CHAPTER

14 High-Frequency Transistor

14.1. INTRODUCTION

It is assumed that at low frequency the transistor responds instantly to variations of the input voltage or current. Actually, it is not so, because the mechanism of the transport of charge carriers from emitter to collector is essentially one of diffusion. So, in order to know the transistor behaviour at high frequencies, it is necessary to have a detailed description of the diffusion mechanism. Such a study is complicated, and the resulting equations are suggestive of those encountered in connection with a lossy transmission line. The time delay in response is mainly contributed by the diffusion process. At low frequencies, this time delay is negligibly small in comparison to the periodic time T (or $1/f$) and may, therefore, be neglected. However, at high frequencies, this time delay becomes a significant part of the periodic time and cannot be neglected in the analysis of the amplifier. Taking into account the mechanism of diffusion and the consequent time delay, we arrive a high-frequency model for transistor. This model is known as high-frequency hybrid-π model. Before considering the high-frequency hybrid-π model we will consider high-frequency T-model of a common base transistor.

14.2. HIGH-FREQUENCY T MODEL

As a first reasonable approximation, the diffusion phenomenon can be taken into account by modifying the basic common-base T-model shown in Fig. 10.25 as follows :

The collector resistor r_c' is shunted by a capacitor C_c, and the emitter resistor r_e' is shunted by a capacitor C_e, as shown in Fig. 14.1. Also, the dependent current generator is made proportional to current i_1 in r'_e and not to the emitter current i_e. The low-frequency alpha is denoted by α_0. If an input current step is applied, then initially this current is bypassed by capacitor C_e and current i_1 remains zero. Hence the output current starts at zero and builds up slowly with time. Such a response is approximately due to diffusion process. A better approximation is to replace capacitor C_e and r'_e by a lumped transmission line consisting of resistance-capacitance sections but such an equivalent circuit is too complicated to be useful.

The physical significance of C_e is not difficult to realize. It represents the sum of the diffusion capacitance C_{D_e} and the transition capacitance C_{T_e} across the emitter

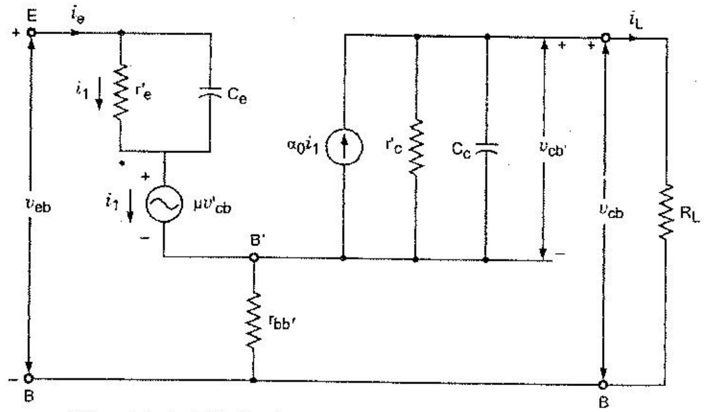


Fig. 14.1. *High-Frequency Transistor T Model*

junction *i.e.*, $C_e = C_{D_e} + C_{T_e}$. As $C_{De} >> C_{T_e}$, except for very small values of emitter current, therefore, C_e is roughly equal to diffusion capacitance C_{D_e} which is directly proportional to the quiescent emitter current. Since the collector junction is reverse biased, the collector diffusion capacitance C_{D_c} is negligible, so that C_c is equal to collector transition capacitance C_{T_c}. Usually, C_e is at least 30 times as large as C_c.

14.2.1. High-Frequency Alpha. Assuming the input excitation sinusoidal of frequency $f = \dfrac{\omega}{2\pi}$ and using capital letters for phasor currents, we have from Fig. 14.2.

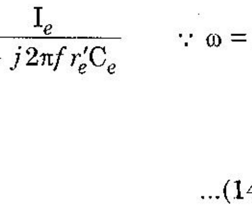


Fig. 14.2

$$I_1 r'_e = \frac{I_e}{\dfrac{1}{r'_e} + j\omega C_e}$$

$$\text{or } I_1 = \frac{I_e}{1 + j\omega r'_e C_e} = \frac{I_e}{1 + j2\pi f r'_e C_e} \qquad \because \omega = 2\pi f$$

$$\text{Let } 2\pi r'_e C_e = \frac{1}{f_\alpha}$$

$$\text{or } f_\alpha = \frac{1}{2\pi r'_e C_e} \qquad ...(14.1)$$

$$\text{So } I_1 = \frac{I_e}{1 + jf/f_\alpha} \qquad ...(14.2)$$

It is possible to consider the current generator to be proportional to the emitter current rather than the current through r'_e provided that the proportionality factor α to be a complex function of frequency. Thus if

$$\alpha_0 I_1 = \alpha I_e \qquad \text{...(14.3)}$$

Then from Eq. (14.2)

$$\alpha = \frac{\alpha_0}{1 + j\dfrac{f}{f_\alpha}} \qquad \text{...(14.4)}$$

The magnitude of the complex or high-frequency alpha α is α_0 at zero frequency and comes down to 0.707 α_0 at $f = f_\alpha$. This frequency f_α is called the *alpha cutoff frequency.* The diffusion equation leads to a solution for α equal to the hyperbolic secant of a complex quantity. By expanding this expression into a power series in the variable f/f_α and retaining only the first two terms we get Eq. (14.4). Thus Eq. (14.4) and the equivalent circuit shown in Fig. 14.1 are valid at frequencies which are appreciably less than f_α (up to perhaps $f_\alpha/2$). General purpose transistors have values of f_α in the range of hundreds of kHz. High-frequency transistors may have alpha cutoff frequencies in the tens, hundreds, or even thousands of MHz. Since $\alpha = -h_{fb}$, the symbol $f_{h_{fb}}$ is sometimes used for f_α.

