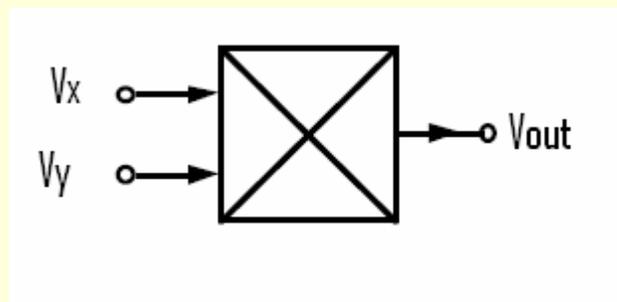


# Microelectronics

## Chapter 2:

( Lectures 2 & 3 )

## BJT and MOS Analog Multiplier



# Analog Multiplier

## ■ Objectives:

1. Introduction
2. Revision of Simple Emitter Coupled Circuit .
3. Gilbert Cell As a 4-quadrant Multiplier
4. Complete Gilbert Multiplier without any restriction on the I/P Signals.
5. The application of the Gilbert Cell to realize a Phase Detector (PD)
6. Gilbert Cell as a DSB-SC Modulator
7. MOS Gilbert Cell

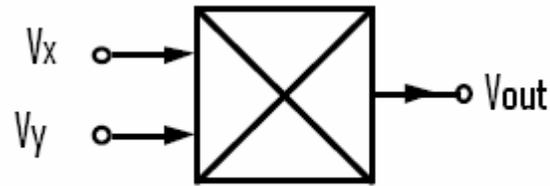


# Introduction

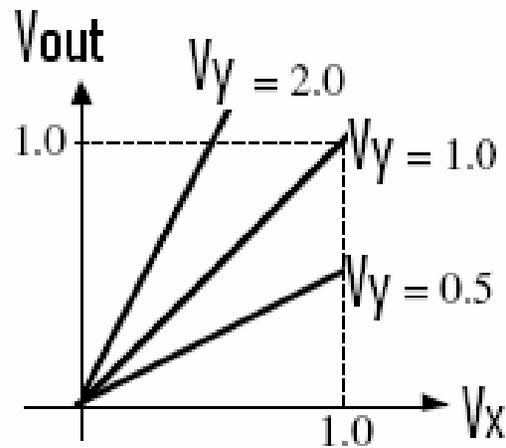
- One of the fundamental building blocks in analog circuit design is the analog multiplier.
- Multipliers are particularly important in communication and signal processing circuits.
- **Applications of Multipliers**
  - Nonlinear analog signal processing
  - Mixing
  - Phase difference detection
  - Modulation and demodulation
  - Frequency translation



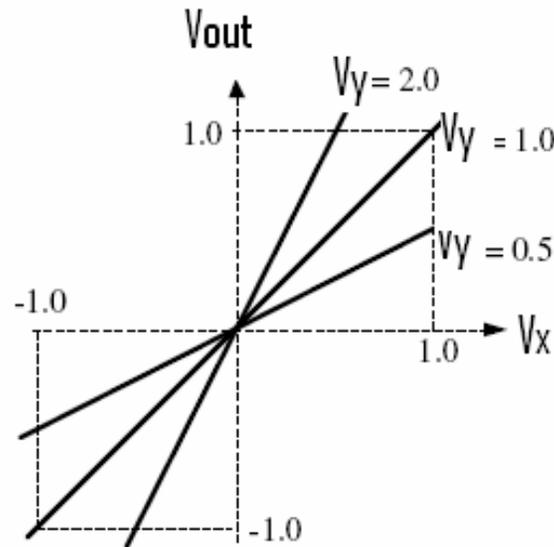
# Introduction



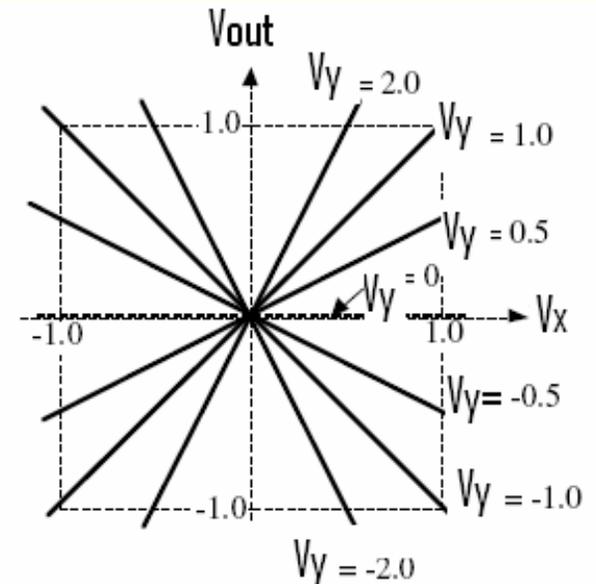
## Types of Multiplication



One-Quadrant Multiplier



Two-Quadrant Multiplier



Four-Quadrant Multiplier



# Simple Emitter Coupled Circuit (ECC)

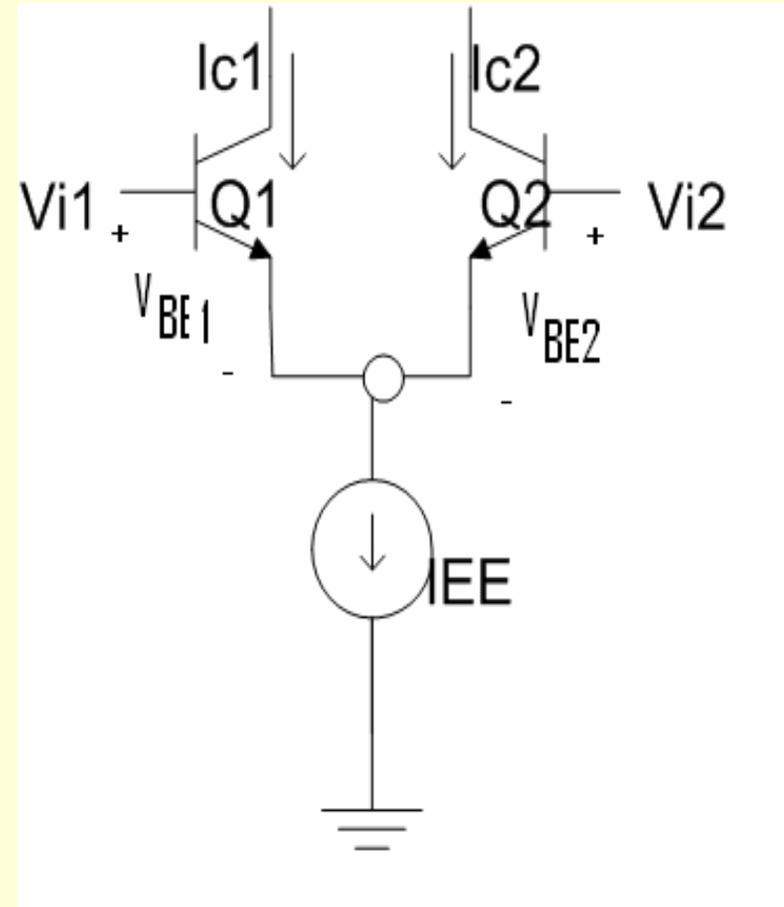
The Basic equation of the ECC is :

$$V_{i1} - V_{BE1} = V_{i2} - V_{BE2}$$

But in the active region, the collector current of the npn transistor is given by:

$$I_C = I_o e^{\frac{V_{BE}}{V_T}}$$

$$\Rightarrow V_{BE} = V_T \ln\left(\frac{I_C}{I_o}\right)$$



# Simple Emitter Coupled Circuit (ECC)

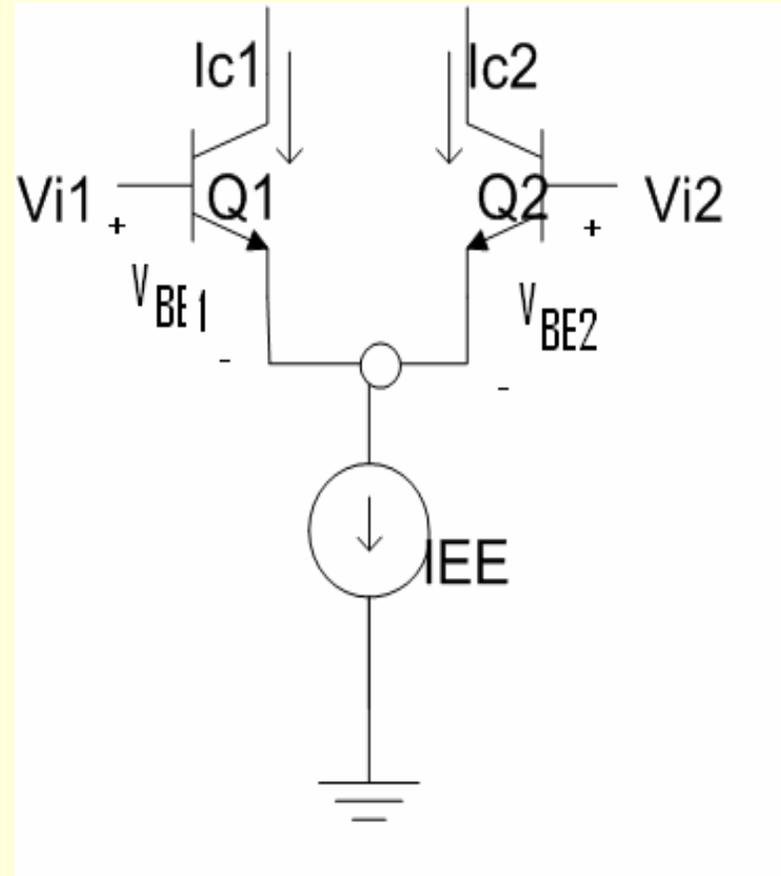
Therefore

$$V_{i1} - V_{i2} = V_{id} = V_T \ln\left(\frac{I_{C1}}{I_{o1}}\right) - V_T \ln\left(\frac{I_{C2}}{I_{o2}}\right)$$

