[redacted]

Rules:

1. Work in a group of two to three persons. Individual work is not allowed.
2. MATLAB codes must be included in an Appendix.
3. Plagiarized work will be rejected. However, moderate collaboration in the form of joint problem solving with one or two other groups is permitted provided your writeup is your own.

Problem 1.

The aim of this exercise is to simulate a single-path Rayleigh fading channel. For this channel, the fading process $a(t)$ is a complex Gaussian process with a specific Doppler power spectrum (or psd) $S(f)$. A way to simulate a Rayleigh fading channel is given as follows:

1. Generate a complex Gaussian process with variance 1 (the real and imaginary part each have variance 1/2).
2. Filter the process with a Doppler filter (e.g., FIR filter) with frequency response $H(f) = \sqrt{S(f)}$ and impulse response $h(t) = \mathbb{F}^{-1}\{H(f)\}$, where $\mathbb{F}^{-1}\{.\}$ denotes inverse Fourier transform. The impulse response is sampled at a rate f_s, to obtain a discrete-time impulse response $h(n)$.
3. Scale the filtered Gaussian noise sequence to obtain the desired average power.

The process is illustrated in Figure 1.

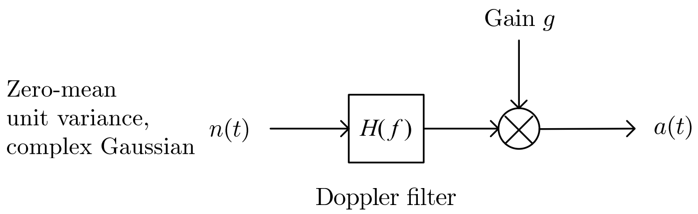

Figure 1: Generation of channel gain processes.

One of the commonly used Doppler power spectrum models is the Jakes Doppler spectrum which is given by:

$$S(f) = \frac{1}{\pi f_d \sqrt{1 - (f/f_d)^2}}, \quad |f| \leq f_d \tag{1}$$

where f_d is the maximum Doppler shift. The corresponding autocorrelation is:

$$R(\tau) = J_0(2\pi f_d \tau) \tag{2}$$

where $J_0(x)$ is the Bessel function of the first kind of order 0. The impulse response of the Doppler filter can be derived as

$$h(t) = \Gamma(3/4)\left(\frac{f_d}{\pi|t|}\right)^{1/4} J_{1/4}(2\pi f_d|t|) \tag{3}$$

where $\Gamma(.)$ is the gamma function. The value of $h(t)$ at $t = 0$ is calculated as $\lim_{t\to 0} h(t) = \Gamma(3/4)/\Gamma(5/4) f_d^{1/2}$.

The discrete-time impulse response used for simulation is a sampled, truncated (to M points), causal (delayed by $M/2$ points) version of $h(t)$, given by

$$\begin{aligned} h(n) &= h((n - M/2)t_s) \\ &= \Gamma(3/4)\left(\frac{f_d}{\pi|(n-M/2)t_s|}\right)^{1/4} J_{1/4}(2\pi f_d|(n-M/2)t_s|) \end{aligned} \tag{4}$$

for $n = 0, 1, ..., M - 1$. To reduce the effect of the Gibbs phenomenon due to truncation, the sampled impulse response $h(n)$ is multiplied by a window, e.g., a Hamming window, to obtain the windowed impulse response of the shaping filter:

$$h_w(n) = h(n)w_H(n), \quad n = 0, 1, ..., M - 1 \tag{5}$$

The windowed impulse response is then normalized sucah that its total power is 1:

$$\bar{h}(n) = \frac{h_w(n)}{\sqrt{\sum_{m=0}^{M-1} |h_w(m)|^2}} \tag{6}$$

By using Matlab, simulate a single-path Rayleigh fading channel with average power 0 dB according to the steps mentioned above. Assume that the sampling interval $t_s = 0.1$ ms and the maximum Doppler shift $f_d = 100$ Hz. Plot and compare the analytical and simulated psd and autocovariance functions of the channel. The autocovariance can be calculated using the `xcov` function, and the power spectrum using the `pwelch` from the Signal Processing Toolbox.

Some useful Matlab m-files for this problem are given in the table below:

m-files	**Description**
`randn`	Gaussian distributed random numbers
`filter`	Filter data with an IIR or FIR filter
`besselj`	Bessel function of the first kind
`gamma`	Gamma functions
`hamming`	Compute a Hamming window

The expected plots are shown in Figure 2.