14.2.2. Approximate CB T-model. If the load resistance R_L is small, the output voltage is v_{cb} and therefore, voltage $v_{cb'}$ will be small. Since $\mu \simeq 10^{-4}$, the Early generator $\mu v_{cb'}$ can be neglected. Under these conditions, the network shown in Fig. 14.1 reduces to that shown in Fig. 14.3, which is known as the approximate CB high-frequency T-model.

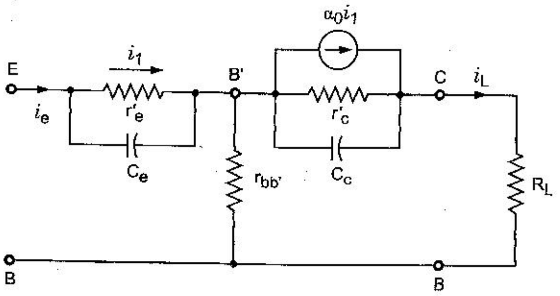


Fig. 14.3. *Approximate CB High-Frequency T-Model*

The order of magnitudes of the parameters in Fig. 14.3 are :

$$r'_e \simeq 20\ \Omega \quad r_{bb'} \simeq 100\ \Omega \quad r'_c \simeq 1\ \text{M}\Omega$$
$$C_c \simeq 1 - 50\ \text{pF and } C_e = 30 - 10{,}000\ \text{pF}.$$

14.3. COMMON-BASE SHORT-CIRCUIT-CURRENT FREQUENCY RESPONSE

Consider a transistor in the CB configuration excited by a sinusoidal current I_e of frequency f. If terminals C and B are connected together in Fig. 14.3, then $r_{bb'}$, r'_c and C_c are placed in parallel. Since $r'_c >> r_{bb'}$, r'_c may be neglected. Usually, $r_{bb'}\ C_c << r'_e C_e$ and under such conditions, the response is determined by the larger time constant $r'_e C_e$. Now C_c can also be omitted from Fig. 14.3. With these simplifications, $I_L = \alpha_0 I_1$ or from Eqs. (14.3) and (14.4), the common-base short-circuit current gain is given by

$$A_{ib} = \frac{I_L}{I_e} = \frac{\alpha_0 I_1}{I_e} = \alpha = \frac{\alpha_0}{1 + jf/f_\alpha} \qquad \text{...(14.5)}$$

The magnitude of α and its phase angle θ are given by

$$|\alpha| = \frac{\alpha_0}{\sqrt{1 + \left(\dfrac{f}{f_\alpha}\right)^2}} \qquad \text{...(14.6a)}$$

$$\theta = -\text{Tan}^{-1}\frac{f}{f_\alpha} \qquad \text{...(14.6b)}$$

If $f = f_\alpha$, $\alpha = \dfrac{\alpha_0}{\sqrt{2}}$, and $20 \log \left|\dfrac{\alpha}{\alpha_0}\right| = -20 \log \sqrt{2} = -3$ dB.

Hence the alpha cutoff frequency f_α is called the -3 dB frequency of the CB short-circuit current gain.

Equation (14.6) also predicts that α has undergone a 45° phase shift as compared to its low-frequency value. The calculated amplitude response is in close agreement with experiment, but the phase shift calculation may well be far off. The reason for such a discrepancy is because the lumped-circuit equivalent representation of the transistor is not accurate enough. It is found, empirically, that the discrepancy between the computed and practical values of θ can be reduced by introducing an "excess-phase" factor in the expression for α, so that Eq. (14.5) becomes

$$\alpha = \frac{\alpha_0}{1 + j\dfrac{f}{f_\alpha}} e^{-jmf/f_\alpha} \qquad \text{...(14.7)}$$

where m is adjustable parameter that ranges from 0.2 for a diffusion transistor to about unity for a drift transistor. **Diffusion transistors** are those in which the base doping is uniform, so that minority carriers cross the base entirely through diffusion. **Drift** transistors are those in which the doping is nonuniform, and an electric field exists in the base that causes a drift of minority carriers which adds to the diffusion current.

14.4. ALPHA CUTOFF FREQUENCY

For high-frequency applications f_α has to be very large. In order to make a transistor with a definite value of f_α, it is necessary to know all the parameters upon which f_α depends. Expression for the emitter capacitance is obtained first so that the desired equation for f_α can be had.

Diffusion Capacitance. Consider Fig. 14.4 representing the injected hole concentration vs. distance in the base region of P-N-P transistor. The base- width W is assumed to be small in comparison to diffusion length L_B of the minority carriers. Since the collector is reverse-biased the

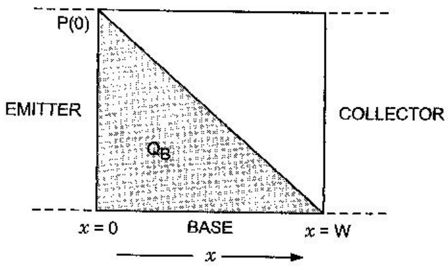


Fig. 14.4. *Minority-Carrier Charge Distribution in Base Region*

injected charge concentration P at the collector junction is essentially zero (Fig. 14.5).