For a matched transistors

$$I_{o1} = I_{o2}$$

$$V_{id} = V_T \ln\left(\frac{I_{C1}}{I_{C2}}\right)$$



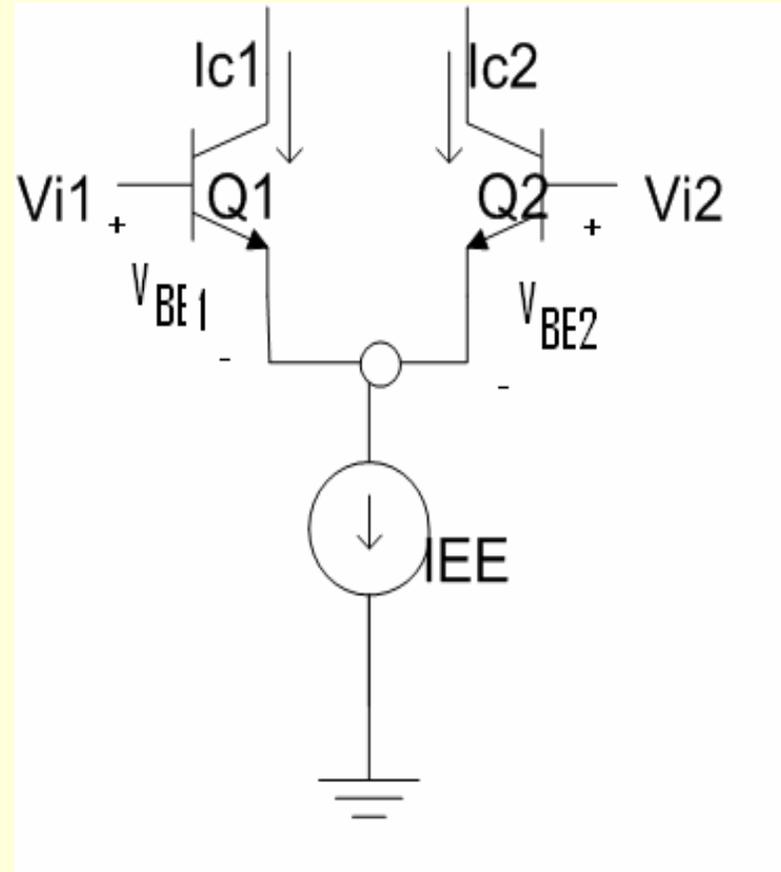
# Simple Emitter Coupled Circuit (ECC)

Therefore:

$$\frac{I_{C1}}{I_{C2}} = e^{\frac{V_{id}}{V_T}} \quad (1)$$

And assuming high  $\beta$

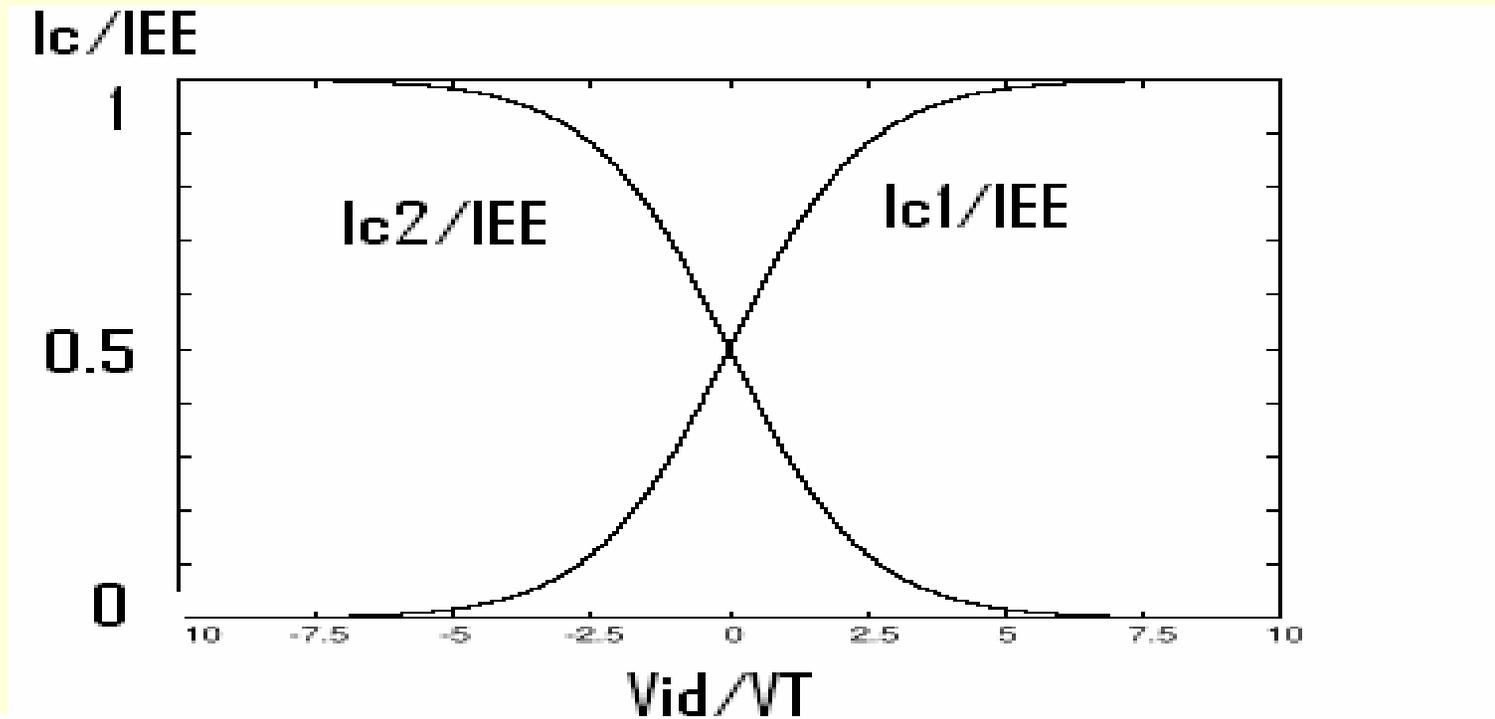
$$I_{C1} + I_{C2} = I_{EE} \quad (2)$$



# Simple Emitter Coupled Circuit (ECC)

From (1) and (2):

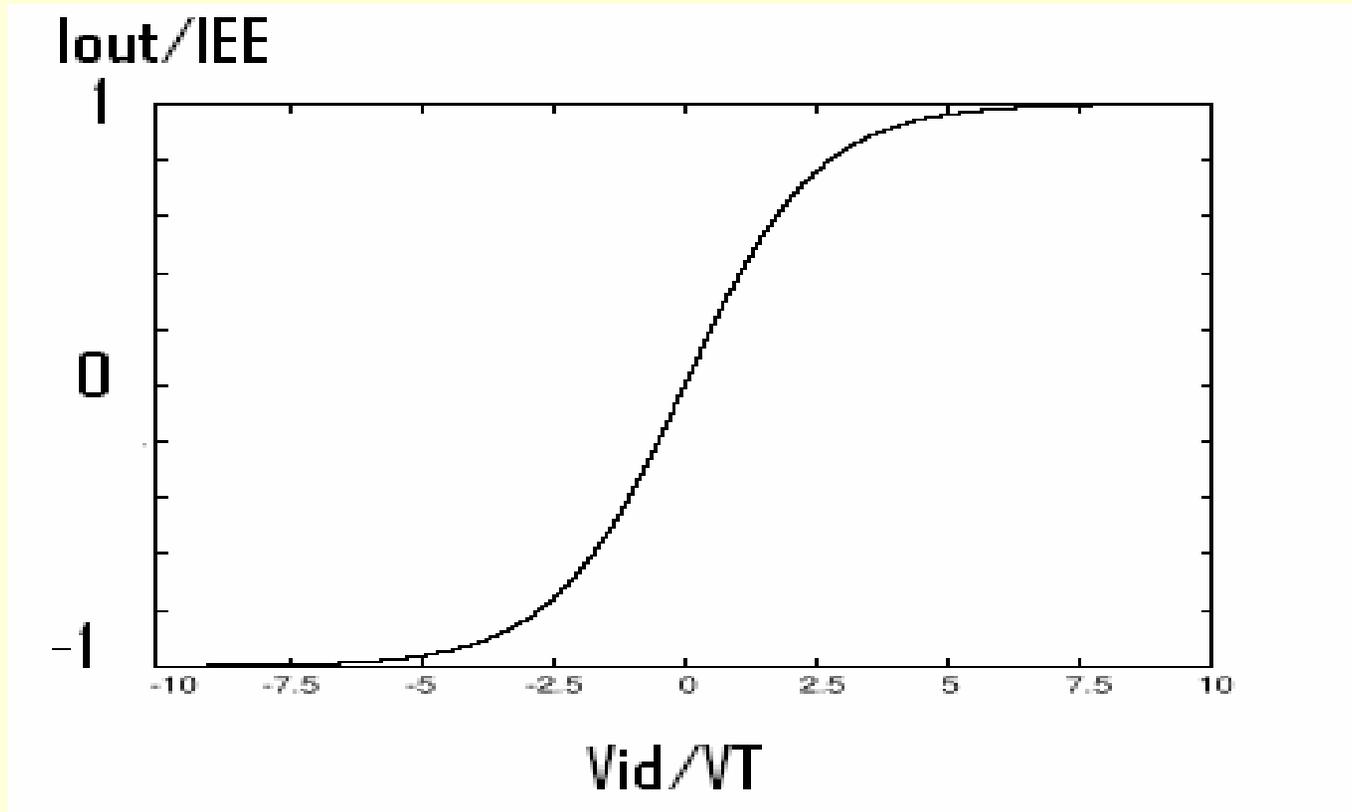
$$I_{C1} = \frac{IEE}{1 + e^{\frac{V_{id}}{V_T}}} \quad \& \quad I_{C2} = \frac{IEE}{1 + e^{-\frac{V_{id}}{V_T}}}$$



