

$$k_9 = \frac{2.0 \sqrt{a b}}{a + b} \quad (4.4.2.11)$$

$$k_8 = \frac{k_9}{\sqrt{1.0 - \frac{(b - a)^2}{(2.0 c)^2}}} \quad (4.4.2.12)$$

These equations assume loose coupling and that the substrate is infinitely thick.

#### REFERENCES

- [1] Bandrand, H., et al., "Bias-Variable Characteristics of Coupled Coplanar Waveguide on GaAs Substrate," *Electronics Letters*, Vol. 23, No. 4, February 13, 1987, pp. 171–172.
- [2] Chang, Ching Ten, and Graham A. Garcia, "Crosstalk Between Two Coplanar Waveguides," *Archiv Für Elektronik und Übertragungstechnik*, Band 43, Heft 1, 1989, pp. 55–58.
- [3] Ghione, Giovanni, and Carlo U. Naldi, "Coplanar Waveguides for MMIC Applications: Effect of Upper Shielding Conductor Backing, Finite-Extent Ground Planes, and Line-to-Line Coupling," *IEEE Transactions on Microwave Theory and Techniques*, Vol. MTT-35, No. 3, March 1987, pp. 260–267.
- [4] Wen, Cheng P., "Coplanar-Waveguide Directional Couplers," *IEEE Transactions on Microwave Theory and Techniques*, Vol. MTT-18, No. 6, June 1970, pp. 318–322.

#### 4.4.3 Edge-Coupled CPWG

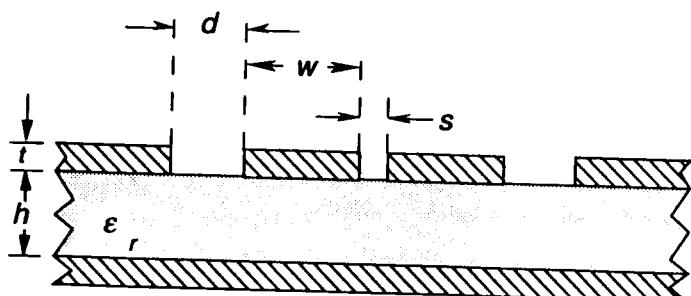


Figure 4.4.3.1: Edge-Coupled CPWG

$$Z_{0,o} = \frac{\eta_0}{\sqrt{\epsilon_{eff,o}}} \left[ \frac{1.0}{2.0 \frac{K(k_o)}{K'(k_o)} + \frac{K(\beta_1)}{K'(\beta_1)}} \right] \quad (\Omega) \quad (4.4.3.1)$$

$$Z_{0,e} = \frac{\eta_0}{\sqrt{\epsilon_{eff,e}}} \left[ \frac{1.0}{2.0 \frac{K(k_e)}{K'(k_e)} + \frac{K(\beta_1 k_1)}{K'(\beta_1 k_1)}} \right] \quad (\Omega) \quad (4.4.3.2)$$

$$\epsilon_{eff,o} = \frac{2.0 \epsilon_r \frac{K(k_o)}{K'(k_o)} + \frac{K(\beta_1)}{K'(\beta_1)}}{2.0 \frac{K(k_o)}{K'(k_o)} + \frac{K(\beta_1)}{K'(\beta_1)}} \quad (4.4.3.3)$$

$$\epsilon_{eff,e} = \frac{2.0 \epsilon_r \frac{K(k_e)}{K'(k_e)} + \frac{K(\beta_1 k_1)}{K'(\beta_1 k_1)}}{2.0 \frac{K(k_e)}{K'(k_e)} + \frac{K(\beta_1 k_1)}{K'(\beta_1 k_1)}} \quad (4.4.3.4)$$

where

$$k_o = \Lambda \frac{-\sqrt{\Lambda^2 - t_c^2} + \sqrt{\Lambda^2 - t_B^2}}{t_B \sqrt{\Lambda^2 - t_c^2} + t_c \sqrt{\Lambda^2 - t_B^2}} \quad (4.4.3.5)$$