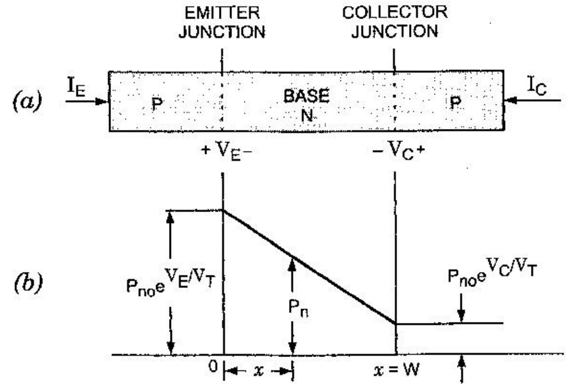


Fig. 14.5. ***Minority-Carrier Density in The Base Region***

If $W << L_B$, then P varies almost linearly from the value P(0) at the emitter to zero at the collector, as shown in Fig. 14.4. The stored base charge Q_B is the average concentration $\frac{P(0)}{2}$ times the volume of the base WA (where A is the base cross-sectional area) times the electronic charge e; that is,

$$Q_B = \frac{1}{2} P(0)AWe \qquad ...(14.8)$$

The diffusion current is given by

$$I = -AeD_B \frac{dP}{dx} = AeD_B \frac{P(0)}{W} \qquad ...(14.9)$$

where D_B is the diffusion constant for minority carriers in the base.

Combining Eqs. (14.8) and (14.9) we have

$$Q_B = \frac{IW^2}{2D_B} \qquad ...(14.10)$$

The emitter diffusion capacitance C_{D_e} is given by the rate of change of Q_B with respect to emitter voltage V *i.e.*,

$$C_{D_e} = \frac{dQ_B}{dV} = \frac{W^2}{2D_B} \cdot \frac{dI}{dV} = \frac{W^2}{2D_B} \cdot \frac{1}{r_e'} \qquad ...(14.11)$$

where $r_e' = \frac{dV}{dI}$, the emitter-junction incremental resistance.

We know that $r_e' = \frac{V_T}{I_E}$

where V_T is the volt-equivalent of temperature and is defined as

$$V_T = \frac{k'T}{e} = kT = \frac{T}{11{,}600}$$

where k' is Boltzmann constant in joules per K and k is Boltzmann constant in eV/K.

Substituting $r'_e = \frac{V_T}{I_E}$ in Eq. (14.11) we have

$$C_{D_e} = \frac{W^2}{2D_B} \cdot \frac{I_E}{V_T} \qquad ...(14.12)$$

This is the required expression for C_{D_e}.

The Eq. (14.12) indicates that the diffusion capacitance is proportional to the emitter bias current I_E. Since D_B is inversely proportional to T, and V_T is proportional to T, the diffusion capacitance C_{De} is almost independent of temperature.

Except for very small values of I_E, the diffusion capacitance C_{D_e} is much greater than the transition capacitance C_{T_e}. Hence

$$C_e = C_{D_e} + C_{T_e} \simeq C_{D_e} \qquad ...(14.13)$$

Dependence of f_α upon Base Width or Transit Time. From Eq. (14.1)

$$f_\alpha = \frac{1}{2\pi r_e' C_e} = \frac{1}{2\pi r_e' C_{D_e}} = \frac{D_B}{\pi W^2}$$

$$\because C_e = C_{D_e} \text{ and from Eq. (4.11) } C_{D_e} = \frac{W^2}{2D_B} \times \frac{1}{r_e'}$$

$$\text{or } \omega_\alpha = 2\pi f_\alpha = \frac{2D_B}{W^2} \qquad ...(14.14)$$

The above equation indicates that the alpha cutoff frequency varies inversely as the square of the base width W.

For a P-N-P germanium transistor with W = 1 mil = 2.54×10^{-3} cm = 25.4 microns, f_α becomes equal to 2.3 MHz. Combining Eqs. (14.10) and (14.14) we have

$$I = Q_B \omega_\alpha \qquad ...(14.15)$$

If t_B is the base-transit time, defined as the time taken by the charge carriers in seconds to cross the base region, then during base-transit time t_B an amount of charge reaching the collector is equal to base charge Q_B.

The resulting current is given by

$$I = \frac{Q_B}{t_B} \qquad ...(14.16)$$

From Eqs. (14.15) and (14.16) we have

$$\omega_\alpha = \frac{1}{t_B} \qquad ...(14.17)$$

The above Eq. (14.17) gives relation between the alpha cutoff (angular) frequency and transit time. It indicates that the alpha cutoff frequency is proportional to the reciprocal of the transit time t_B.

14.5. COMMON-EMITTER SHORT-CIRCUIT-CURRENT FREQUENCY RESPONSE

The T-model given in Fig. 14.3 is applicable to the CE configuration if E is grounded, the input signal is applied to base B and the load is connected between collector C and emitter E.

The CE short-circuit current gain A_{ie} is obtained by shorting the collector terminal C to emitter terminal E as shown in Fig. 14.6. When terminals C and E are short-circuited, the circuit is simplified to that shown in Fig. 14.7.

