Trade-offs for low power integrated pulse oximeters

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Abstract—**The paper presents detailed trade-offs encountered in the design of low-power integrated pulse oximeters(PO). The significance and interdependence of parameters such as the switching frequency, duty cycle, settling speed, stability and capacitor area and their role in system power and noise are discussed thoroughly. Quantitative relations between these parameters are derived and numerical examples with typical values given.**

Index Terms— **SpO_2, pulse oximetry, oximeters, low-power**

I. Introduction

The design of low-cost, portable, preferably disposable, multi-sensor monitoring systems for continuous or intermittent monitoring of a patient's vital signals is becoming a strategic goal in improving the level of provided healthcare. For such systems, an increased level of integration to improve usability and lower the cost as well as a reduced power consumption that will allow longer operation from a battery are key requirements. Blood oxygen saturation as measured noninvasively by pulse oximeters (SpO_2) is considered by the physicians as one of these vital signals.

Although several efforts have been made to improve the sensor used in pulse oximeters (PO), a limited number of researchers have attempted to lower their power consumption by improving the design of the sensor front-end circuitry. Asada et al. developed a ring sensor with notably low power consumption resorting to low power components and optimizing the clock frequency of the microprocessor used, by considering some of the power trade-offs at system level[1]. Tavakoli et al. designed a single chip pulse oximeter front-end which claims a remarkably low power consumption for the sensor employed, achieved by carefully designing the transimpedance amplifier of the front-end[2]. Finally, Aguillar-Pelaez et al. have attempted a mathematical description of the trade-offs involved in the design of a low-power pulse oximeter[3].

Both [2] and [3] discuss trade-offs encountered in the design of PO. However, in [2] resolution considerations as well as the effect of reduced duty cycle on the signal amplitude are disregarded, leading to underestimation of the significance of noise. Whereas [3] takes these facts into consideration, parameters such as system stability and noise of the integrated filters are not considered in detail. Moreover, the fundamental trade-off between LED and photodetector power is ignored.

This paper attempts to advance these efforts, by carrying out an analysis starting from application specifications and going down to circuit level. Its goal is to provide a more complete illustration of the design trade-offs that will be of use to engineers and researchers of the field. It also attempts to estimate some fundamental limits on the achievable performance based on physical constraints and typical parameter values.

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The paper is organized as follows: Section II reviews the form of the PO signal. Section III concentrates on sensitivity and resolution specifications for the system. Section IV discusses trade-offs in the design of the LED-photoreceptor pair of the front-end. Section V reports on noise considerations. Section VI concludes the paper.

II. The signal in pulse oximetry

Pulse oximetry measures the percentage of haemoglobin molecules in the blood that are bound to oxygen. In the most common setup this is achieved by shining light from a red and an infrared LED onto a highly perfused skin tissue area. A photodetector detects the light reflected from or transmitted through the tissue. The detected signal (usually a current) has a constant (dc) and a time-varying (ac) component. Typically the ac component is 0.25%-1% of the dc component in amplitude[2]. Furthermore, as it is related to heart pulsation, it contains frequencies in the band of 0.5-5 Hz approximately.

In the most widely used approach SpO_2 is estimated by measuring the ratio Λ of the ratios of the ac to the dc components for red and infrared light:

$$\Lambda = \frac{(i_{ac} / I_{DC})_R}{(i_{ac} / I_{DC})_{IR}} = \frac{c_R}{c_{IR}} \tag{1}$$

where c will from now on be termed the signal *contrast*.
The relation between SpO_2 and Λ is then established by experimental calibration[4]. Due to the light attenuation by the tissue, the LEDs are required to have a high peak current drive, usually 1-50 mA[4]. Thus, to save power the LED drive is commonly modulated with a square wave with a low duty cycle D. This ranges from 10-20% in older PO[4] down to around 1% in some modern ones. Unfortunately, modulation also proportionally decreases the amplitude of the signal. To see that, the detected signal can be formulated as a tone $c\alpha I_{LED}\sin(2\pi f_o t)$, approximating the narrowband pulsatile signal, added to a dc level of αI_{LED}, where I_{LED} is the peak LED current, α is a factor representing the attenuation of the incident light by the tissue, and c is the contrast of the signal.This is modulated by a pulse train with a (switching) frequency f_{sw} and a duty cycle D, with Fourier series representation:

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$$m(t) = D + 2\sum_{n=1}^{\infty} \frac{\sin(n\pi D)}{n\pi} \cos(2\pi n f_{sw} t) \quad (2)$$

