hat{E}_z(a) = \sum_{l=1}^{l_2} D_l V_l(a) \quad (6b)$$

where

$$\hat{U}_l(a) = \frac{\pi W}{2} \{J_0[(Y_l - a)W] + J_0[(Y_l + a)W]\}$$

$$\hat{V}_l(a) = \frac{\pi W}{2} \{J_0[(Y_l - a)W] - J_0[(Y_l + a)W]\}$$

where $J_0(a)$ is the Bessel function of zero order of the argument a . When (6) is substituted into (5), (5) may be written as

$$\hat{J}_x = jY_{11}^0 \sum_{l=0}^{l_1} C_l \hat{U}_l(a) - Y_{12}^0 \sum_{l=1}^{l_2} D_l \hat{V}_l(a) \quad (7a)$$

$$\hat{J}_z = jY_{21}^0 \sum_{l=1}^{l_1} C_l \hat{U}_l(a) - Y_{22}^0 \sum_{l=1}^{l_2} D_l \hat{V}_l(a) \quad (7b)$$

We take inner products of the above equations with $\hat{U}_m(a)$, $\hat{V}_m(a)$ for different values m .

According to Parseval's relation, the left-hand sides of (7) can be eliminated because $\hat{U}_m(X)$, $\hat{V}_m(X)$ and $\hat{J}_x(X)$, $\hat{J}_z(X)$ are nonzero only in the complementary regions. we obtain

$$\sum_{l=0}^{l_1} C_l \delta P_{ml} - \sum_{l=1}^{l_2} D_l Q_{ml} = 0 \quad m=0, \dots, l_1 \quad (8a)$$

$$\sum_{l=0}^{l_1} C_l \delta R_{ml} - \sum_{l=1}^{l_2} D_l S_{ml} = 0 \quad m=1, \dots, l_2 \quad (8b)$$

where the matrix elements are

$$P_{ml} = \int_{-W}^W \hat{U}_l(a) Y_{11}^0(a) \hat{U}_m(a) da$$

$$Q_{ml} = \int_{-W}^W \hat{V}_l(a) Y_{12}^0(a) \hat{U}_m(a) da$$

$$R_{ml} = \int_{-W}^W \hat{U}_l(a) Y_{21}^0(a) \hat{V}_m(a) da$$

$$S_{ml} = \int_{-W}^W \hat{V}_l(a) Y_{22}^0(a) \hat{V}_m(a) da$$

By assuming $\delta D_l = D'_l$, the resulting set of linear equations may be written in matrix form as follows

$$\begin{bmatrix} P & Q \\ R & S \end{bmatrix} \begin{bmatrix} C_0 \\ \vdots \\ C_{l_1} \\ D'_1 \\ \vdots \\ D_{l_2} \end{bmatrix} = 0$$

The dispersion relation is obtained by solving for values of guide wavelength that render the determinant of the PQRS matrix to zero for a given free-space wavenumber.

NUMERICAL RESULTS

Fig.5 shows the computed results for the variation of guide wavelength against the z -coordinate. Results are compared with those reported in the paper (2). The agreement is seen to be very good.

REFERENCES

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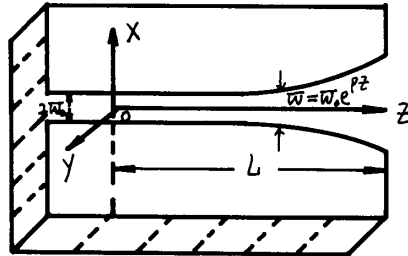


Fig.1 The principal view of a Vivaldi antenna

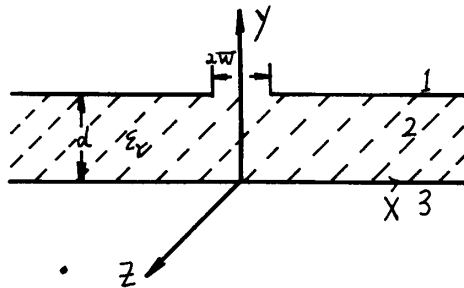


Fig. 2 A cross-sectional view

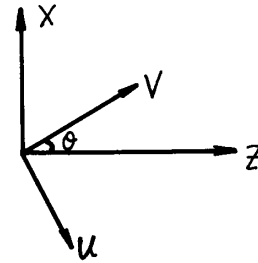


Fig. 3

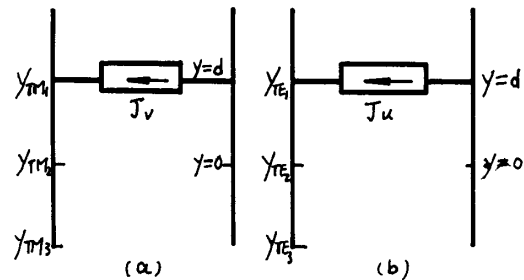


Fig. 4

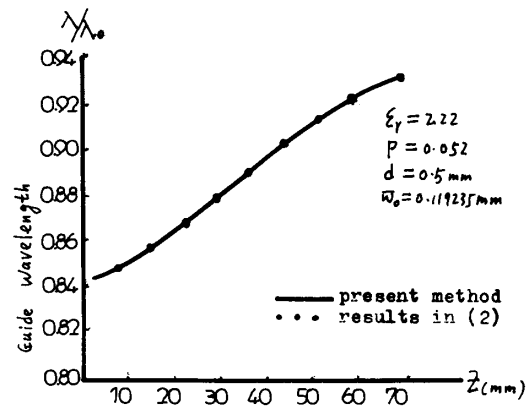


Fig. 5 Guide wavelength against the z -coordinate