PSoC Today Presents:

Chris Keeser, Synchronous Detection Theory

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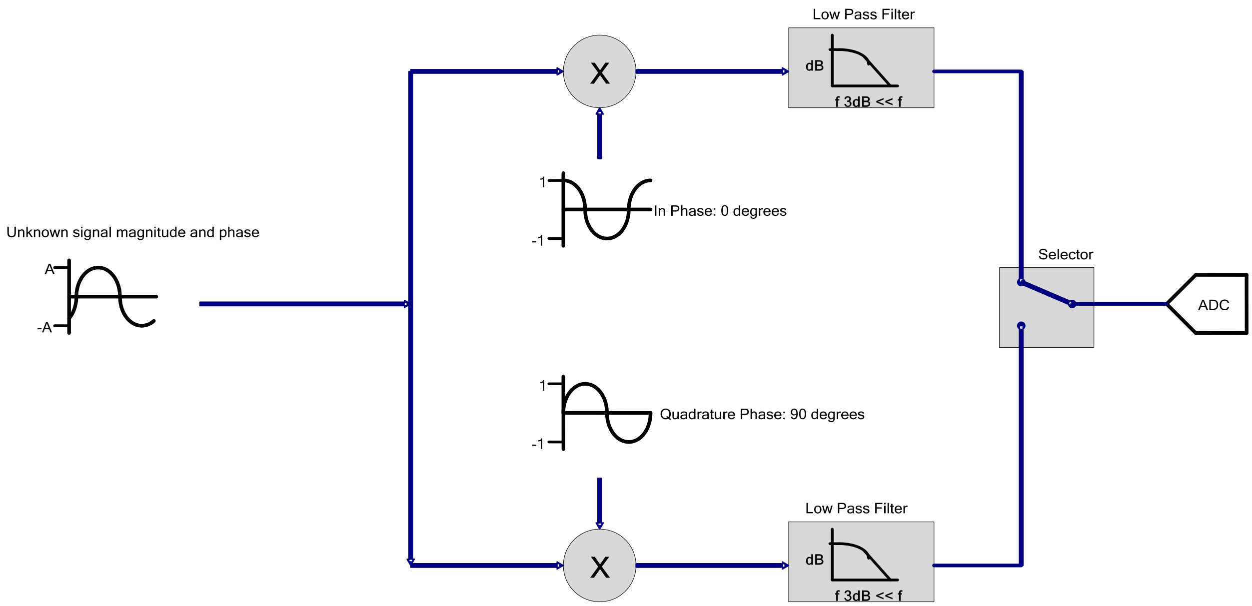
Note * There is also an MPG movie to assist in understanding the synchronous detection process. The movie is named: SynchronousDetection_good.mpg

Synchronous detection is commonly used in demodulating an amplitude modulated (AM) signal. It began to see widespread use in AM receivers because it boosted the performance of the receiver and allowed signal reception in conditions that would have been difficult otherwise. However, synchronous detection is not limited to demodulating AM signals; with the proper application, it can produce the magnitude and phase information of an input signal. The magnitude and phase can then be used in other applications which require that data from the input signal. The advantage of using synchronous detection for extracting magnitude and phase over using an FFT is significantly reduced CPU load, and with an averaging converter, very low sample rate requirements on the ADC.

Synchronous detection works by mixing an input signal with an in phase signal (represents 0 degree phase). The mixed signal is then low pass filtered and the resulting output represents the magnitude and phase relationship of the original signal with respect to the mixed in phase signal.