$$k_e = \Lambda' \frac{-\sqrt{\Lambda'^2 - t'_c^2} + \sqrt{\Lambda'^2 - t'_B^2}}{t'_B \sqrt{\Lambda'^2 - t'_c^2} + t'_c \sqrt{\Lambda'^2 - t'_B^2}} \quad (4.4.3.6)$$

$$\Lambda = \sinh^2 \left[ \frac{\pi (s / 2.0 + w + d)}{2.0 h} \right] \quad (4.4.3.7)$$

$$t_c = \sinh^2 \left[ \frac{\pi (s / 2.0 + w)}{2.0 h} \right] - \Lambda \quad (4.4.3.8)$$

$$t_B = \sinh^2 \left( \frac{\pi s}{4.0 h} \right) - \Lambda \quad (4.4.3.9)$$

$$\Lambda' = \frac{\cosh^2 \left[ \frac{\pi (s / 2.0 + w + d)}{2.0 h} \right]}{2.0} \quad (4.4.3.10)$$

$$t'_c = \sinh^2 \left[ \frac{\pi (s / 2.0 + w)}{2.0 h} \right] - \Lambda' + 1.0 \quad (4.4.3.11)$$

$$t'_B = \sinh^2 \left[ \frac{\pi s}{4.0 h} \right] - \Lambda' + 1.0 \quad (4.4.3.12)$$

To guarantee coplanar propagation,

$$s + 2.0 w + 2.0 d \leq h.$$

The analysis is a conformal mapping technique, so the equations are exact for low frequency calculation.

#### REFERENCES

- [1] Chang, Ching Ten, and Graham A. Garcia, "Crosstalk Between Two Coplanar Waveguides," *Archiv Für Elektronik und Übertragungstechnik*, Band 43, Heft 1, 1989, pp. 55–58. (Edge-coupled CPW's separated by ground plane.)
- [2] Hanna, Victor Fouad, "Parameters of Coplanar Directional Couplers with Lower Ground Plane," *15th European Microwave Conference Proceedings*, 1985, pp. 820–825. ([2] dropped primes in (8), corrected here.)

## 4.5 COUPLED MICROSTRIP LINES

### 4.5.1 Edge-Coupled Microstrip Lines

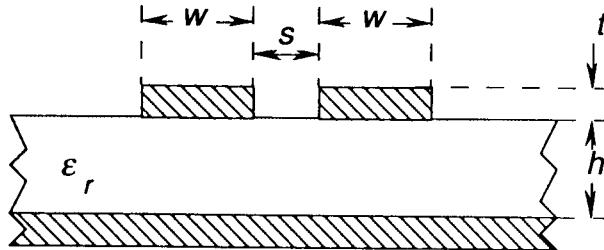


Figure 4.5.1.1: Coupled Microstrip Line

Kirschning and Jansen [8] analyzed this structure with a rigorous spectral-domain hybrid mode calculation. The results of this were then fit numerically. These equations are:

$$Z_{0,e}(0) = \frac{Z_0(0) \sqrt{\frac{\epsilon_{eff}(0)}{\epsilon_{eff,e}(0)}}}{1.0 - \frac{Z_0(0)}{\eta_0} \sqrt{\frac{\epsilon_{eff}(0)}{\epsilon_{eff,e}(0)}} Q_4} \quad (4.5.1.1)$$

$$Z_{0,o}(0) = \frac{Z_0(0) \sqrt{\frac{\epsilon_{eff}(0)}{\epsilon_{eff,o}(0)}}}{1.0 - \frac{Z_0(0)}{\eta_0} \sqrt{\frac{\epsilon_{eff}(0)}{\epsilon_{eff,o}(0)}} Q_{10}} \quad (4.5.1.2)$$

where  $Z_0(0)$  is the zero-frequency impedance of a single, isolated strip (3.5.1.1) and

$$Q_1 = 0.8695 \mu^{-0.194} \quad (4.5.1.3)$$

$$Q_2 = 1.0 + 0.7519 g + 0.189 g^{2.31} \quad (4.5.1.4)$$

$$Q_3 = 0.1975 + [16.6 + (8.4 / g)^{6.0}]^{-0.387} + \frac{\ln \left[ \frac{g^{10}}{1.0 + (g / 3.4)^{10}} \right]}{241} \quad (4.5.1.5)$$