# Simple Emitter Coupled Circuit (ECC)

And also:

$$I_{out} = IEE \tanh\left(\frac{V_{id}}{2V_T}\right)$$



# Simple Emitter Coupled Circuit (ECC)

Note That:

For  $|V_{id}| \ll V_T$

The output current of the ECC is given by:

$$I_{out} \cong K (V_{id}) * (IEE)$$

Therefore, the circuit can operate as a multiplier with two inputs  $V_{id}$  and  $IEE$

Where  $K$  is the constant of the proportionality and is given by:

$$K = \frac{1}{2V_T} \quad [V^{-1}]$$



# Simple Emitter Coupled Circuit (ECC)

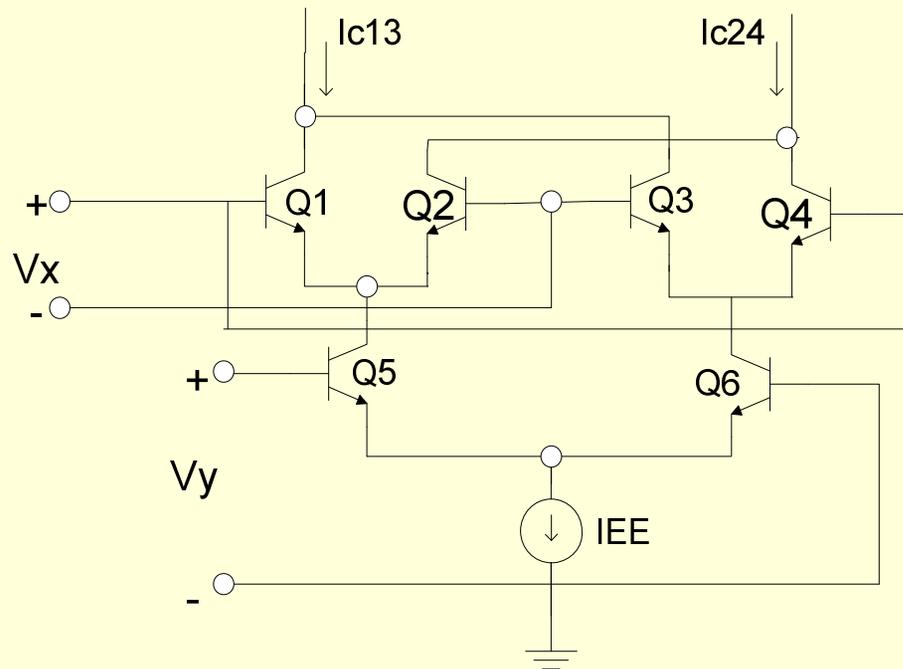
## ■ The disadvantages of the ECC as an multiplier:

1. The circuit operates as 2- quadrant multiplier  
(  $V_{id}$  may be positive or negative but  $I_{EE}$  must be positive)
2. The two inputs of the multiplier are not the same type  
( one of the inputs is voltage  $V_{id}$  and the other is current)
3. The circuit operates as 2- quadrant multiplier under the restriction:  $|V_{id}| \ll V_T$



# Gilbert Cell As a 4-quadrant Multiplier

The Gilbert Cell is a circuit used to overcome the disadvantages of the ECC.



The output differential current of the Gilbert Cell is defined as:

$$I_{out} = (I_{C13} - I_{C24}) = (I_{C1} + I_{C3}) - (I_{C2} + I_{C4})$$



# Gilbert Cell As a 4-quadrant Multiplier

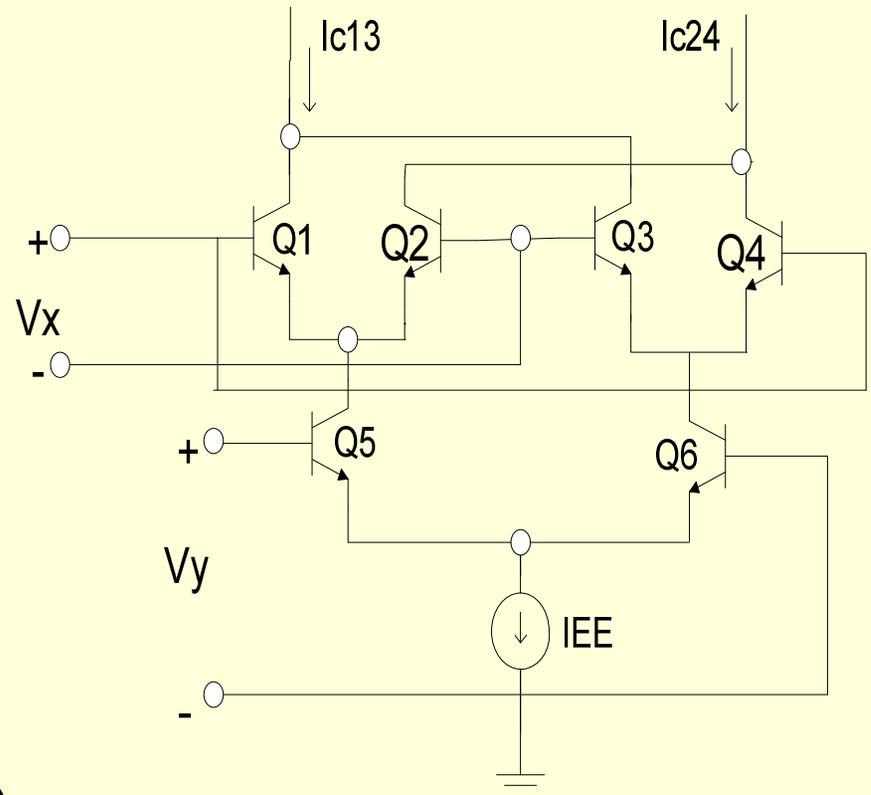
The output current can be written as:

$$I_{out} = (I_{C1} - I_{C2}) + (I_{C3} - I_{C4})$$

$$I_{out} = I_{C5} \tanh\left(\frac{V_X}{2V_T}\right) + I_{C6} \tanh\left(\frac{-V_X}{2V_T}\right)$$

$$I_{out} = (I_{C5} - I_{C6}) \tanh\left(\frac{V_X}{2V_T}\right)$$

$$I_{out} = IEE \tanh\left(\frac{V_Y}{2V_T}\right) \tanh\left(\frac{V_X}{2V_T}\right)$$



# Gilbert Cell As a 4-quadrant Multiplier

Note that:

From the expression of the output current of the Gilbert Cell:

$$I_{out} = IEE \tanh\left(\frac{V_Y}{2V_T}\right) \tanh\left(\frac{V_X}{2V_T}\right)$$

For

$$|V_X| \& |V_Y| \ll V_T \quad \Rightarrow \quad I_{out} \cong K (V_X * V_Y)$$

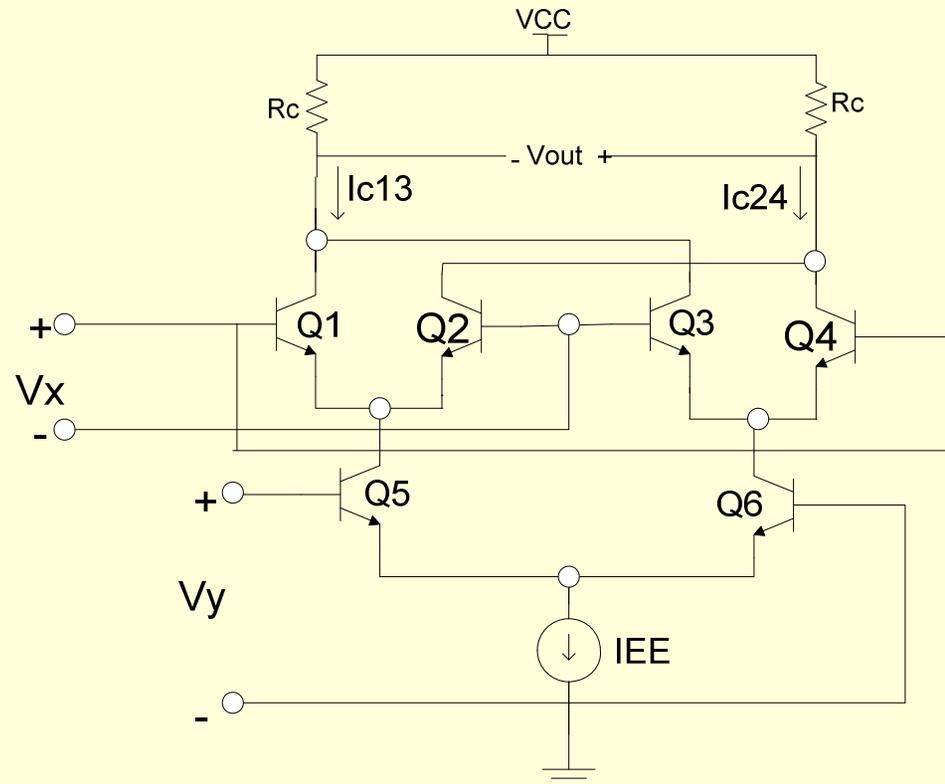
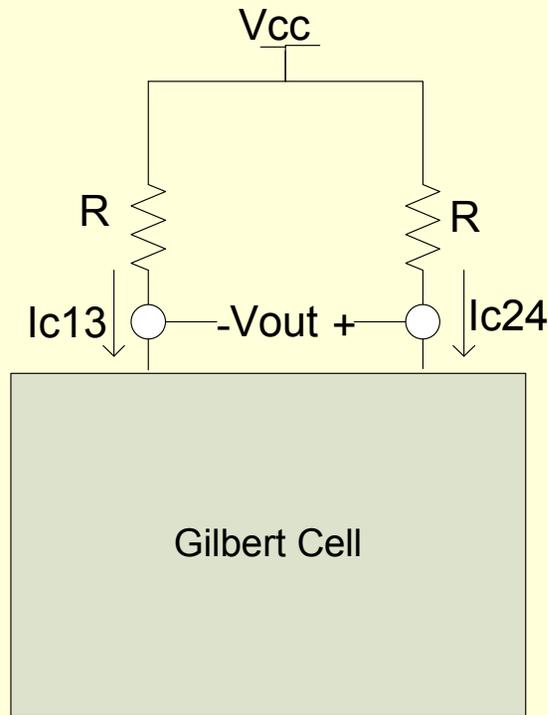
Therefore Gilbert Cell operates as 4-quadrant multiplier because  $V_x$  &  $V_y$