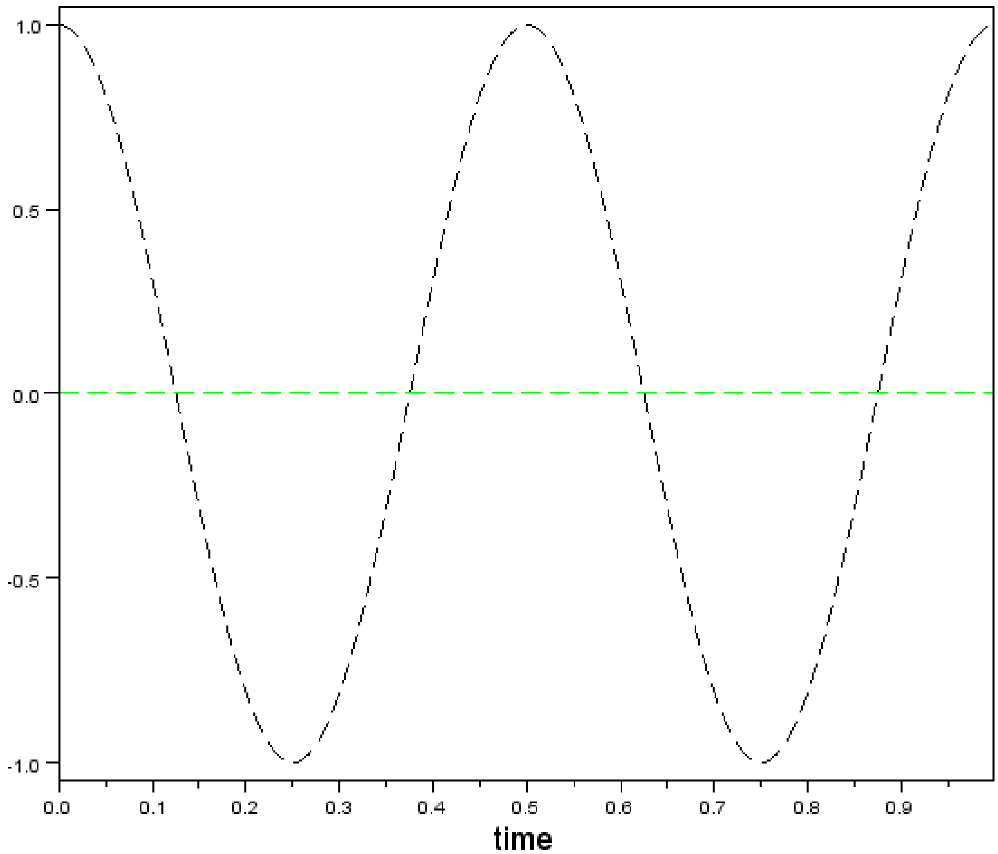
A second quadrature phase (represents 90 degree phase) signal is also generated and mixed with the input signal. This mixed signal is also low pass filtered and the resulting output represents the magnitude and phase relationship of the original signal with respect to the mixed quadrature phase signal.

These two results (in phase result and quadrature phase result) are combined to provide the magnitude of the original signal as well as its phase relationship to our 0 phase (in phase) signal.



Confused?

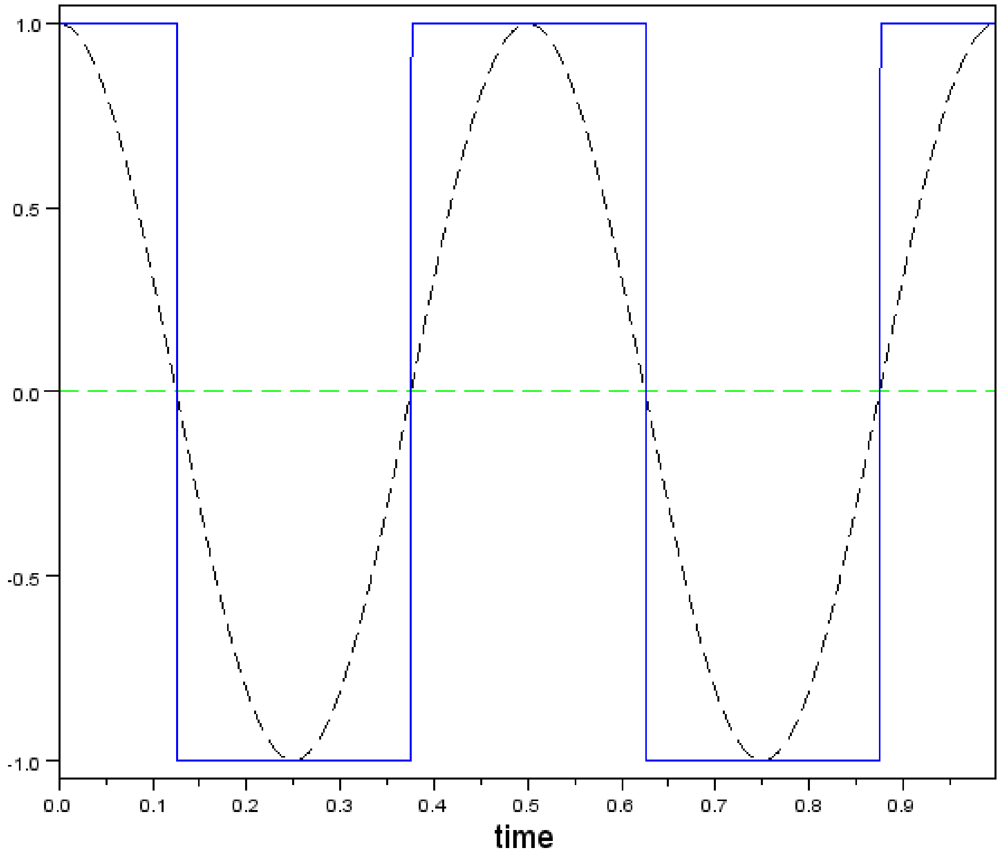
The best way to facilitate understanding of synchronous detection is to step through an illustrated example. We will begin with a cosine which we will define as $a\cos(\omega t-\theta)$ = $\cos(4\pi t-\theta)$ where a=1 and $\omega = 2\pi f = 2\pi * 2$. This will be our input signal, and is plotted below with ground represented as a dashed green line at 0:



We will construct our in phase signal by generating a square wave, at the same frequency as the input signal. When the input signal frequency is not known, this requires a PLL or other frequency detection circuitry, which is not covered in this application note. For the impedance meter, we are generating the drive signal so we know its frequency exactly.

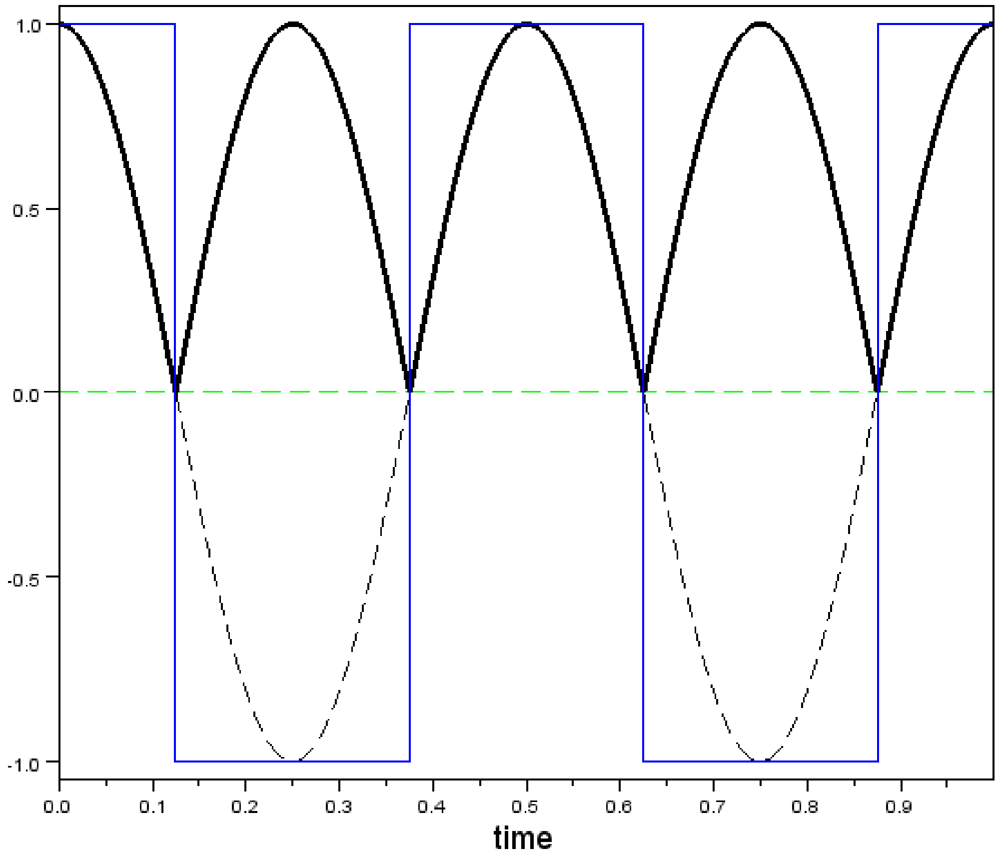
The phase of the in phase signal with respect to the input is not important, but this 'in phase' signal that we have created will be what we define as '0' degrees from here on out. The in phase signal becomes our reference phase for all other signals, and we will describe the phase of our input signal with respect to our in phase signal. For illustration purposes, I have chosen our in phase signal to correspond exactly with a cosine with 0 degrees of phase shift.