The resulting signal at the photodetector is then:

$$I_{ph}(t) = DaI_{LED}\{1 + c\sin(2\pi f_o t) + 2\sum_{n=1}^{\infty} \mathrm{sinc}(n\pi D)\cos(2\pi n f_{sw} t) + \quad (3)$$

$$+ c\sum_{n=1}^{\infty} \mathrm{sinc}(n\pi D)(\sin(2\pi(f_o - nf_{sw})t) + \sin(2\pi(f_o + nf_{sw})t)\}$$

There are two points to note in (3). Firstly, the ac component of the signal at f_o has an amplitude 1/c times smaller than the dc and the (unwanted) components at the harmonics of f_{sw}. Secondly, its amplitude is reduced by the duty cycle D. This was to be expected, since modulating the LED drive reduces respectively the signal energy. Both of these facts introduce limitations in the design of the PO system.

III. Resolution

As will be shown later, one of the main limitations in PO design is noise. Therefore, estimations of the smallest signal (threshold or sensitivity) as well as of the minimum change in the signal (resolution) that need to be detectable are imperative.

The contrast value of the measured signal changes between subjects and with SpO_2. A typical value for the smallest observable (amplitude) value is 0.25%[2][1].

Estimating the resolution requires knowledge of the SpO_2-Λ relationship, which is usually determined ad-hoc experimentally. However, by using a simplified model of the tissue and the Beer-Lambert law it is possible to derive a relation that approximates the calibration curves used in oximeters as[4]:

$$SpO_2 = 100 * \frac{0.81 - 0.18\Lambda}{0.73 + 0.11\Lambda} \ [\%] \quad (4)$$

Using (4) we can evaluate the relative variation in Λ from a variation in the measured SpO_2 value:

$$\frac{\Delta\Lambda}{\Lambda} = \frac{d\Lambda}{dSpO_2}\frac{1}{\Lambda}\Delta SpO_2 \quad (5)$$

Commercial POs are usually specified to have a standard deviation of 2 digits (2%) in a range of saturations 70-100%. The maximum $d\Lambda/dSpO_2$ in this range corresponds to SpO_2=70% and is 3.3384. Considering Λ=1.1634 (4) and ΔSpO_2=2%, the acceptable relative deviation in Λ is calculated to be ~5.7%. Recall that Λ is determined by individually measuring the detected contrast for two different wavelengths. Due to noise, each of these values can have an error Δc:

$$\Lambda + \Delta\Lambda = \frac{c_R + \Delta c_R}{c_{IR} + \Delta c_{IR}} \quad (6)$$

Assuming that the Δc deviations are due to random noise and considering the worst case, the relative deviation in Λ is:

$$\frac{\Delta\Lambda}{\Lambda} = \sqrt{(\frac{\Delta c_R}{c_R})^2 + (\frac{\Delta c_{IR}}{c_{IR}})^2} \quad (7)$$

Assuming finally that the noise is almost equal in the measurement of both signals we obtain:

$$\frac{\Delta c}{c} \leq 4\% \quad (8)$$

In practice, the ac and dc components are usually measured separately. However, since the ac component is much smaller than the dc one, the relative error in its measurement is the dominant one. Thus, it will be adequate to require a measurement error smaller than 4% for the ac component.

This resolution requirement can be used to acquire an estimate of the required SNR at the detector[3]:

$$SNR_{ac} = \frac{i_{ac}^2}{\Delta i_{ac}^2} \sim \frac{c^2}{\Delta c^2} \quad (9)$$

which using (8) can be calculated to be 625 or 28dB.

IV. Photoreceptor and Duty Cycle

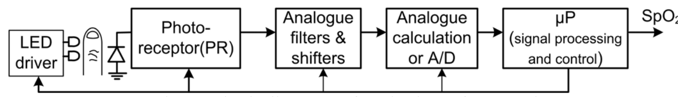

Fig. 1 Block diagram of typical PO front-end

A conventional PO front-end resembles that of Fig. 1. The detected signal is the current generated by a photodiode. This is subsequently converted to a voltage by a transimpedance amplifier (TIA) or as alternatively termed, a photoreceptor (PR). The signal is then conditioned by means of lowpass and/or bandpass filtering, as well as, level shifting circuitry to optimally fit in the range of the A/D converter. The system is usually controlled by a microprocessor.