can take positive and negative values, Where  $K = \frac{IEE}{(2V_T)^2} [mA/V^2]$



# Gilbert Cell As a 4-quadrant Multiplier with Output Voltage

Note that : The output of the Gilbert Multiplier is a current, and if it is required to have an output voltage, a differential current to a voltage converter is needed. This can be realized by using the two resistors connected to  $V_{CC}$  as shown.



# Gilbert Cell As a 4-quadrant Multiplier with Output Voltage

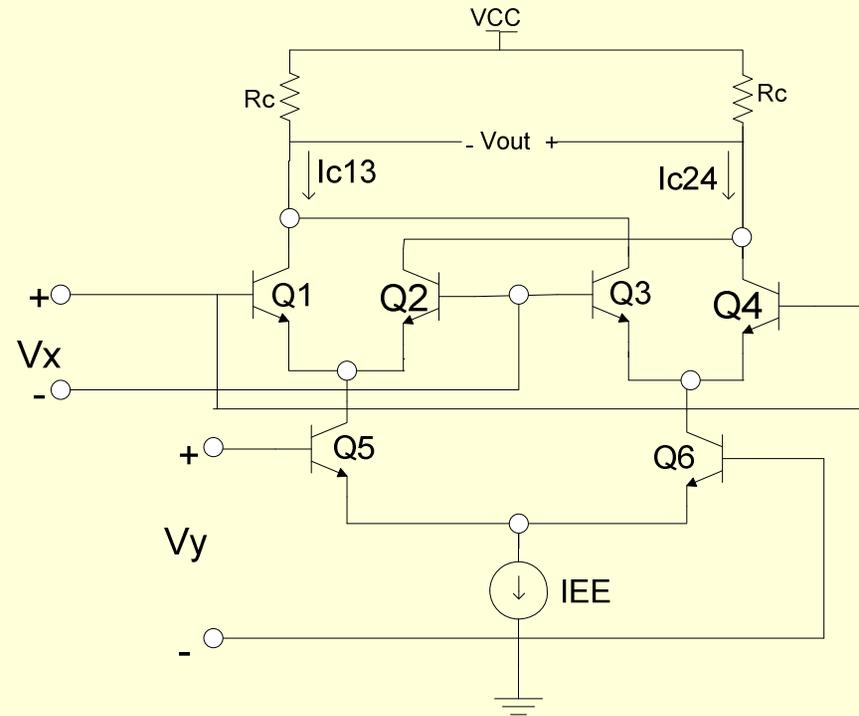
$$V_{out} = (V_{CC} - I_{C24}R_C) - (V_{CC} - I_{C13}R_C)$$

$$V_{out} = (I_{C13} - I_{C24})R_C$$

$$V_{out} = I_{EE}R_C \tanh\left(\frac{V_X}{2V_T}\right) \tanh\left(\frac{V_Y}{2V_T}\right)$$

$$V_{out} \cong K(V_X * V_Y) \quad \text{For } |V_X| \& |V_Y| \ll V_T$$

$$\text{Where } K = I_{EE}R_C / (2V_T)^2 [V^{-1}]$$

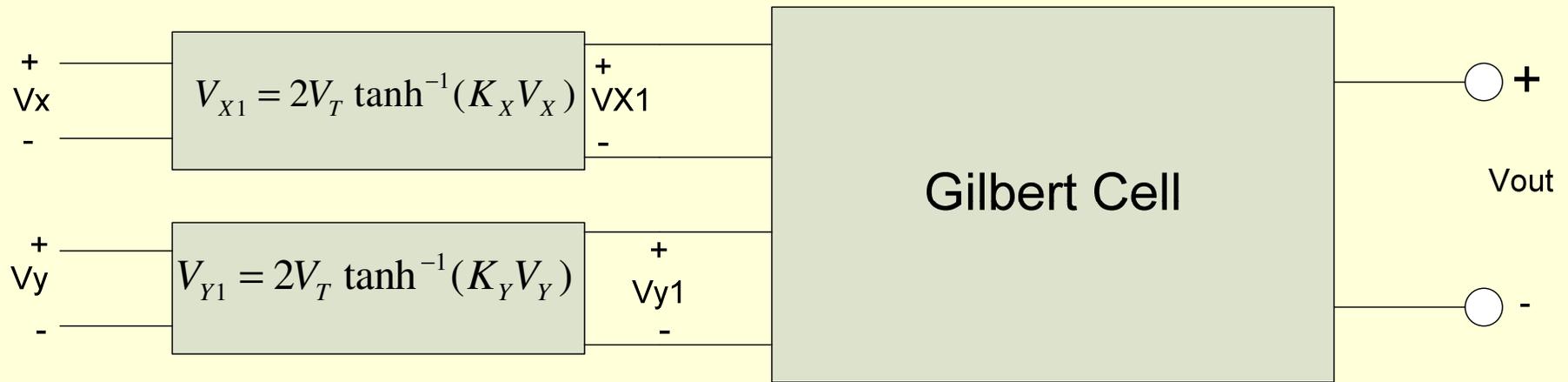


**Note that: For large signals, the Gilbert cell is not operate correctly as a multiplier, therefore we try to make the I/P signals pass through certain circuits that eliminate the nonlinearity of the tanh function.**



# Complete Gilbert Multiplier without any restriction on the I/P Signals

## Basic Idea



$$V_{out} = I_{EE} R_C \tanh\left(\frac{V_{X1} = 2V_T \tanh^{-1}(K_X V_X)}{2V_T}\right) \tanh\left(\frac{V_{Y1} = 2V_T \tanh^{-1}(K_Y V_Y)}{2V_T}\right)$$

$$V_{out} = K (V_X * V_Y) \quad \text{where } K = I_{EE} R_C K_X K_Y [V^{-1}]$$



# $\tanh^{-1}$ circuit

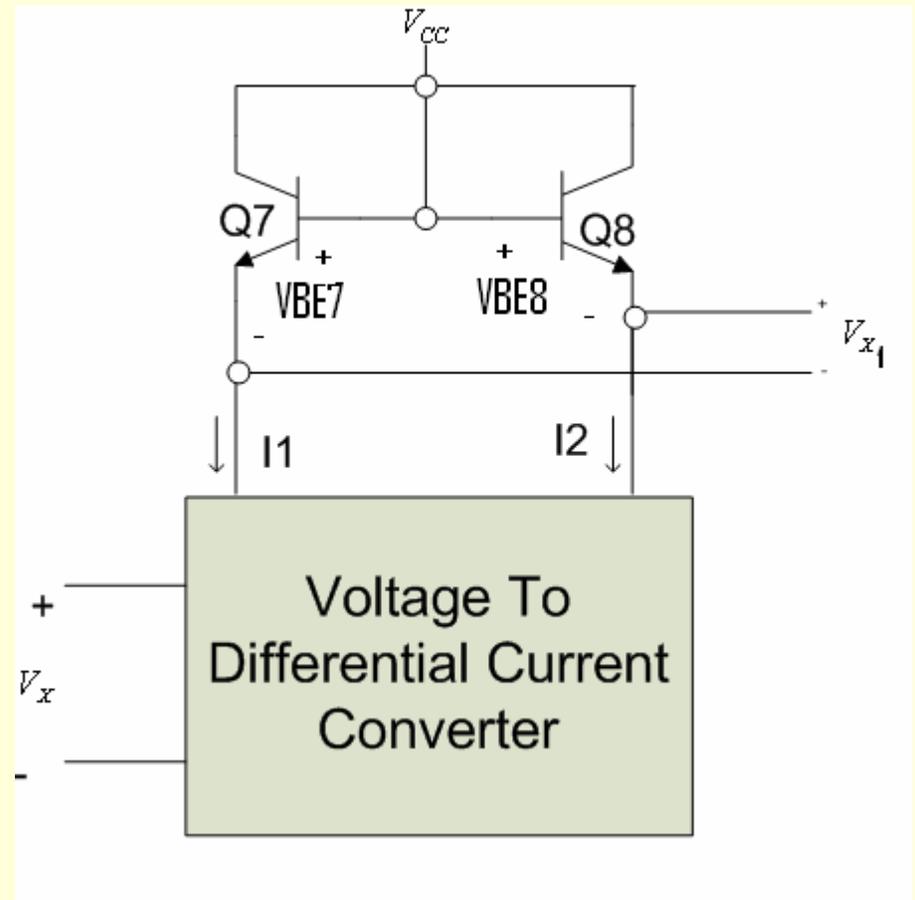
From the shown circuit:

$$V_{X1} = V_{BE7} - V_{BE8}$$

$$V_{X1} = V_T \ln\left(\frac{I_1}{I_{o7}}\right) - V_T \ln\left(\frac{I_2}{I_{o8}}\right)$$