Here we display the in phase signal on top of the input cosine:



We mix this in phase signal with the input signal and get the following output from the mixer. Mixing is the same as multiplying the input by +1 or -1. Remember these multiplication rules:

+1 x +a = +a
+1 x -a = -a
-1 x +a = -a
-1 x -a = +a

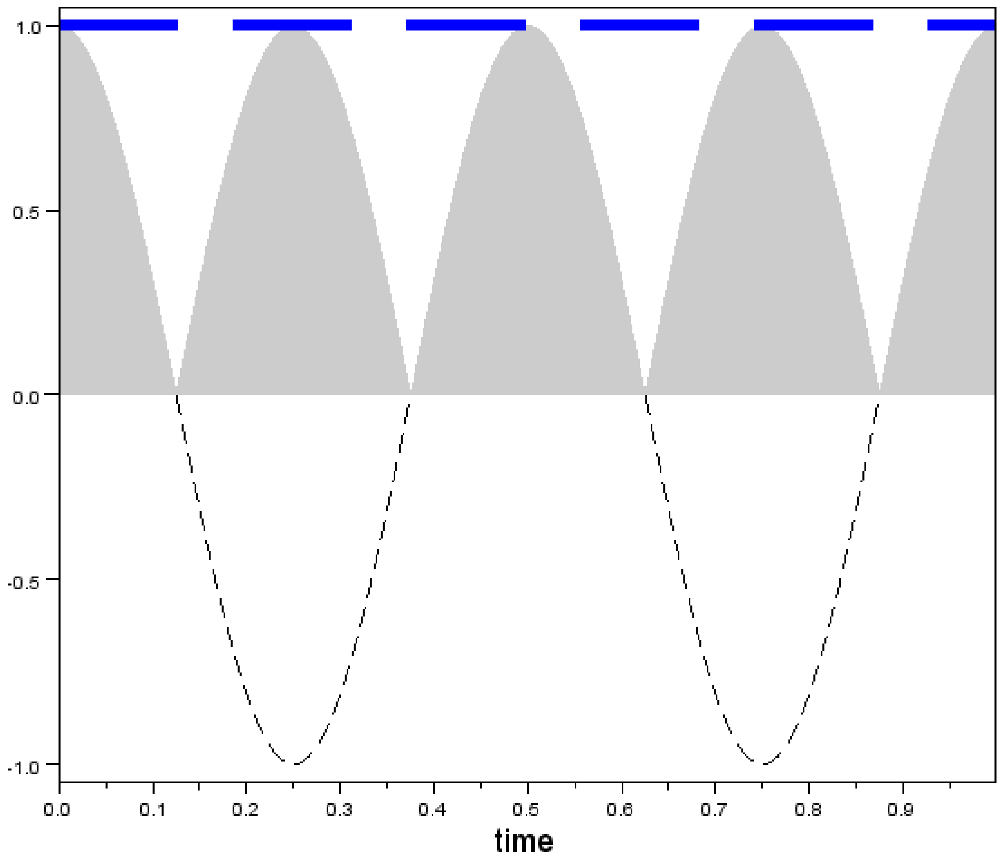


The dashed line represents the original signal, and the solid line represents the mixer output. In the case shown above, the solid line looks like a rectified version of the input signal.

The next step is to low pass filter the output. We will accomplish this in the example by taking the average of the mixed outputs. To recover something meaningful from the average, we multiply the average by $\pi/2$*. We will call this scaled average the 'in-phase output'.