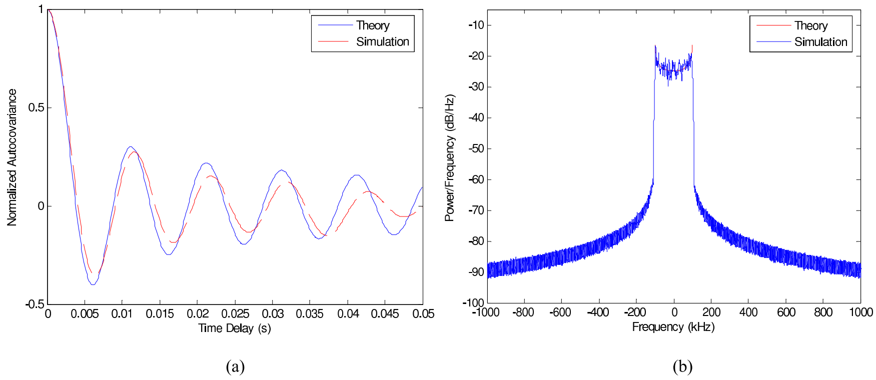

Figure 2: (a) Autocovariance (real part) and (b) power spectrum of the complex fading process for a Rayleigh channel with a Jakes Doppler spectrum.

Problem 2.
The aim of this exercise is to derive and simulate the error rate performance of binary PSK when the signal is transmitted over a single-path (or frequency non-selective) slowly fading channel. The frequency-nonselective channel results in multiplicative distortion of the transmitted signal $s(t)$. In addition, the condition that the channel fades slowly implies that the multiplicative process may be regarded as a constant during at least one signaling interval. Therefore, the received equivalent lowpass signal in one signaling interval is

$$r(t) = \alpha e^{j\phi} s(t) + z(t) \quad 0 \leq t \leq t_s \tag{7}$$

where $z(t)$ represents the complex-valued white Gaussian noise process with power spectral density $\frac{N_0}{2}$.

Assume that the channel fading is sufficiently slow that the phase shift ϕ can be estimated from the received signal without error. Then, we can achieve ideal coherent detection of the received signal. The received signal can be processed by passing it through a matched filter. Show that for a fixed (time-invariant) channel, i.e., for a fixed attenuation α, the error rate of binary PSK as a function of the received SNR γ_b is

$$P_b(\gamma_b) = Q(\sqrt{2\gamma_b}) \tag{8}$$

where $\gamma_b = \alpha^2 \varepsilon_b / N_0$ and ε_b is the energy of the transmitted signal per bit.

To obtain the error probability when α is random, we must average $P_b(\gamma_b)$ over the pdf of γ_b. That is

$$P_b = \int_0^\infty P_b(\gamma_b) p(\gamma_b) d\gamma_b \tag{9}$$

where $p(\gamma_b)$ is the pdf of γ_b.

When α is Rayleigh-distributed, show that α^2 has a chi-square pdf with two degrees of freedom and consequently γ_b is also chi-square distributed, i.e.,

$$p(\gamma_b) = \frac{1}{\bar{\gamma_b}} e^{-\gamma_b/\bar{\gamma_b}}, \quad \gamma_b \geq 0 \tag{10}$$

where $\bar{\gamma_b}$ is the average signal-to-noise ratio, defined as

$$\bar{\gamma_b} = \frac{\varepsilon_b}{N_0} E(\alpha^2) \tag{11}$$

The term $E(\alpha^2)$ is simply the average value of α^2.

Carry out the integration in (9) and show that

$$P_b = \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma_b}}{1+\bar{\gamma_b}}}\right) \tag{12}$$

By using the results of Problem 1, write a simulation program using Matlab to simulate the error rate of the binary PSK and compare it to the analytical result in (12). The expected plots are shown in Figure 3.

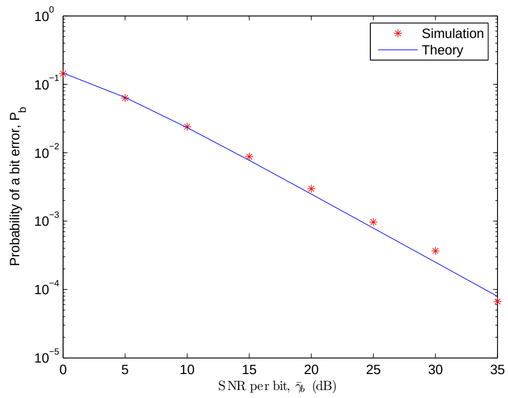

Figure 3: Performance of binary PSK on a frequency-nonselective Rayleigh fading channel.