Examine figure 2 on page 172 of the pdf.

The top drawing is a side view of the experiment. Note how the magnet is close to the disk and the magnetic field line go through the disk. Since the disk is so close to the magnet, the magnetic field is uniform throughout the disk.

The bottom drawing is a top view, looking down on the Faraday Disk.

As described in a prior post, the arrows next to the charge Q represent the velocity, v, at that instant of position and time. "e" is the elementary charge, 1.6×10^{-19} coulombs. The "-" sign indicates the charge on an electron.

"-e**v**×**B**" is the force on the electron due to its motion in the magnetic field. The magnetic field points in the "+z" direction, that is, perpendicular to the page, and toward the reader. "**v**" points perpendicular to a radius, consistent with circular motion. The arrow associated with "-e**v**×**B**" points toward the center of the disk, is the direction resulting from taking the vector product of the two vectors, **v** and **B**, and accounting for the "-" charge of the electron.

"-e**E**" is the electric force acting on the electron, with the arrow indicating the direction, radially away.

"ω" is the angular velocity and the curved arrow indicates the direction of rotation. The relationship between the linear velocity v and the angular velocity ω is v = ωr. while the entire disk rotates at the same rate, ω, particles close to the axis have smaller velocities, while particles further out have larger velocities.

This is a statement of fact. Each of the four items with equals signs between them are different expressions for the same thing.

$$V = -\int_a^b E_r dr = +\frac{\omega}{2\pi}\int_a^b 2\pi r B_z dr = \frac{\omega}{2\pi}\Phi_B$$

"E_r" is the electric field in the radial direction. The subscript is part of the designation, not a separate entity. We do not know E, and therefore cannot do the integral.

"B_z" is the magnetic field in the vertical direction. The subscript is part of the designation, not a separate entity.

We know, however, the relationship between E_r and B_z. The magnetic force and the electric force acting on the electron are equal, so $E_r = -\omega B_z r$, as in equation 2 on page 173.

Therefore, we exchange E_r with $\omega B_z r$, and multiply by $2\pi/2\pi$ to get the third item in the equation, so we have:

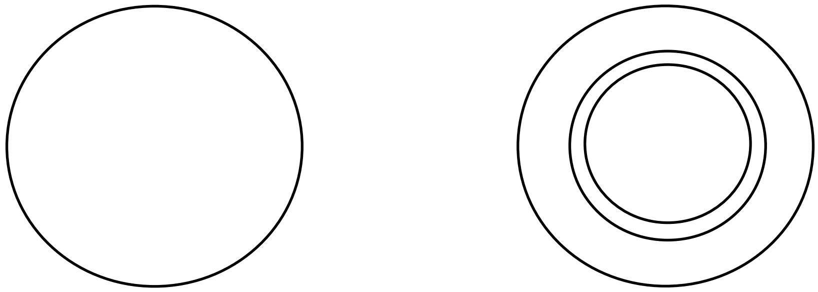
$$V = +\frac{\omega}{2\pi}\int_a^b 2\pi r B_z dr$$

Now, do this integral.

As noted above, B_z is constant at the disk, so we can take it out of the integral: $V = +\frac{\omega}{2\pi} B_z \int_a^b 2\pi r dr$

Now we want to do the integral:
$\int_a^b 2\pi r dr$

On the left is a disk of radius r, and on the right, the same disk with a concentric annular ring.



The circumference of the annular ring is $2\pi r$. The thickness of the annular ring is dr. This is the same dr as in the integral. It means the annular ring is extremely thin. The drawing is only a representation so we can see what's going on. Therefore, $2\pi r\, dr$ is the area of the annular ring. When we integrate from a=0 to b=R, the radius of the disk, we get πr^2, the area of the disk.

In our problem, we integrate from a to b, so we get $2\pi(b^2-a^2)$, the area of the Faraday Disk. That is, the area of a disk with a hole at the center for the axel.

Now we need to put back all the constants we took out of the integral: $\frac{\omega}{2\pi} B_z$

Therefore: $V = +\frac{\omega}{2\pi} B_z\, 2\pi\, (b^2 - a^2)$

Since $2\pi(b^2-a^2)$ is the area of the Faraday Disk, the magnetic flux, Φ_B is $\Phi_B = B_z 2\pi\, (b^2 - a^2)$

Therefore: $V = +\frac{\omega}{2\pi} \Phi_B$

Replace V with curly E and one has equation 3.