**There is an explanation in the appendix for the reason the result of the average is multiplied by* $\pi/2$.

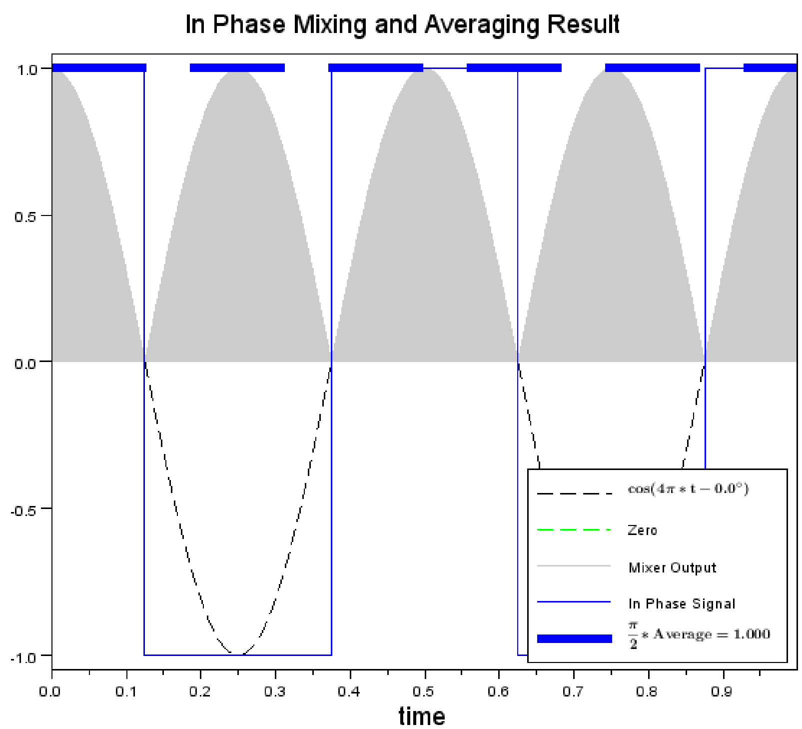
The figure below shows the original signal (thin dashed black line) the mixer output (grey signal with the area under the curve shaded to emphasize what will be averaged) and the result of the scaled average (thick dashed blue line):



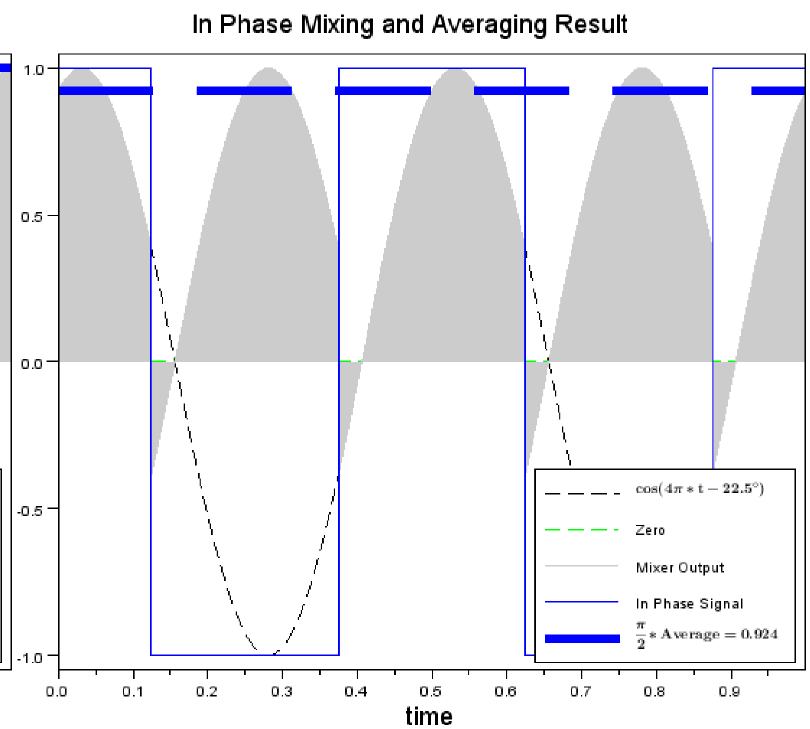
In this case, the output of the scaled average is 1.000. Not particularly impressive by itself. So how do we extract something meaningful from this?