Assuming Q7 and Q8 are matched

$$V_{X1} = V_T \ln\left(\frac{I_1}{I_2}\right)$$



# $\tanh^{-1}$ circuit

Assuming the voltage to differential current converter produces two Output currents given by:

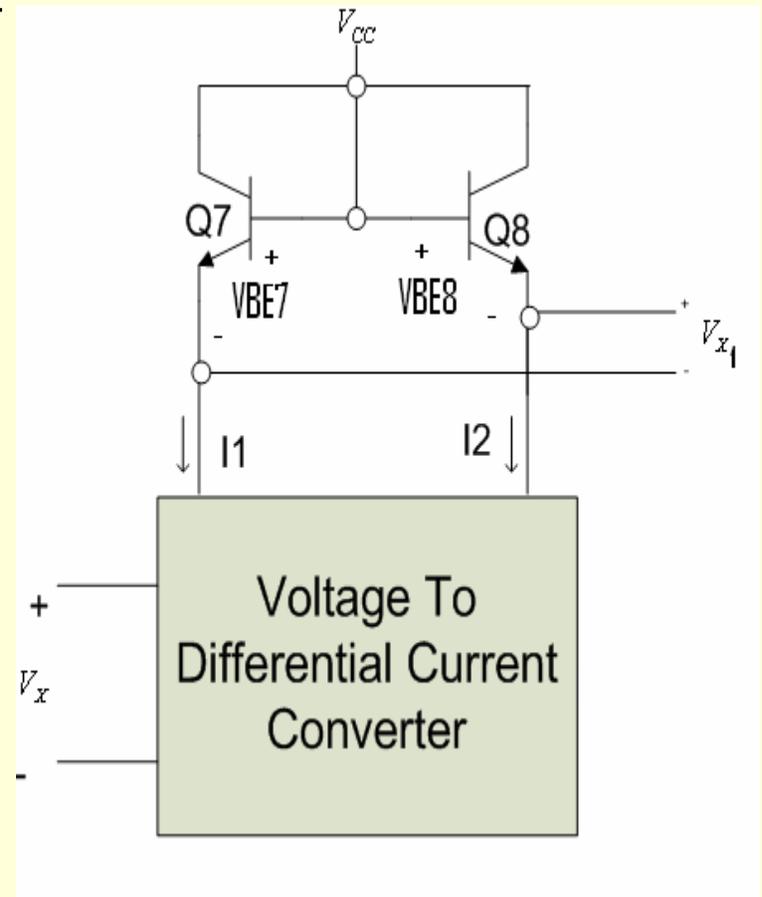
$$I_1 = I_{dc} (1 + K_X V_X) \quad \&$$

$$I_2 = I_{dc} (1 - K_X V_X)$$

Therefore, the output voltage of the  $\tanh^{-1}$  circuit is given by:

$$V_{X1} = V_T \ln\left(\frac{1 + K_X V_X}{1 - K_X V_X}\right)$$

$$\Rightarrow V_{X1} = 2V_T \tanh^{-1}(K_X V_X)$$



# Voltage to Differential Current Converter

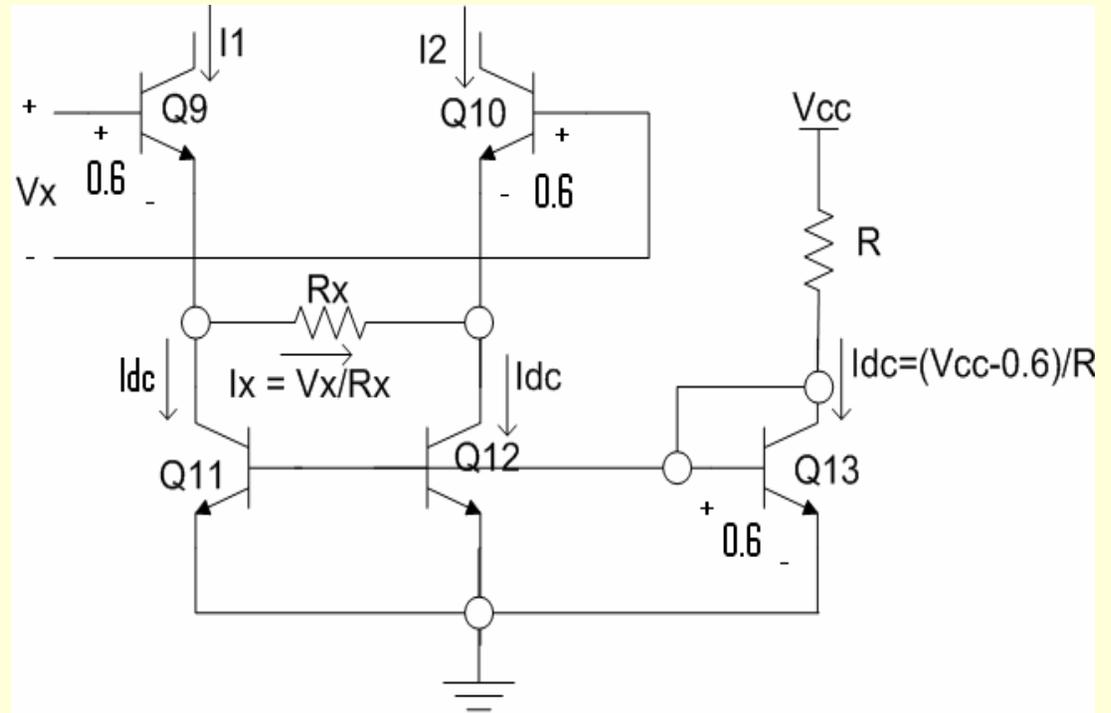
From the shown circuit, we have:

$$I_1 = I_{dc} + \frac{V_X}{R_X}$$

$$I_1 = I_{dc} \left( 1 + \frac{V_X}{I_{dc} R_X} \right)$$

$$\Rightarrow I_1 = I_{dc} (1 + K_X V_X)$$

and 
$$K_X = \frac{1}{I_{dc} R_X}$$



Similarly 
$$I_2 = I_{dc} (1 - K_X V_X)$$



# Report

Draw the complete circuit of 4-quadrant multiplier using only one current source to bias the overall circuit ( Gilbert Cell and  $\tanh^{-1}$  circuits) and find the constant of proportionality  $K$  of the multiplier, the matched conditions and the range of operation.



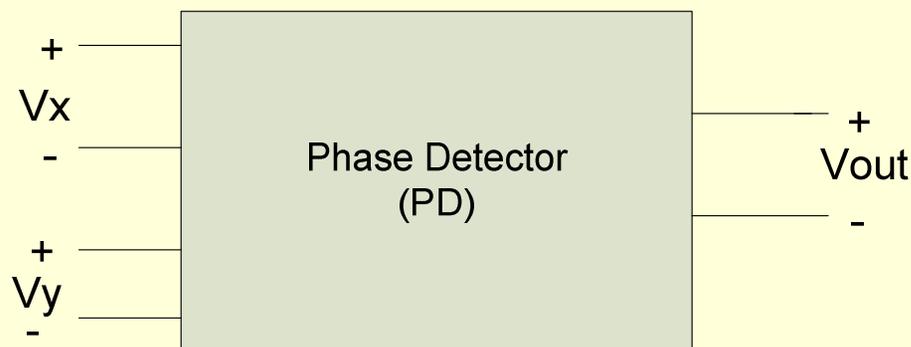
# The application of the Gilbert Cell to realize a Phase Detector (PD)

Phase Detector is a circuit provide output voltage proportional to the phase difference between two input signals.

Types of PD:

**Linearized phase detector:** The output is linearly proportional to the phase difference between the two input signal.

**Sinusoidal phase detector:** The output is proportional to the Sin or Cosine the phase difference between the two input signal.