Since $r_c' >> r_e'$ and $C_e >> C_c$, the parallel elements r_c' and C_c may be neglected. The circuit is further simplified as shown in Fig. 14.8.

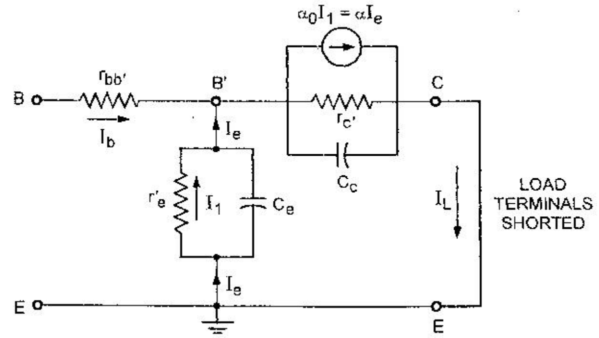


Fig. 14.6. ***The T-Circuit in The CE Configuration Under Short-Circuit Conditions***

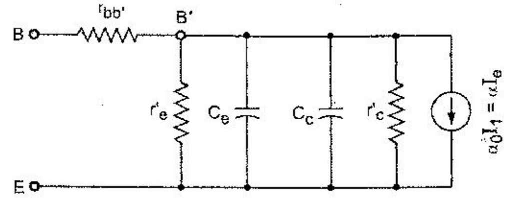


Fig. 14.7

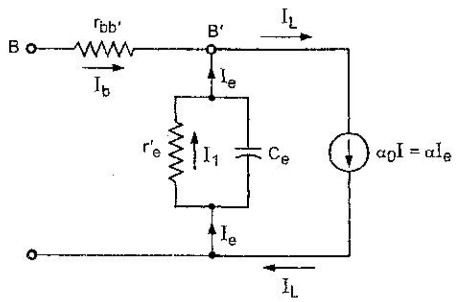


Fig. 14.8

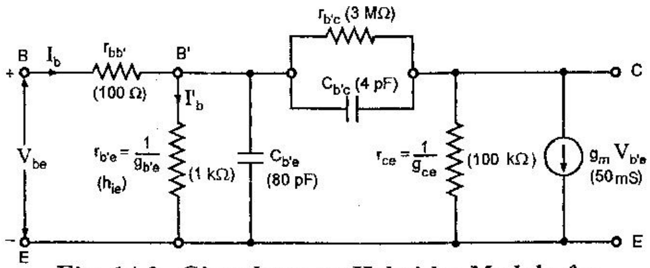


Fig. 14.9. ***Giacoletto or Hybrid-π Model of a Transistor in CE Configuration***

Applying Kirchhoff's current law to node B' we have

$$I_L = I_b + I_e$$

$$\text{or } I_e - I_L = -I_b$$

$$\text{or } I_e(1-\alpha) = -I_b \qquad \because I_L = \alpha_0 I_1 = \alpha I_e$$

$$\text{Finally } A_{ie} = \frac{I_L}{I_b} = \frac{\alpha I_e}{I_b} = \frac{\alpha I_e}{-(1-\alpha)I_e} = \frac{-\alpha}{(1-\alpha)}$$

Since the value of α depends on frequency ω, so it may be expressed as $\alpha(\omega)$

$$\therefore A_{ie} = \frac{-\alpha(\omega)}{1-\alpha(\omega)} \qquad \text{...(14.18)}$$

Using Eq. (14.4), A_{ie} may be put in the form

$$A_{ie} = \frac{-\beta_0}{1 + \dfrac{jf}{f_\beta}} \qquad \text{...(14.19)}$$

$$\text{where } \beta_0 = \frac{\alpha_0}{1-\alpha_0} \qquad \text{...(14.20)}$$

$$\text{and } f_\beta = f_\alpha(1-\alpha_0) \qquad \text{...(14.21)}$$

At zero frequency the CE short-circuit current amplification is $\beta_0 = h_{fe}$ and the corresponding CB parameter is $\alpha_0 = -h_{fb}$

The CE 3 dB frequency, or the beta cutoff frequency, is f_β which is also denoted as f_{hfe} or $f_{\alpha e}$.

From Eqs. (14.20) and (14.21) we have

$$\beta_0 f_\beta = h_{fe} f_\beta = \alpha_0 f_\alpha \qquad \text{...(14.22)}$$

Since α_0 is approximately equal to unity, the high-frequency response for CE configuration is much worse than that for the CB configuration. But the amplification for the CE configuration is much larger than that for the CB configuration. The *short-circuit-current gain-bandwidth product* (amplification times 3 dB frequency) is the same for both configurations.

14.6. HIGH-FREQUENCY HYBRID-pi (π) COMMON-EMITTER TRANSISTOR MODEL

Since common-emitter circuit is considered the most important practical configuration, we seek a CE model suitable for high frequencies. Hybrid-π or Giacoletto common emitter transistor model is given in Fig. 14.9. This circuit is quite simple and analyses of circuits using this model are not too difficult and give results which are in excellent agreement with experiment at all frequencies for which the transistor gives reasonable amplification. Furthermore, the resistive components in this circuit may be derived from the low frequency h-parameters. All parameters (resistances and capacitances) in the model are assumed frequency invariant. Parameters may vary with the quiescent operating point, but under given bias conditions they are reasonably constant for small signal variations. Typical values of the parameters are indicated in the Fig. 14.9. For high frequency analysis, the transistor is replaced by this high frequency hybrid π-model and voltage gain, current gain, input impedance etc. are determined.