In this section we will mainly consider the LED and the PR stages, as these commonly dissipate most power[2, 4]. This power is:

$$P = DI_{LED}V_{LED} + I_{PR}V_{PR} \quad (10)$$

with V_{PR} and V_{LED} the respective power supply voltages and I_{PR} the bias current of the photoreceptor.

To estimate I_{PR} we need to consider the PR in more detail. In almost all PO implementations this consists of an impedance element around a high-gain voltage amplifier. The feedback element can either be a resistor (linear-TIA) or a diode (log-TIA). In our analysis we will consider the feedback element to have an incremental resistance R. In the log-TIA case this just needs to be substituted for the diode's incremental resistance $1/g_m$.

This TIA topology has been thoroughly studied (e.g.[5]). It is effectively a two-pole closed-loop system. Its dominant pole is set by the large output capacitance of the photodiode C_d (typical values 100pF-5nF[2, 4]), together with the small-signal input impedance of the TIA. This creates a dominant open loop pole at $f_{dOL}=1/2\pi RC_d$. Due to the shunt-shunt feedback this pole is sped-up by the open-loop gain A of the amplifier, when the loop is closed. Thus the dominant closed-loop pole is: $f_{dCL}=A/2\pi RC_d$.

The non-dominant pole of the open-loop gain transfer function is the dominant pole of the amplifier:

$$f_{ndOL} = \frac{GBW_{amp}}{A} = \frac{g_{m_i}}{2\pi AC_o} \quad (11)$$

with g_{mi} the transconductance of the input stage of the amplifier and C_o the load or the compensation capacitance.

[1] Note that [2] discusses transmittance pulse oximetry with a finger tip sensor. Values may be different in other setups and should be substituted accordingly in the calculations.

The amplifier needs to have a large gain to speed-up the dominant pole and allow for fast settling of the pulsed photocurrent. However, as expected, excessively increasing the loop gain brings the two poles together and leads to instability or ringing that increases the settling time. To avoid ringing, the closed-loop system will be designed to have real poles[2]. For reasonable gain values (A>10) and separation of f_{dOL} and f_{ndOL} this requires f_{ndOL} to be at least 4 times larger than the crossover frequency of the open-loop transfer function, which equals f_{dCL}:

$$f_{ndOL} > 4 f_{dCL} \Rightarrow g_{mi} > \frac{4A^2 C_o}{RC_d} \tag{12}$$

Assuming that the input stage of the amplifier operates in strong inversion to achieve a large g_m and the ideal case where most of I_{PR} is used at the PR's input stage (12) becomes:

$$I_{PR} > 2(V_{GS} - V_{TH})\frac{A^2 C_o}{RC_d} \tag{13}$$

Eq. (13) describes the basic speed-power trade-off inherent in the TIA: beyond a certain point, to ensure the presence of real poles in the system, the PR power needs to increase quadratically with the required input pole speed-up A.

The input pole speed-up needs to be such that the (pulsed) PR output can settle within certain accuracy before further processing is carried out. Considering a dominant pole approximation for the closed-loop system, the settling time is[6]:

$$T_{settle} = RC_d \frac{\ln(1/\varepsilon)}{A} \tag{14}$$

with ε the required settling accuracy. Assuming that the measured photocurrent when a LED is off is much smaller than the current level αI_{LED} when a LED is on, the step size in the photocurrent will approximately be αI_{LED}. As shown earlier, the ac component needs to be measured with an error smaller than 4%. For a smallest physiologically relevant contrast of 0.25% this requirement translates to 0.04*0.0025=0.01% of the step size and thus $\ln(1/\varepsilon)=9.21$. Remembering that $D=t_{ON}f_{sw}$ and $t_{ON}>T_{settle}$ the minimum required amplifier gain is:

$$A > \frac{RC_d \ln(1/\varepsilon)}{D} f_{sw} \tag{15}$$

Combining (10), (13) and (15) we can obtain a relationship that expresses a lower limit for the power consumption as a function of duty cycle, R and I_{LED}:

$$P = V_{LED} D I_{LED} + 2[\ln(1/\varepsilon)]^2 (V_{GS} - V_{TH}) V_{PR} f_{sw}^2 \frac{RC_o C_d}{D^2} \tag{16}$$

Eq. (16) shows that the duty cycle of the system must be carefully selected to minimize the power of the combination of the LED and the PR. A too large D eases the PR design but leads to excessive power consumption in the LEDs. Conversely, too small a D would lead to the PR consuming more power than the LEDs to provide the necessary speed while remaining stable.