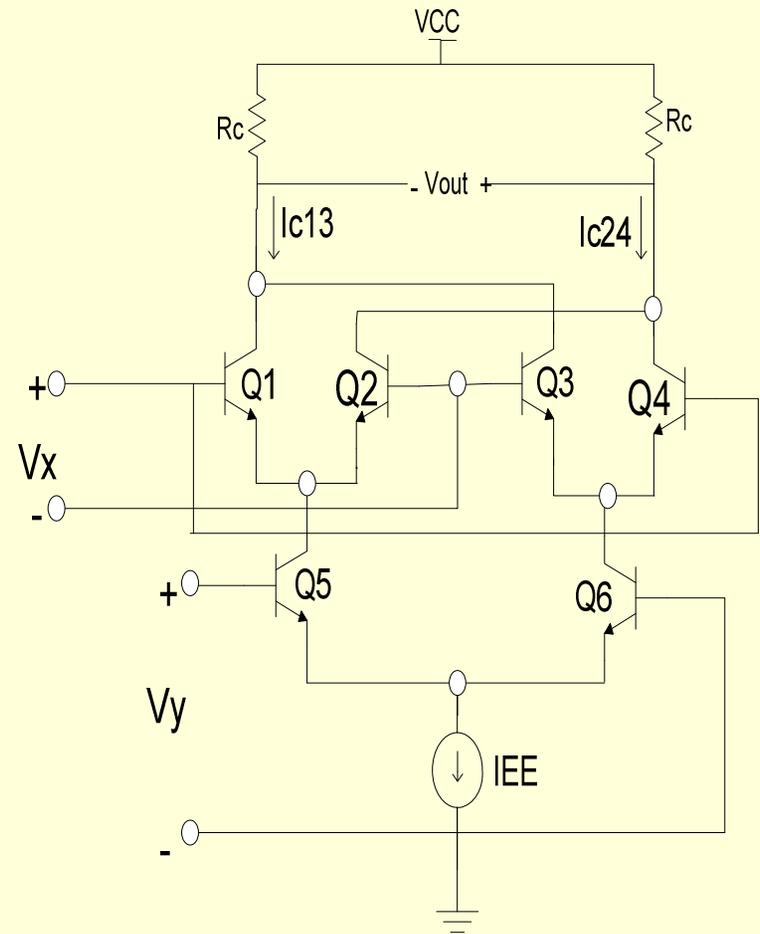
# Linearized PD

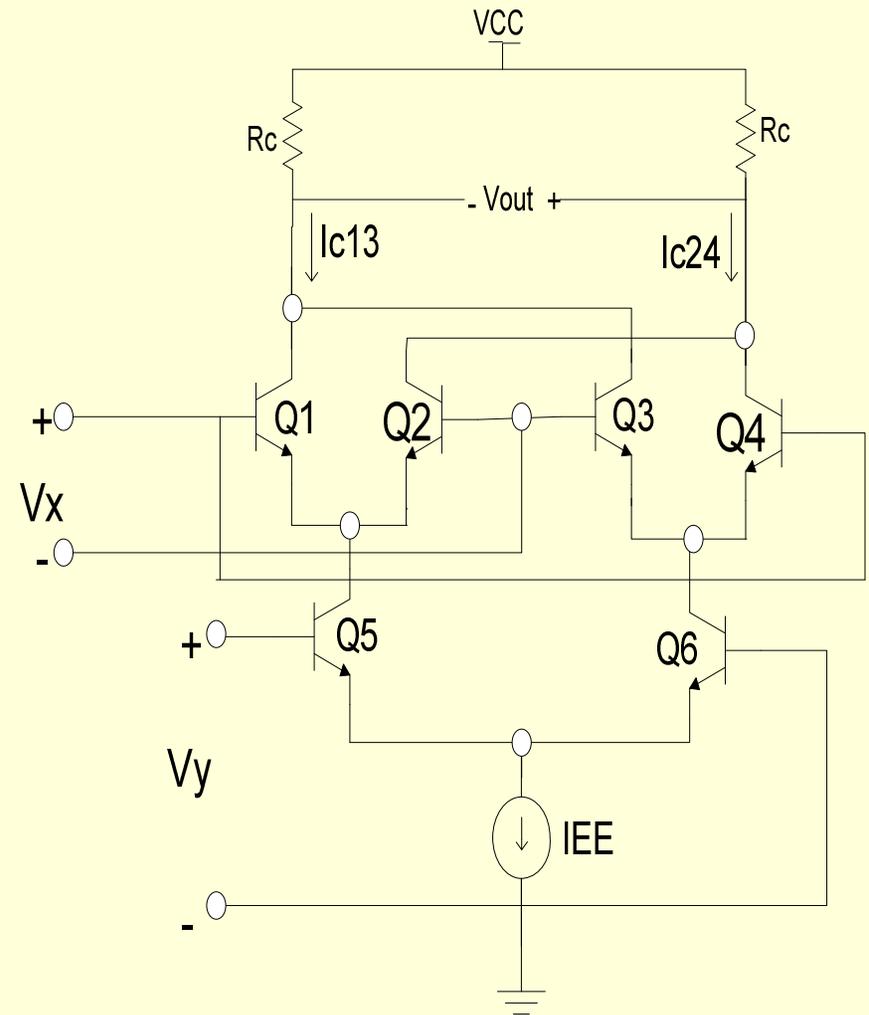
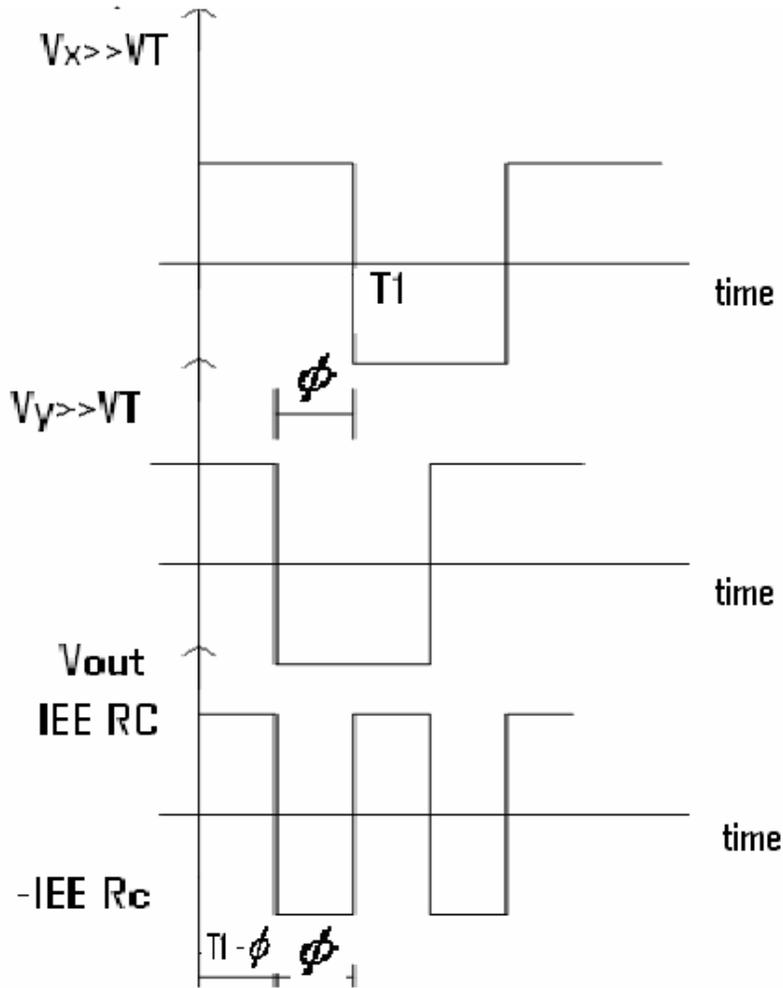
Condition on the inputs for proper operations:

The amplitudes of  $|V_x|$  &  $|V_y| \gg V_T$

Therefore, the transistors operate as a switch (ON/OFF)

Example: Draw the output waveform and calculate its average value for the input signal waveforms shown below:





$$\bar{V}_{out} = \int_0^{T_1} v_{out}(t) dt = -\frac{1}{T_1} [I_{EE} R_C \phi - I_{EE} R_C (T_1 - \phi)]$$

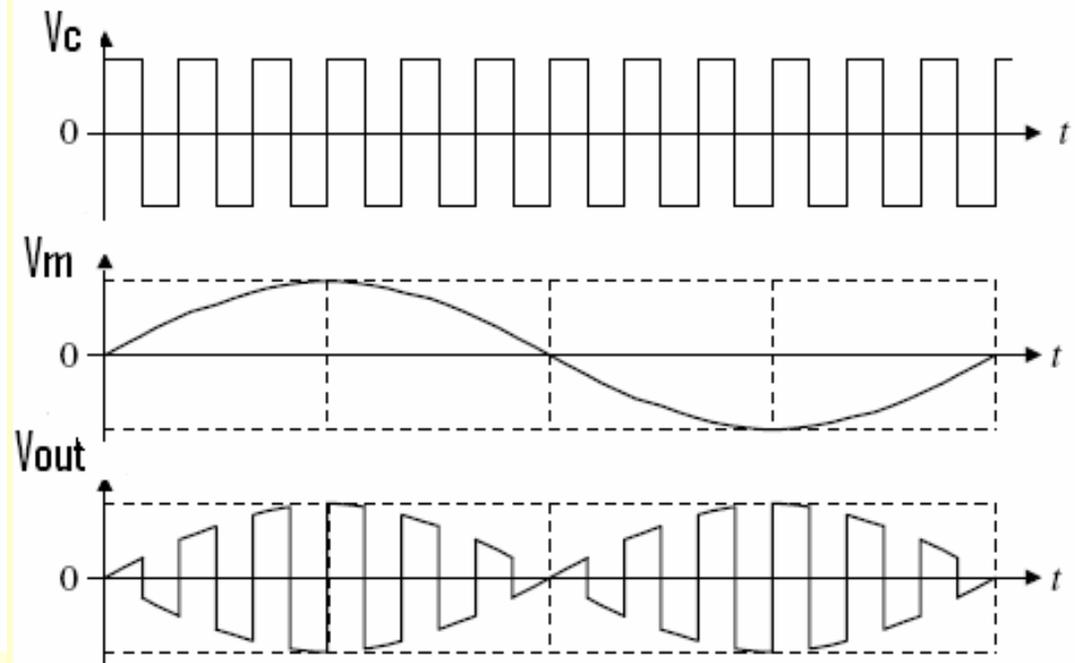
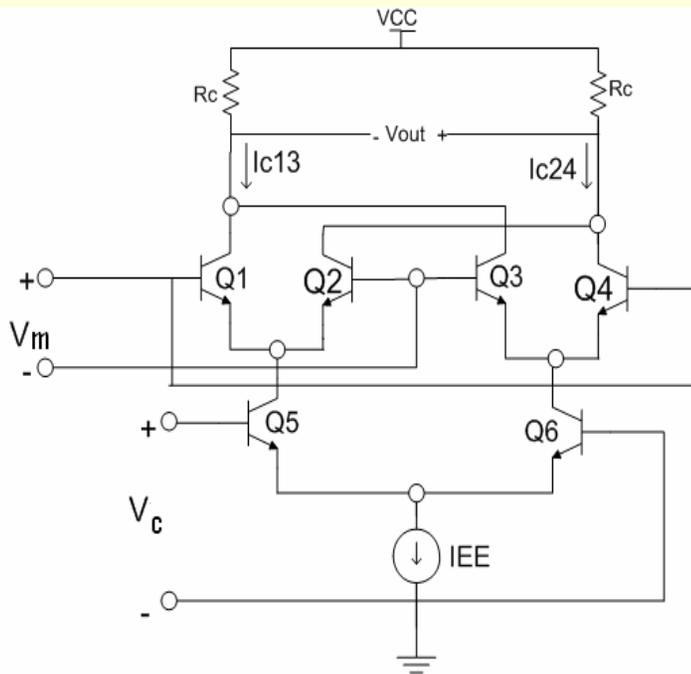
$$\bar{V}_{out} = -I_{EE} R_C [2\phi / T_1 - 1]$$