Explanation of Parameters. The internal node B' is not physically accessible. The *ohmic base-spreading resistance* $r_{bb'}$, is represented as a lumped parameter between the external base terminal and B'. This resistance $r_{bb'}$, includes the base contact, base bulk and base spreading resistances. The first is due to the actual connection to the base, the second includes the resistance from the external terminal to the active region of the transistor, while the last is the actual resistance within the active base region.

The increase in minority carriers in the base results in increased recombination base current, and this effect is taken into account by inserting a resistance $r_{b'e}$ between internal node B' and E.

The excess-minority carrier storage in the base is accounted for by the diffusion capacitance $C_{b'e}$ connected between B' and E.

The Early effect indicates that the varying voltage across the collector to-emitter junction results in *base-width modulation*. A change in the effective base width makes the emitter and collector currents to vary because of change in the slope of the minority-carrier distribution in the base. The feedback effect between output and input is taken into account by connecting $r_{b'c}$ between B' and C.

Resistance r_{ce} is the resistance present between collector and emitter.

For small variations in the voltage $V_{b'e}$, across the emitter junction, the excess minority-carriers concentration injected into the base is proportional to $V_{b'e}$ and therefore the resulting small-signal collector current with the collector shorted to the emitter is proportional to $V_{b'e}$. This effect accounts for the current generator $g_m V_{b'e}$ in the circuit.

Lastly, the collector-junction barrier capacitance is included in the capacitance $C_{b'c}$. It is to be noted that $C_{b'c}$ is a transition capacitance while $C_{b'e}$ is a diffusion capacitance.

For the typical h-parameters given in Table 10.3, at $I_C = 1.3$ mA and room temperature, component values for a germanium transistor are

$g_m = 50$ mA/V ; $r_{b'e} = 1$ kΩ; $r_{bb'} = 100$ Ω, $r_{b'c} = 4$ MΩ; $r_{ce} = 80$ kΩ; $C_c = 3$ pF, $C_e = 100$ pF

14.7. HYBRID-pi CONDUCTANCES IN TERMS OF LOW-FREQUENCY *h*-PARAMETERS

All the resistive components in the hybrid-π model can be had from the *h*-parameters in the CE configuration. These *h*-parameters are discussed briefly now.

1. Transistor Conductance g_m. Figure 14.10 depicts P-N-P transistor in the CE configuration with the collector shorted to the emitter for time-varying signals. In the active region, the collector current given by expression $I_C = -\alpha I_E + I_{CO}$ is applicable here with $\alpha_N = \alpha_0$.

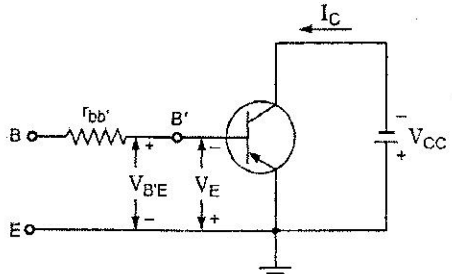


Fig. 14.10. ***P-N-P Transistor in CE Configuration For The Derivation of Transconductance (g_m)***

The transconductance g_m is given by

$$g_m = \left.\frac{\partial I_C}{\partial V_{B'E}}\right|_{V_{CE} = \text{constant}} = -\alpha_0 \frac{\partial I_E}{\partial V_{B'E}} = \alpha_0 \frac{\partial I_E}{\partial V_E} \quad ...(14.23)$$

In the above we have assumed that α_N is independent of V_E. For a P-N-P transistor

$V_E = -V_{B'E}$, as indicated in Fig. 14.10

If the emitter diode resistance is r'_e, then

$$r'_e = \frac{\partial V_E}{\partial I_E} \text{ and, therefore}$$

$$g_m = \frac{\alpha_0}{r'_e} = \frac{\alpha I_E}{V_T} \quad ...(14.24) \because r'_e = \frac{V_T}{I_E}$$

$$= \frac{I_{CO} - I_C}{V_T} \quad ...(14.25) \because \alpha I_E = I_{CO} - I_C$$

For a P-N-P transistor I_C is negative and for an N-P-N transistor I_C is positive. But for the foregoing analysis with $V_E = +V_{B'E}$ we have

$$g_m = \frac{I_C - I_{CO}}{V_T}$$

Hence for either type of transistor g_m is positive. Since $|I_C| >> |I_{CO}|$, g_m is given by

$$g_m \simeq \frac{|I_C|}{V_T} \quad ...(14.26)$$

$$\text{We know that } V_T = \frac{T}{11{,}600}$$

$$\text{So } g_m = \frac{11{,}600\,|I_C|}{T}$$

At room temperature of 27°C (300 K), the value of V_T is 26 mV, so the value of g_m is given by

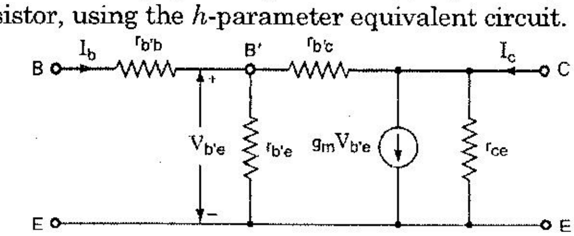
$$g_m = \frac{|I_C| \text{ in mA}}{26} \quad ...(14.27)$$

For $I_C = 1.3$ mA, $g_m = 0.05$ S (mho) = 50 mA/V

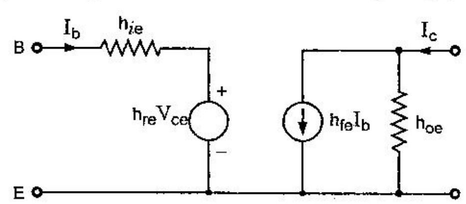
and for $I_C = 10$ mA, $g_m = 400$ mS or 400 mA/V

The above values are much larger than the transconductance obtained with tubes and FETs.