An optimum value of D that minimizes the power consumption can easily be found:

$$(D_P)_{opt} = \sqrt[3]{\frac{2(\ln\varepsilon)^2 (V_{GS} - V_{TH}) V_{PR} f_{sw}^2 RC_o C_d}{V_{LED} I_{LED}}} \tag{17}$$

[2] A small amount of ringing could lead to a somewhat faster system. However, clarity in the trade-off presentation would be sacrificed.

Using typical values found in Table I, V_{GS}-V_{TH} = 0.2 and $V_{LED} = V_{PR}$, D_{Popt}=0.15%. However, power minimization is not the only constraint that affects the choice of D. A too small value of D may lead to inadequate SNR at the detector. This will be discussed in the next section.

V. SIGNAL CONDITIONING AND NOISE

In all PO designs the output signal of the PR needs to be properly conditioned to ensure that it either optimally fits the range of the A/D converter which follows (most cases) or obtains a proper form for the analogue calculation block([2]). We will mainly concentrate on the ac part of the signal, as this sets the stringiest requirements on the design. The conclusions can be easily extended to the dc one.

The SNR at the PR input is calculated to be (recall (3)):

$$SNR = \frac{D^2 c^2 a^2 I_{LED}^2}{N_t BW_{sig}} \tag{18}$$

with N_t the total input referred current noise spectral density and BW_{sig} the signal bandwidth. Only white noise sources will be considered in this work. Given the narrow signal bandwidth (<5Hz), the effect of flicker noise will be to make the design specifications somewhat tighter than the ones described here.

Noise is mainly contributed by three sources: the PR, the photodiode shot noise and the noise of the filters. Referred to the input of the PR these are (A^2/Hz):

$$N_{PR} = \frac{4kT}{R},\ N_{pd} = 2q(aI_{LED}D^2 + I_{dark}),\ N_f = \frac{\theta kT}{G^2 R^2 C_f} \tag{19}$$

The PR noise has been considered to be predominantly due to the feedback resistance, as is usually the case in frequencies as low as the signal bandwidth[5]. Note that in the subsequent analysis we will assume a linear-TIA for simplicity. Similar analysis can be applied in the case of a log-TIA, with the difference that the value of R is then also constrained by I_{LED}. The photodiode noise contains both photon shot noise and noise due to dark current I_{dark}. The noise of the filters is considered to be proportional to kT/C_f, where C_f the total filter capacitance. G is the passband gain of the filter and θ a constant of proportionality that depends on the filter specifications and implementation[7].

It is clear from (19) that to minimize noise the feedback resistor should be as large as possible. However, the maximum resistor value is limited by the required settling time, the available amplifier gain A (see (15)), the power supply ($R<V_{DD}/\alpha I_{LED}$) and the integration area. Commonly selected values range between 100kΩ and a few MΩ. In what follows, considering a typical measured photocurrent of αI_{LED}=1μA and V_{DD}=3V, we will assume R=1MΩ.

To estimate θ we will use some of the results of [7], for Gm-C filters. Other integrated filter types are expected to give results of the same order of magnitude. In [7] an upper bound for the SNR at the output of an integrated filter with transfer function H(f) is shown to be:

$$SNR \le \frac{\pi C_f V_{\max}^2}{4kT\xi} \frac{G^2}{\int_0^\infty f\left|H'(f)\right|^2 df} = \frac{C_f}{\theta kT}\frac{V_{\max}^2}{2} = \frac{V_{\max}^2}{2G^2 R^2 N_f} \tag{20}$$

where V_{max} is the maximum signal swing in the filter and ξ the transconductors' excess noise factor. We will consider ξ=1 (ideal case). Then a lower bound for θ can be calculated from the filter specifications according to eq. (54) in [7]:

$$\theta = \frac{2\xi}{\pi}\frac{\int_0^{\infty} f\left|H'(f)\right|^2 df}{G^2} > \frac{2}{\pi G^2}\left(\frac{f_1}{f_2-f_1}+\frac{f_3}{f_4-f_3}\right)(G-M_s)^2 \quad (21)$$

where f_1,f_4 and f_2,f_3 are the limits of the stopbands and passband respectively, and G and M_s the passband and stopband gains (Fig.2).