# Gilbert Cell as a DSB-SC Modulator

Condition on the inputs for proper operations:

The Gilbert Cell can be used as a switching modulator under the condition, One of the input signal( Carrier ) is  $\gg V_T$  and the another input( modulating Signal) is  $\ll V_T$



# MOS Multiplier: NMOS Differential Pair

The output current of the NMOS Differential pair is given:

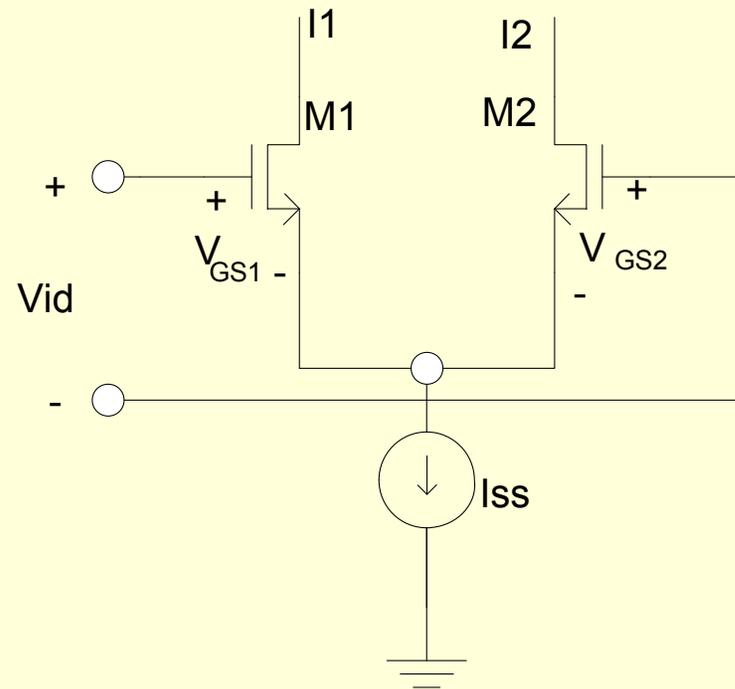
$$I_{out} = I_1 - I_2$$

Where the currents of M1 and M2 in the saturation region are given by:

$$I_1 = \frac{K_1}{2} (V_{GS1} - V_T)^2 = \frac{K_1}{2} A^2$$

$$I_2 = \frac{K_2}{2} (V_{GS2} - V_T)^2 = \frac{K_2}{2} B^2$$

And assuming M1 and M2 are matched ( $K_1 = K_2 = \mu_n C_{ox} \frac{W}{L}$ )



# MOS Multiplier: NMOS Differential Pair

Therefore:

$$I_{out} = \frac{K}{2} (A^2 - B^2) = \frac{K}{2} (A - B)(A + B)$$

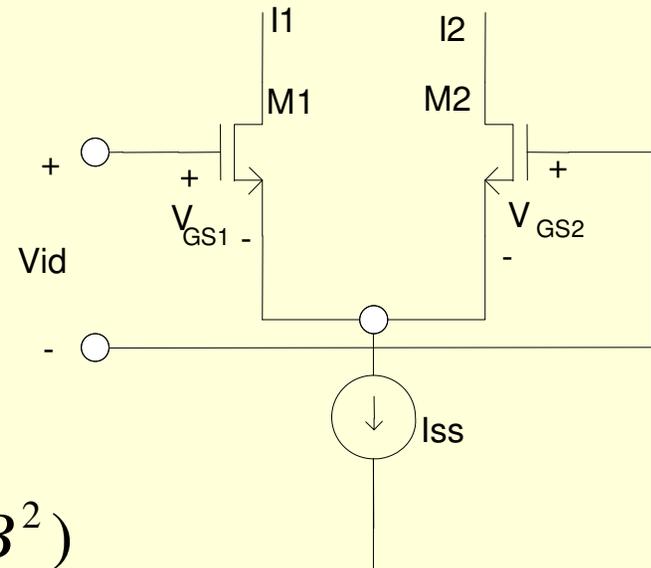
$$\text{And } I_1 + I_2 = \frac{K}{2} (A^2 + B^2) = I_{SS}$$

$$\text{Where } A - B = V_{GS1} - V_{GS2} = V_{id}$$

$$\text{Note: } (A + B)^2 + (A - B)^2 = 2(A^2 + B^2)$$

$$(A + B)^2 = 4 \frac{I_{SS}}{K} - V_{id}^2 \quad \Rightarrow \quad (A + B) = \sqrt{4 \frac{I_{SS}}{K} \left( 1 - \frac{V_{id}^2}{4I_{SS} / K} \right)}$$

$$I_{out} = \sqrt{I_{SS} K} V_{id} \left( \sqrt{1 - \frac{V_{id}^2}{4I_{SS} / K}} \right)$$



# MOS Multiplier: NMOS Differential Pair

Note:

$$I_{out} = I_1 - I_2 = \sqrt{I_{SS} K} V_{id} \left( \sqrt{1 - \frac{V_{id}^2}{4I_{SS} / K}} \right)$$

and

$$I_1 + I_2 = I_{SS}$$

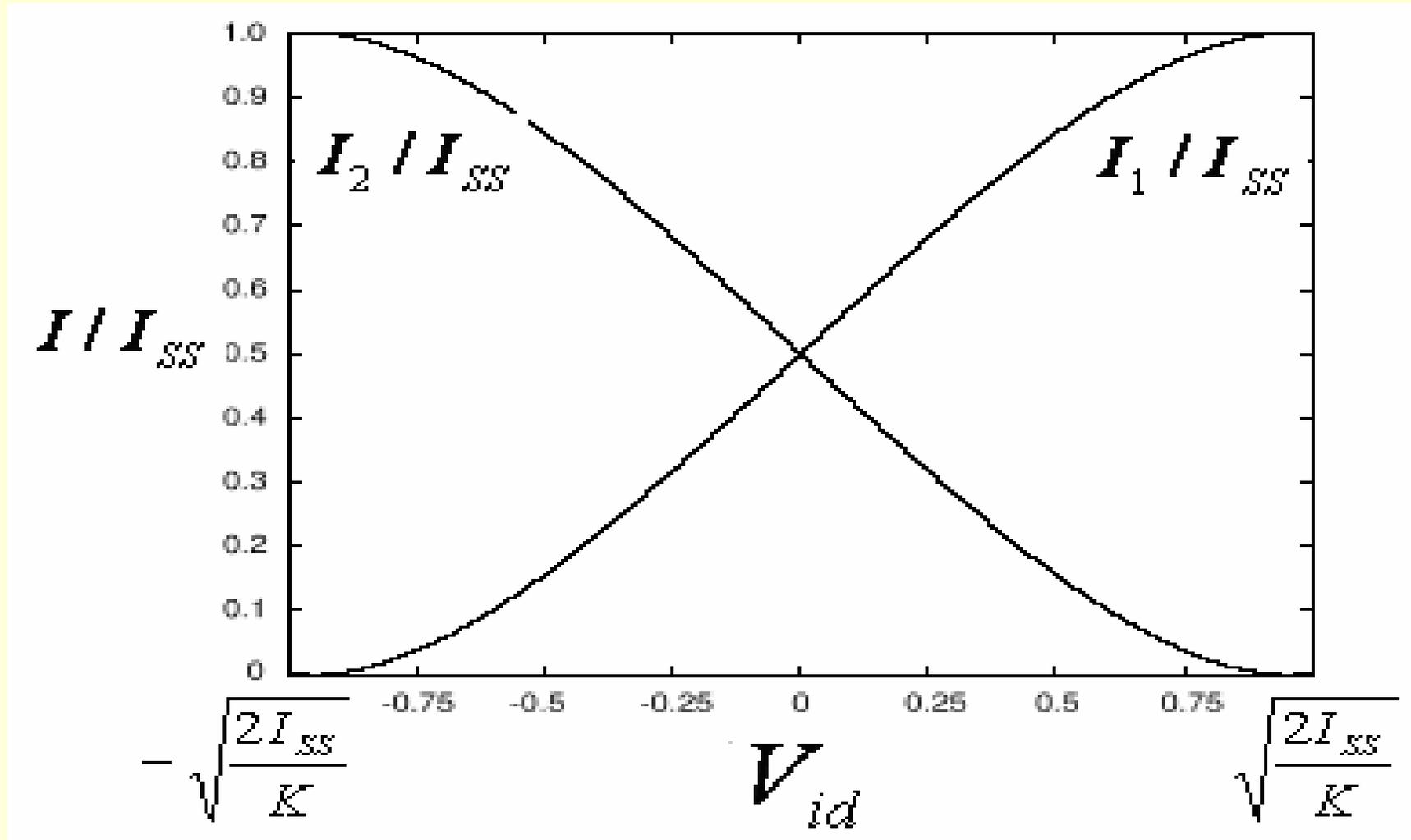
Therefore:

$$I_1 = \frac{I_{SS}}{2} + \frac{1}{2} \sqrt{I_{SS} K} V_{id} \left( \sqrt{1 - \frac{V_{id}^2}{4I_{SS} / K}} \right)$$