2. Input Conductance $g_{b'e}$. Figure 14.11 (*a*) depicts the hybrid-π model valid at low frequencies, where all capacitances are negligible. Figure 14.11 (*b*) depicts the same transistor, using the *h*-parameter equivalent circuit.



(a) The Hybrid π Model at Low Frequencies



(b) The h-Parameter Model at Low Frequencies

Fig. 14.11

From the component values given in Art. 14.6, we note that $r_{b'c} >> r_{b'e}$. Hence almost the complete base current I_b flows into $r_{b'e}$

Thus $V_{b'e} \approx I_b r_{b'e}$ ...(14.28) and the short-circuit collector current I_c is given by

$$I_c = g_m V_{b'e} \approx g_m I_b r_{b'e}$$

The short-circuit gain h_{fe} is defined as

$$h_{fe} = \left.\frac{I_c}{I_b}\right|_{V_{CE} = \text{Constant}} = g_m\, r_{b'e}$$

$$\text{or } r_{b'e} = \frac{h_{fe}}{g_m} = \frac{h_{fe} V_T}{|I_c|}$$

$$\text{or } g_{b'e} = \frac{g_m}{h_{fe}} \qquad \text{...(14.29)}$$

The above equation is required expression for the input conductance.

Thus, over the range of currents for which h_{fe} remains fairly constant, $r_{b'e}$ is directly proportional to temperature and inversely proportional to current.

Since $g_m = \dfrac{\alpha_0}{r'_e}$, and $h_{fe} \simeq \dfrac{\alpha_0}{1 - \alpha_0}$, $r_{b'e}$ may be expressed in terms of the T-model emitter resistance r'_e as

$$r_{b'e} = \frac{h_{fe}}{g_m} = \frac{r'_e}{1 - \alpha_0} \qquad \text{...(14.30)}$$

3. Feedback Conductance $g_{b'c}$. With the input open-circuited, h_{re} is defined as the reverse voltage gain, or from Fig. 14.11 (a) with $I_b = 0$,

$$h_{re} = \left.\frac{V_{b'e}}{V_{ce}}\right|_{\text{For } I_b = 0} = \frac{r_{b'e}}{r_{b'e} + r_{b'c}} \qquad \text{...(14.31)}$$

or $r_{b'e}(1 - h_{re}) = h_{re} \cdot r_{b'c}$

Since $h_{re} << 1$, then to a good approximation

$$r_{b'e} = h_{re}\, r_{b'c} \quad \text{or} \quad g_{b'c} = \frac{h_{re}}{r_{b'e}} \qquad \text{...(14.32)}$$

Since $h_{re} \simeq 1 \times 10^{-4}$, Eq. (14.32) verifies that $r_{b'c} >> r_{b'e}$

It is found that h_{re} is quite insensitive to current and temperature. Therefore $r_{b'c}$ has the same dependence on $|I_C|$ and T as does $r_{b'e}$.

4. Base-Spreading Resistance $r_{bb'}$. Parameter h_{ie} forms the input resistance of the transistor with the output shorted. With reference to the circuit given in Fig. 14.11(a), with output shorted, $r_{b'c}$ is in parallel with $r_{b'e}$. Hence

$$h_{ie} = r_{bb'} + r_{b'e} \,||\, r_{b'c} = r_{bb'} + r_{b'e} \qquad \text{...(14.33)}$$

$\because r_{b'c} >> r_{b'e}$ and therefore $r_{b'e} \,||\, r_{b'c} = r_{b'e}$

or Base-spreading resistance

$$r_{bb'} = h_{ie} - r_{b'e} = h_{ie} - \frac{h_{fe}}{g_m} \qquad \text{...(14.34)}$$

$\because$ From Eq. (14.30) $r_{b'e} = \dfrac{h_{fe}}{g_m}$

From Eqs. (14.29) and (14.33) it may be noted that the short-circuit input impedance h_{ie} varies with current and temperature in the following manner :

$$h_{ie} = r_{bb'} + \frac{h_{fe} V_T}{|I_C|} \qquad \text{...(14.35)}$$

5. Output Conductance g_{ce}. Parameter h_{oe} constitutes the output conductance with open-circuited input

$$i.e.,\ h_{oe} = \left.\frac{I_C}{V_{ce}}\right|_{\text{For } I_b = 0}$$

From the circuit shown in Fig. 14.11(a) with $I_b = 0$, we have

$$I_C = \frac{V_{ce}}{r_{ce}} + \frac{V_{ce}}{r_{b'c} + r_{b'e}} + g_m V_{b'e} \qquad \text{...(14.36)}$$

