

# MEASURED ERROR TERMS FOR THE THREE-ANTENNA GAIN-MEASUREMENT TECHNIQUE

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## Abstract

**This paper will detail the implementation and results of a gain calculation performed on standard gain horns (SGHs) in the LS and XN microwave bands. The three-antenna method was used to ensure the highest accuracy possible, and extensive efforts were made to minimize the error budget. The measurement was performed in a large anechoic chamber, with the receive and transmit antennas placed 4.6 meters high in opposing corners. The resulting fifteen meters of aperture separation (approximately  $10D^2/\lambda$  for LS band and  $15D^2/\lambda$  for XN band) eliminated all measurable aperture interactions and greatly reduced multipath interference from chamber reflections. Rigorous analysis of the error terms proved this method to be both accurate and reliable. Typical values of measured error terms will be presented.**

Keywords: Antenna Gain, Mismatch, Multipath Proximity Effect

## 1. Introduction

The well-established three-antenna method was recently employed to perform accurate gain calibration measurements for standard gain horns (SGH) at MI-Technologies, LLC. The measurements were taken in the far-field region, utilizing a temperature controlled anechoic chamber, at a distance of greater than fifteen meters.

This particular setup was chosen in lieu of the extrapolation method developed by Newell et al. [1] and currently used by the National Institute of Standards and Technology (NIST) in Boulder, Colorado [2] and by the National Physical Laboratory (NPL) in Teddington, Middlesex, UK [3] due to availability of a large anechoic

chamber. A non-linear regression method (detailed later) was used to reduce errors caused by reflections.

The largest contributing error terms that are introduced when performing the gain measurements in the far-field region are the proximity effects and impedance mismatch.

## 2. Measurement Setup

The receive and transmit antennas are placed on 4.6 meter towers in a 9.2 meter high chamber. The towers are located in opposite corners of the chamber, which is 15 meters wide by 15 meters deep. The entire chamber is covered with absorber material providing maximum protection from scattering.

The source and power meter are placed on the respective transmit and receive towers allowing for the smallest possible length of low-loss cable for the RF path. Both of the SGHs were securely fastened to stable mounting brackets that allowed for translation along the antenna axis and rotation both parallel and perpendicular to the horizon. Slots in the mounting brackets allowed for polarization adjustment, where a precision level was used to align the antenna edge with gravity.

The gain calibration included the waveguide adapter with the breakpoint at the coaxial termination. This allowed for ease of substitution once the setup was established. Horizontal polarization of the SGHs was chosen in order to orient the E-field of the antenna pattern in the same plane as the diagonal dimension of the chamber. Finally, a self-leveling laser was used to accurately align the transmit and receive antennas mechanically.

The first antenna set for each band was mounted, leveled and electrically boresighted. The azimuth and elevation axes were then locked so that the second and third antenna sets could be substituted and leveled without interfering with the known alignment.

### 3. Measurement Procedure

The measurement procedure follows closely that which was described by Hollis et al. [4].

A source and power meter are used to measure transmitted power of the system both before and after the antenna measurements are performed. This is done by connecting the transmit and receive cables together at the point where they normally attach to the antennas (the coaxial port of the waveguide adapter).

The antennas are measured in the following sequence:

<u>Tx</u>	<u>Rx</u>
1 and 2	
1 and 3	
2 and 3	

Once the antennas are mounted, mechanically and electrically aligned, first one and then both antennas are translated along the antenna axis to quantify scattering and reflections. Each position is carefully recorded in conjunction with the respective absolute power reading for each frequency of interest. The second and third antenna sets are then mounted and, in a single position, absolute power readings are recorded for each frequency.

VSWR measurements are made using a network analyzer and a small anechoic chamber for each SGH waveguide adapter input port and on the generator and load. These values are recorded and later used for calculating mismatch correction factors and error terms.

### 4. Error Analysis

Several uncertainties are associated with each antenna measurement. How these uncertainties combine to yield an overall uncertainty for a particular antenna gain can be approximated using a Taylor Series. First, let us take the case of a single error:

$$f(x + \Delta x) \approx f(x) + f'(x) \cdot \Delta x + \dots + \frac{f''(x) \cdot \Delta x^2}{2!} + \frac{f'''(x) \cdot \Delta x^3}{3!} + \dots + \frac{f^{(n)}(x) \cdot \Delta x^n}{n!} + R_n$$

If  $\Delta x$  is small, the first few terms are usually adequate to approximate the series:

$$f(x + \Delta x) \approx f(x) + f'(x) \cdot \Delta x.$$

Now let  $\Delta x$  represent an error in  $x$ . Rewriting the previous equation approximates the error this generates in  $f(x)$  as

$$f(x + \Delta x) - f(x) \approx f'(x) \cdot \Delta x.$$

Usually what is required is not the magnitude of the error but the percent error. Notice we can divide both sides of the equation by  $f(x)$  to yield

$$\frac{f(x + \Delta x) - f(x)}{f(x)} \approx \frac{f'(x) \cdot \Delta x}{f(x)},$$

which gives us the equation for the percent error in  $f(x)$

$$e_{f(x)} \approx \frac{f'(x) \cdot \Delta x}{f(x)}.$$

For a function of several variables (i.e. several uncertainties), say  $f(x,y)$ , the percent error is expressed as

$$e_{f(x,y)} \approx \frac{\partial f}{\partial x} \cdot \frac{\Delta x}{f(x,y)} + \frac{\partial f}{\partial y} \cdot \frac{\Delta y}{f(x,y)},$$

assuming  $x$  and  $y$  are independent.