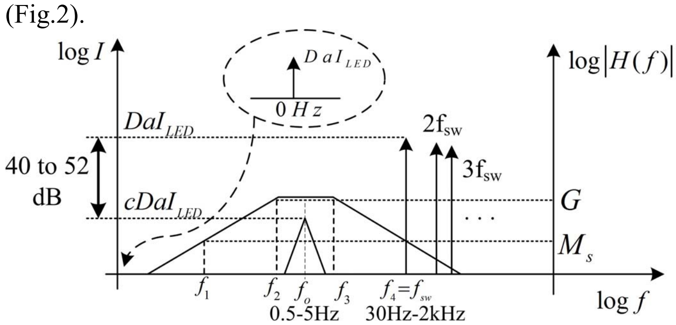

Fig. 2 Frequency spectra of PO signal and required filter. All axes have logarithmic scaling, but absolute (linear) values are annotated. f_o denotes the fundamental frequency of the measured signal and f_{sw} the switching frequency of the LEDs (see (3)).

The filter's passband is set by f_2=0.5Hz and f_3=5Hz. Given a desired SNR of around 30dB, the stopband attenuation should be at least 70-80 dB. The lower stopband edge is not critical as long as dc is rejected. We will assume f_l=10mHz, to allow a broad transition band. The edge of the upper stopband is set by the switching frequency f_{sw}. Previously selected values for f_{sw} range from 30Hz to 2kHz. In what follows we investigate trade-offs for low and high f_{sw}.

A low f_{sw} of 100Hz (as in [2]) facilitates the PR design, as low duty cycles can be achieved with moderate settling times of a few hundreds of μs. However, the filter gain G then has to be low. To see why, consider that some transfer function from the filter's input to an internal node will have a minimum 1st order roll-off. Given the 5Hz cut-off frequency, if a G higher than 20 is used, the switching components (up to 52 dB larger than the signal of interest – Fig. 2) could be amplified internally leading to potentially excessive distortion. As an example G=10 is selected. Then using (21) and the typical values of Table I, θ is calculated to be 0.0465. Assuming a maximum integrated capacitor value of C_f=100pF, the noise densities of (19) become:

$$N_{PR}=1.658{*}10^{-26},\ N_f=1.9{*}10^{-26}$$

$$N_{pd}=3.2{*}10^{-25}D^2+3.2{*}10^{-19}I_{dark} \quad [\mathrm{A}^2/Hz]$$

As can be seen, the noise of the PR and the filter are much larger than that of the photodiode for realistic values of D and I_{dark}. It should be stressed here that N_f as given by (20) is the lowest possible. In practice it will be at least a few times larger. Then, from (18)-(20), the minimum D, as dictated by noise, is:

$$(D_N)_{\min} > \sqrt{SNR\left(\frac{\theta kT}{G^2R^2C_f}+\frac{4kT}{R}\right)\frac{BW_{sig}}{c^2a^2I_{LED}^2}} \quad (22)$$

For the values in Table I, $D_{N\min}$=0.42%. Observe that this value is higher than the one calculated from power optimization concerns (see (17)).

To lower this value further, the noise needs to be reduced. As the maximum resistor value is limited by the power supply, only the filter noise can be lowered. This can be done either by resorting to larger, off-chip capacitances (as in [2]), or by increasing the filter gain. As discussed, the latter would require increasing f_{sw}, to avoid excessive distortion due to amplification of the switching components. When for example f_{sw}=1kHz, then a gain of as high as G=100 can be selected. It should not be overlooked however that increasing the switching frequency, requires a proportionally increased settling speed for the PR to achieve the same duty cycle, something that, as was seen in (16), is directly translated to higher power consumption at the PR, if stability is to be guaranteed. The optimal exchange of PR power for settling speed (or bandwidth) is a question open to further investigation (see for example [2],[5] and [8]).

VI. Conclusions

This work identified and discussed in detail the trade-offs present in the design of integrated POs, when designed for low power consumption. Useful general relationships were derived and used in indicative calculations with realistic design values. The investigation presented here is not exhaustive. However, it shows that the realisation of a low power integrated PO poses an interesting challenge, and provides some guidelines to tackle it.

TABLE I: PARAMETER VALUES IN A PO SYSTEM

	Typical Range	Value used in calculations
α	10^{-4}-10^{-3}	10^{-3}
c	0.25-1%	0.25%
D	1-25%	1%
R	100kΩ-5MΩ	1MΩ
C_d	100pF-5nF	1nF
C_o	10fF-1pF	10fF
f_{sw}	30Hz-2kHz	100 Hz
I_{LED}	1-50mA	1mA
I_{dark}	<1nA	1nA
SNR	-	28 dB

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