With $I_b = 0$, we have, from Eq. (14.31), $V_{b'e} = h_{re} V_{ce}$ and from Eq. (14.36), we have

$$h_{oe} = \frac{I_c}{V_{ce}} = \frac{1}{r_{ce}} + \frac{1}{r_{b'c}} + g_m h_{re} \qquad \text{...(14.37)}$$

Substituting the values of g_m and h_{re} from Eqs. (14.29) and (14.32) in Eq. (14.37) and assuming $r_{b'c} >> r_{b'e}$ we have

$$h_{oe} = g_{ce} + g_{b'c} + g_{b'e} h_{fe} \frac{g_{b'c}}{g_{b'e}} = g_{ce} + (1 + h_{fe}) g_{b'c} \qquad \text{...(14.38)}$$

but if $h_{fe} >> 1$, Eq. (14.38) may be written as

$$g_{ce} \simeq h_{oe} - g_m\, h_{re} \qquad \text{...(14.39)}$$

6. Feedback Conductance $g_{b'c}$:

$$g_{b'c} = h_{re}\, g_{b'e} \qquad \text{...(14.40)}$$

If the CE h-parameters at low frequencies are known at a given collector current I_c, the conductances or resistances in the hybrid-π model can be calculated from the following equations :

$$g_m = \frac{|I_C|}{V_T} \qquad \text{...(14.41)}$$

$$r_{b'e} = \frac{h_{fe}}{g_m} \qquad \text{...(14.42)}$$

$$r_{bb'} = h_{ie} - r_{b'e} \qquad \text{...(14.43)}$$

$$r_{b'c} = \frac{r_{b'e}}{h_{re}} \qquad \text{...(14.44)}$$

$$g_{ce} = h_{oe} - (1 + h_{fe})\, g_{b'c} = \frac{1}{r_{ce}} \qquad \text{...(14.45)}$$

For the typical h-parameters given in Table 10.3, at $I_C = 1.3$ mA and room temperature, the component values obtained are listed on page 316.

Hybrid-π Capacitances. The basic high-frequency hybrid π model, discussed in Art. 14.6, includes two capacitances (i) $C_{b'c}$ or C_c and (ii) C_e.

1. Collector-Junction Capacitance C_c. The collector-junction capacitance C_c or $C_{b'c}$ is the measured CB output capacitance with the input open ($I_E = 0$), and is specified by the manufacturers as C_{ob}. Since in active region the collector junction is reverse-biased, then C_c is a transition capacitance, and therefore varies as V_{CE}^{-n}, when $n = \dfrac{1}{2}$ or $\dfrac{1}{3}$ for an abrupt or gradual junction respectively.

Since $C_e = C_{b'e}$ represents, mainly the diffusion capacitance across the emitter junction, it is directly proportional to the current and is almost independent of temperature. Experimentally, C_e is determined from a measurement of frequency f_T at which the CE short-circuit current gain falls to unity. C_e is given approximately by the following relation

$$C_e = \frac{g_m}{2\pi f_T} \quad ...(14.46)$$

Reasonable values for $C_{b'c}$ and $C_{b'e}$ are 3 pF and 100 pF respectively.

Example 14.1. A BJT has h_{ie} = 6 kΩ and h_{fe} = 224 at I_C = 1 mA with f_T = 80 MHz and $C_{b'c}$ = 12 pF. Determine (i) g_m (ii) $r_{b'e}$ (iii) $r_{bb'}$ and (iv) $C_{b'e}$ at room temperature and a collector current of 1 mA. [*U.P. Technical Univ. Solid State Devices and Circuits 2006-07*]

Solution : (*i*) From Eq. (14.41);

$$g_m = \frac{I_C}{V_T} = \frac{1\text{ mA}}{26\text{ mV}} = \frac{1}{26}\text{S or } 38.46\text{ mS} \textbf{ Ans.}$$

(*ii*) From Eq. (14.42);

$$r_{b'e} = \frac{h_{fe}}{g_m} = \frac{224}{1/26} = 224 \times 26\ \Omega = 5.824\text{ k}\Omega \textbf{ Ans.}$$

(*iii*) From Eq. (14.43);

$$r_{b'b} = h_{ie} - r_{b'e} = 6 \times 10^3 - 5.824 \times 10^3$$
$$= 0.176\text{ k}\Omega \text{ or } 176\ \Omega. \textbf{ Ans.}$$

$$(iv)\ C_{b'e} = \frac{g_m}{2\pi f_T} - C_{b'c} = \frac{38.46 \times 10^{-3}}{2\pi \times 80 \times 10^6} - 12 \times 10^{-12}\text{ F}$$
$$= 76.5 \times 10^{-12} - 12 \times 10^{-12}\text{ F}$$
$$= 64.5\text{ pF. } \textbf{Ans.}$$

Example 14.2. The following low-frequency parameters are known for a given transistor at I_C = 10 mA and V_{CE} = 10 V and at room temperature h_{ie} = 500 Ω; h_{oe} = 10^{-5} A/V; h_{fe} = 100 and h_{re} = 10^{-4}.

At the same operating point f_T = 50 MHz and C_{ob} 3 pF. Calculate the values of all the hybrid parameters.