This technique is then incorporated in the determination of how the various error terms

associated with the measurements contribute to the overall uncertainty of the calculated gain.

The three-antenna method, based on the Friis transmission formula, requires measuring three combinations of three antennas to provide three equations with three unknowns. The gain of antenna 1 can be expressed as

$$g_1 = \sqrt{\frac{p_{r1} \cdot p_{r2} \cdot l_{s3}}{p_t \cdot p_{r3} \cdot l_{s1} \cdot l_{s2}}}$$

where:  $g_1$  is the gain of antenna 1  
 $p_{r1}$  is the absolute power for 1 and 2  
 $p_{r2}$  is the absolute power for 1 and 3  
 $p_{r3}$  is the absolute power for 2 and 3  
 $l_{s1}$  is the combined losses for  $p_{r1}$   
 $l_{s2}$  is the combined losses for  $p_{r2}$   
 $l_{s3}$  is the combined losses for  $p_{r3}$

To determine the contribution of uncertainty from  $p_{r1}$  on the overall uncertainty of  $g_1$ , the partial derivative with respect to  $p_{r1}$  is taken.

$$\frac{\partial g_1}{\partial p_{r1}} = \frac{1}{\sqrt{\frac{p_{r1} \cdot p_{r3} \cdot p_t \cdot l_{s1} \cdot l_{s3}}{p_{r2} \cdot l_{s3}}}}$$

The contribution of uncertainty from  $p_{r1}$  can then be calculated by

$$e_{pr1} \approx \frac{1}{2 \cdot \sqrt{\frac{p_{r1} \cdot p_{r3} \cdot p_t \cdot l_{s1} \cdot l_{s3}}{p_{r2} \cdot l_{s3}}}} \cdot \Delta p_{r1},$$

which simplifies to

$$e_{pr1} \approx \frac{\Delta p_{r1}}{2 \cdot p_{r1}}.$$

Once the error contribution from each term has been determined, the Root Sum Square (RSS) is taken for an overall uncertainty of the gain measurement.

*Error Terms:* Details of most of the error terms considered in the error budget have been examined in the previously cited references as

well as many others. These specifics will not be repeated here. Error terms considered to contribute to the overall uncertainty of the gain measurement are listed in Table 1, while typical values of gain and uncertainty are listed in Tables 2 and 3.

*Reflections:* A non-linear regression technique is used in order to correct for scattered signals.

Amplitude data for the translated set of antennas was evaluated for periodic variation about the  $1/r^2$  space-loss curve. If no variation is observed, amplitude data from any of the positions may be used for the gain calculation. If, however, a periodic pattern is observed, it is evaluated using a Gauss-Newton method for non-linear regression [5]. In general, this method is based on determining coefficients to a proposed equation that minimize the sum of the squares of the residuals. The proposed equation is chosen on a trial and error basis and is of the form:

$$a_0 + a_1 \cdot \cos(a_2 \cdot x_i + a_3) + a_4 \cdot \cos(a_5 \cdot x_i + a_6),$$

where  $x_i$  is the translated position, and each cosine term represents a reflected signal. Since a third term is generally negligible, trial and error between one and two terms will yield the desired results.

The relationship between the nonlinear equation and the data can be expressed as

$$y_i = f(x_i; a_0, a_1, \dots, a_n) + e_i$$

Where  $y_i$  is the measured amplitude data,  $f(x_i; a_0, a_1, \dots, a_n)$  represents the proposed function and  $e_i$  is a random error to be used in the overall error calculation.

Initial guesses for coefficients  $a_1$ - $a_7$  are chosen and partial derivatives of the function are evaluated with respect to each coefficient, which produces a new set of coefficients. The change in the coefficients is added to the previous values and the process is repeated until an acceptable delta is achieved implying that the solution has converged.

If the solution does not converge, a new set of initial values must be chosen. Also, the fitted curve should be plotted against the data points to check for aliasing.

Once a solution is found, the DC term (or  $a_0$ ) is used for the gain calculation, and the error term ( $e_i$ ) is added to the error budget.

*Proximity Effect:* A worst-case proximity error was estimated by simulating the radiated pattern of the transmitting antenna across the aperture of the receiving antenna. The resulting tapered distribution was compared to the radiation pattern at infinity in order to determine a correction factor.

## 5. Results and Conclusions

Table 1. Summary of Error Terms

Uncertainty	MAX (dB)
Measurement Mismatch	0.04
Correction Error	
Noise Power of Power Sensor	0.00
Zero error of Power Sensor	0.00
Power Meter Linearity	0.04
Space Loss Measurement Error	0.01
Multipath Curve Fitting Random Error	0.04
Proximity Effect Correction Error	0.05

Table 2. Typical gain and error results for a LS-Band SGH

Frequency	Gain (dB)	RSS Error (dB)	MAX Error (dB)
f1	16.52	0.09	0.20
f2	16.54	0.09	0.20
f3	16.56	0.09	0.20

Table 3. Typical gain and error results for a XN-Band SGH

Frequency	Gain (dB)	RSS Error (dB)	MAX Error (dB)
f1	22.05	0.07	0.18
f2	22.07	0.07	0.18
f3	22.09	0.07	0.18

The two major uncertainties that correspond to performing these measurements in the far-field are caused by proximity effects and by multipath interference. We have quantified these effects providing correction factors and corresponding error terms that are included in the overall error budget.

## References

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