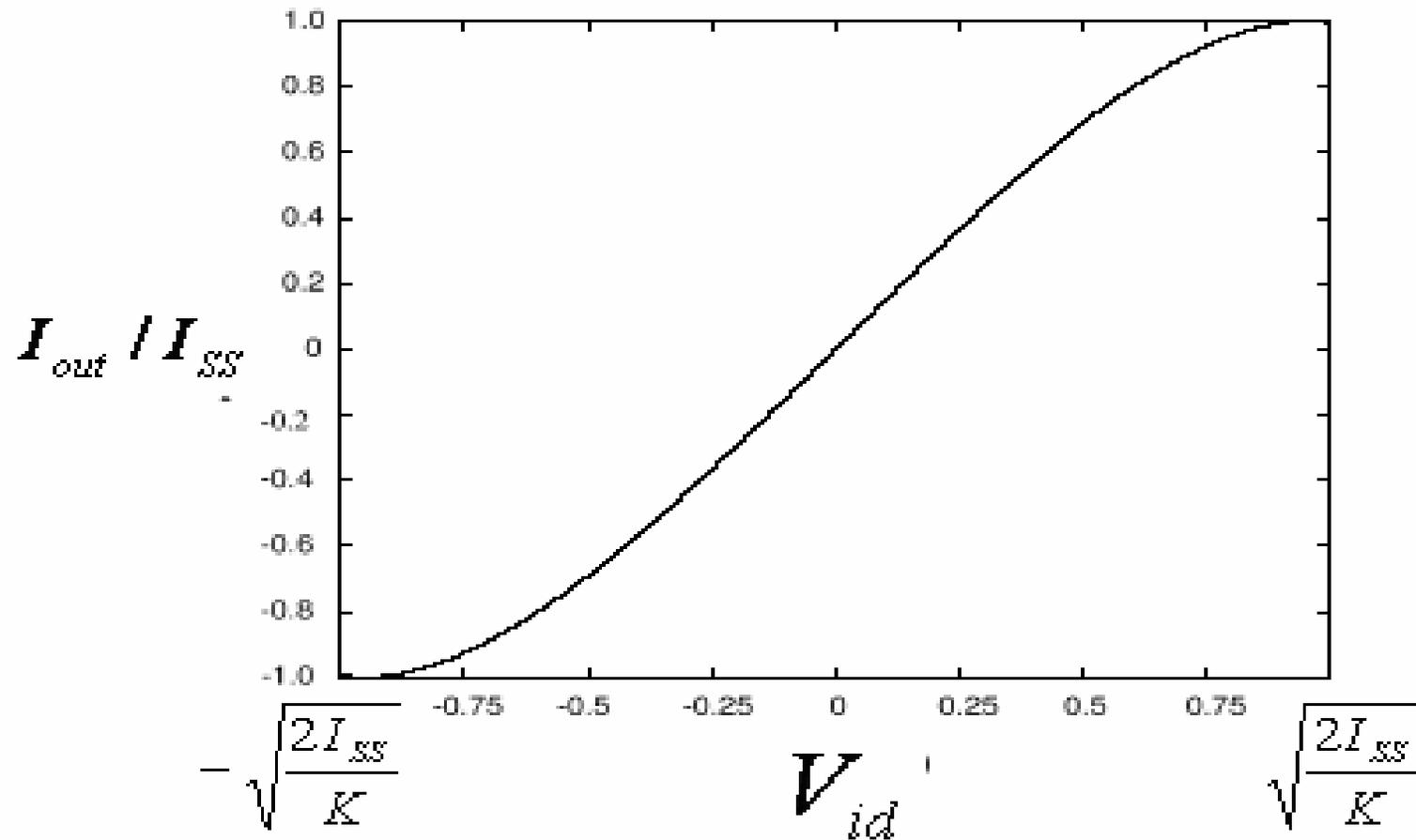
$$I_2 = \frac{I_{SS}}{2} - \frac{1}{2} \sqrt{I_{SS} K} V_{id} \left( \sqrt{1 - \frac{V_{id}^2}{4I_{SS} / K}} \right)$$



# MOS Multiplier: NMOS Differential Pair



# MOS Multiplier: NMOS Differential Pair



# MOS Multiplier: NMOS Differential Pair

Note: if  $V_{id} < \sqrt{\frac{2I_{SS}}{K}}$

The nonlinear term of output current can be neglected and the output current is given by:

$$I_{out} \cong \sqrt{I_{SS} K} V_{id}$$

and

$$I_1 \cong \frac{I_{SS}}{2} + \frac{1}{2} \sqrt{I_{SS} K} V_{id}$$

$$I_2 \cong \frac{I_{SS}}{2} - \frac{1}{2} \sqrt{I_{SS} K} V_{id}$$



# MOS Gilbert Cell

The output current can be written as:

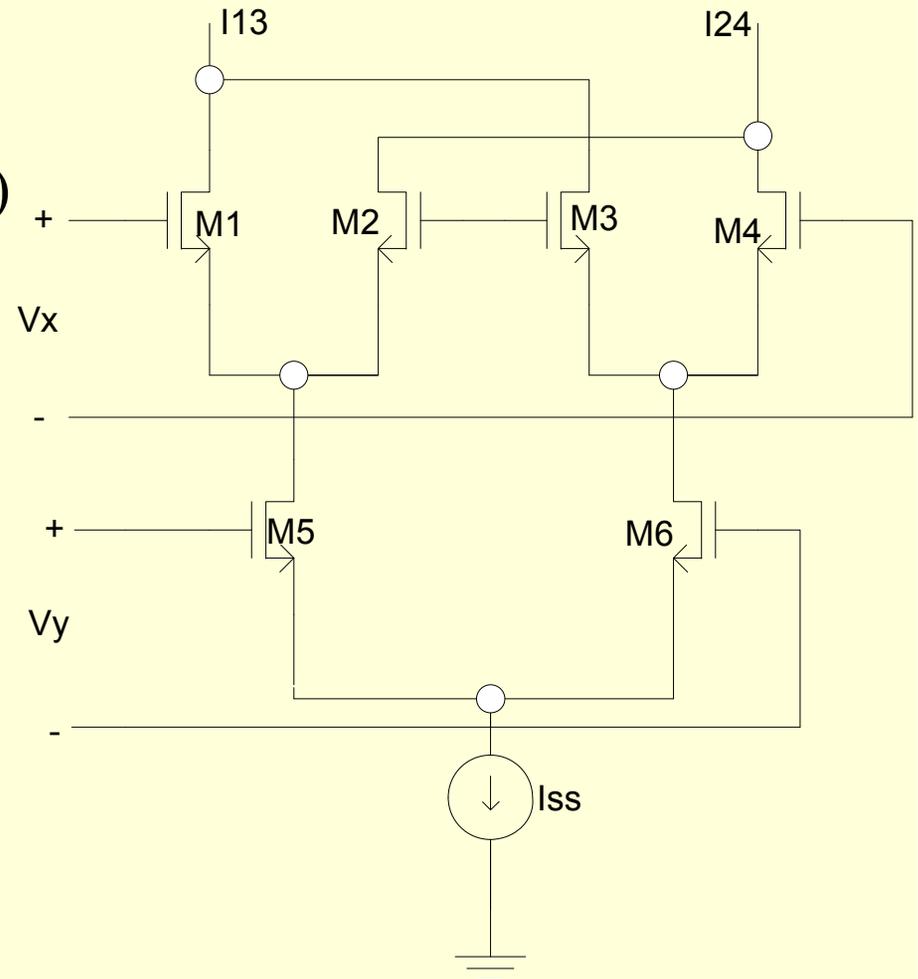
$$I_{out} = I_{13} - I_{24} = (I_1 - I_2) + (I_2 - I_4)$$

$$I_{out} \cong \sqrt{I_5 K} V_X - \sqrt{I_6 K} V_X$$

$$I_{out} \cong \sqrt{K} (\sqrt{I_5} - \sqrt{I_6}) V_X$$

and  $(\sqrt{I_5} - \sqrt{I_6}) = \sqrt{\frac{K}{2}} V_y$

$$\Rightarrow I_{out} \cong \frac{K}{\sqrt{2}} (V_X * V_y)$$



# BJT and MOS Analog Multiplier

## END of Chapter 2