[*U.P. Technical Univ. Solid State Devices and Circuits 2004-05*]

Solution : From Eq. (14.41);

$$g_m = \frac{I_C}{V_T} = \frac{10\text{ mA}}{26\text{ mV}} = 384.6\text{ mS } \textbf{Ans.}$$

(*ii*) From Eq. (14.42);

$$r_{b'e} = \frac{h_{fe}}{g_m} = \frac{100}{384.6 \times 10^{-3}} = 260\ \Omega \textbf{ Ans.}$$

(*iii*) From Eq. (14.43);

$$r_{b'b} = h_{ie} - r_{b'e} = 500 - 260 = 240\ \Omega \textbf{ Ans.}$$

(*iv*) From Eq. (14.44);

$$r_{b'c} = \frac{r_{b'e}}{h_{re}} = \frac{260}{10^{-4}} = 2.6 \times 10^6\ \Omega \text{ or } 2.6\text{ M}\Omega \textbf{ Ans.}$$

(*v*) From Eq. (14.45);

$$g_{ce} = h_{oe} - (1 + h_{fe})g_{b'c}$$
$$= 10^{-5} - (1 + 100) \times \frac{1}{2.6 \times 10^6}$$
$$= 1 \times 10^{-5} - 0.2575 \times 10^{-5} = 0.7425 \times 10^{-5}\text{ S. } \textbf{Ans.}$$

$$C_{b'c} = C_{bo} = 3\text{ pF. } \textbf{Ans.}$$

$$C_{b'e} = \frac{g_m}{2\pi f_T} - C_{b'c} = \frac{384.6 \times 10^{-3}}{2\pi \times 50 \times 10^6} - 3 \times 10^{-12}$$
$$= 1221 \times 10^{-12}\text{ F or } 1{,}221\text{ pF. } \textbf{Ans.}$$

Example 14.3. The following transistor measurements made at I_C = 5 mA and V_{CE} = 10 V and at room temperature. h_{fe} = 100; h_{ie} = 600 Ω; $|A_i|$ = 10 at 10 MHz and C_c = 3 pF. Find f_B, f_T, C_e, $r_{b'e}$, $r_{bb'}$. [*U.P. Technical Univ. Solid State Devices and Circuits 2005-06*]

Solution : From Eq. (14.41);

$$g_m = \frac{I_C}{V_T} = \frac{5\text{ mA}}{26\text{ mV}} = \frac{5}{26}\text{S or } 192.3\text{ mS}$$

$$\text{From Eq. (14.42); } r_{b'e} = \frac{h_{fe}}{g_m} = \frac{100}{5/26} = 520\ \Omega \textbf{ Ans.}$$

$$\text{From Eq. (14.43); } r_{bb'} = h_{ie} - r_{b'e} = 600 - 520 = 80\ \Omega \textbf{ Ans.}$$

$$f_T = |A_i| \times f = 10 \times 10\text{ MHz} = 100\text{ MHz } \textbf{Ans.}$$

$$f_\beta = \frac{f_T}{\beta} = \frac{f_T}{h_{fe}} = \frac{100}{100}\text{ MHz} = 1\text{ MHz } \textbf{Ans.}$$

$$C_e = \frac{g_m}{2\pi f_T} - C_{b'c} = \frac{192.3 \times 10^{-3}}{2\pi \times 100 \times 10^6} - 3 \times 10^{-12}$$
$$\because C_{b'c} = C_c = 3 \times 10^{-12}\text{ F}$$
$$= 303 \times 10^{-12} \text{ for } 303\text{ pF. } \textbf{Ans.}$$

Example 14.4. A BJT is found to have f_T = 500 MHz, h_{fe} = 100, $r_{bb'}$ = 100 Ω, $r_{b'e}$ = 900 Ω and $C_{b'c}$ = 5 pF. It is used as a CE amplifier with R_s = 1 kΩ and R_L = 500 Ω. Determine for the amplifier (i) midband voltage gain $A_{vs} = \frac{v_0}{v_s}$ and (ii) the upper 3 dB cutoff frequency f_β.

$$\text{Solution : From Eq. (14.42) } g_m = \frac{h_{fe}}{r_{b'e}} = \frac{100}{900} = \frac{1}{9}\text{ S}$$

Midband voltage gain,

$$A_v = -\frac{V_{ce}}{V_{b'e}} = \frac{-g_m V_{b'e} R_L}{V_{b'e}} = -g_m R_L = \frac{1}{9} \times 500 = -55.55$$

Midband voltage gain taking R_S into account

$$(i)\ A_{vs} = \frac{A_v \times r_{b'e}}{R_s + r_{bb'} + r_{b'e}} = \frac{-55.55 \times 900}{1{,}000 + 100 + 900} = -25 \textbf{ Ans.}$$

$$(ii)\text{ Upper 3 dB cutoff frequency, } f_B = \frac{f_T}{h_{fe}} = \frac{500}{100} = 5\text{ MHz } \textbf{Ans.}$$

14.8. CE SHORT-CIRCUIT CURRENT GAIN OBTAINED WITH HYBRID-pi MODEL

In order to obtain some idea of the transistor's high-frequency capability and what transistor to choose for a given application, we examine the way in which a transistor's CE short-circuit forward-current transfer ratio or current gain varies with frequency.

Consider a single-stage CE transistor amplifier or the last stage of a cascade configuration. The load R_L on this stage is the collector-circuit resistor, so that $R_C = R_L$. Here we short the output terminals C and E so that $R_L = 0$ and we obtain short-circuit current gain. To obtain the frequency response of the transistor amplifier, we make use of hybrid-π model given in Fig. 14.9, which