The following graphics illustrate an input signal $\cos(4\pi t - \theta)$ being phase delayed from 0 to 180 degrees. The gray shaded area represents the area under the curve that is being averaged. The result of the averaging, scaled by $\pi/2$, is displayed by the thick blue dashed line.

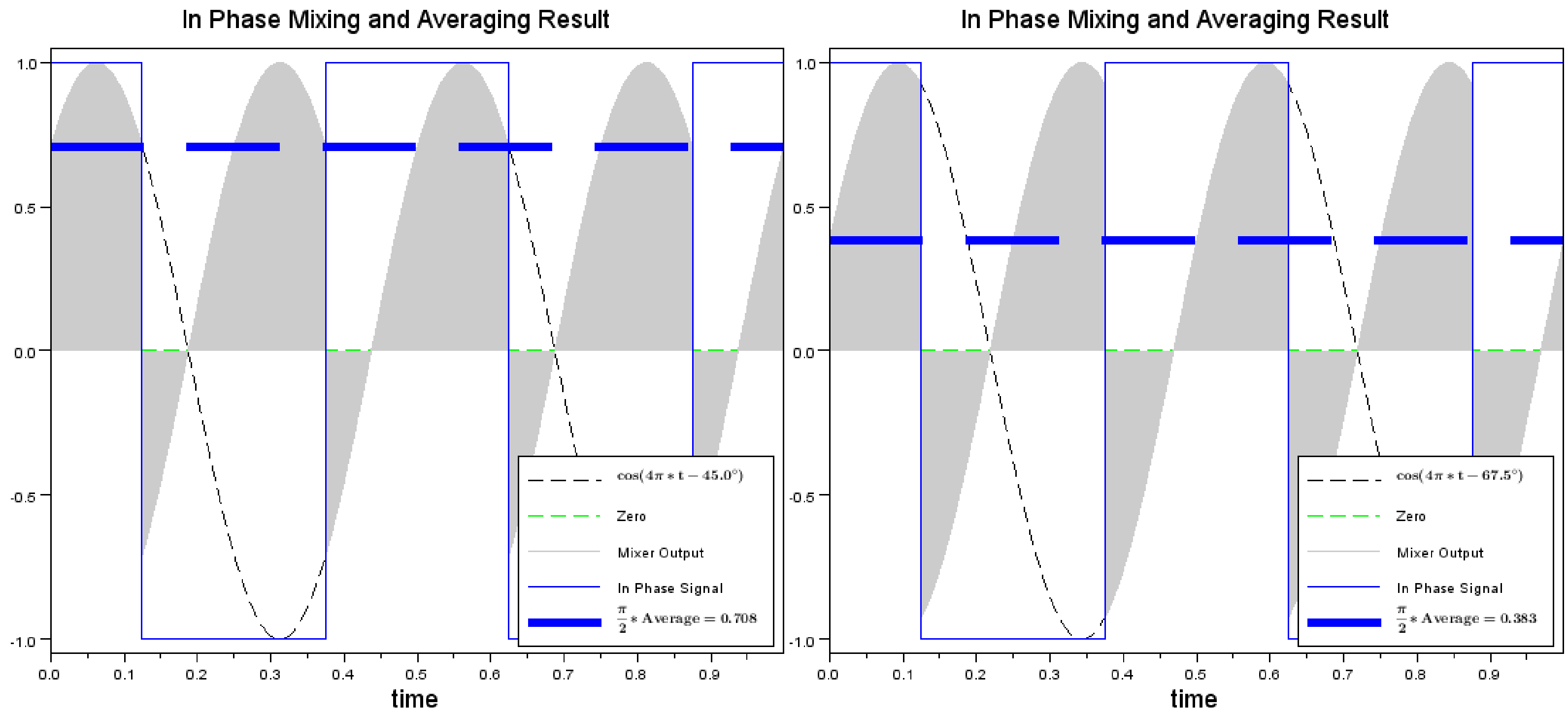
Observe the change in the averaged output as the input signals phase changes relative to the in phase signal:

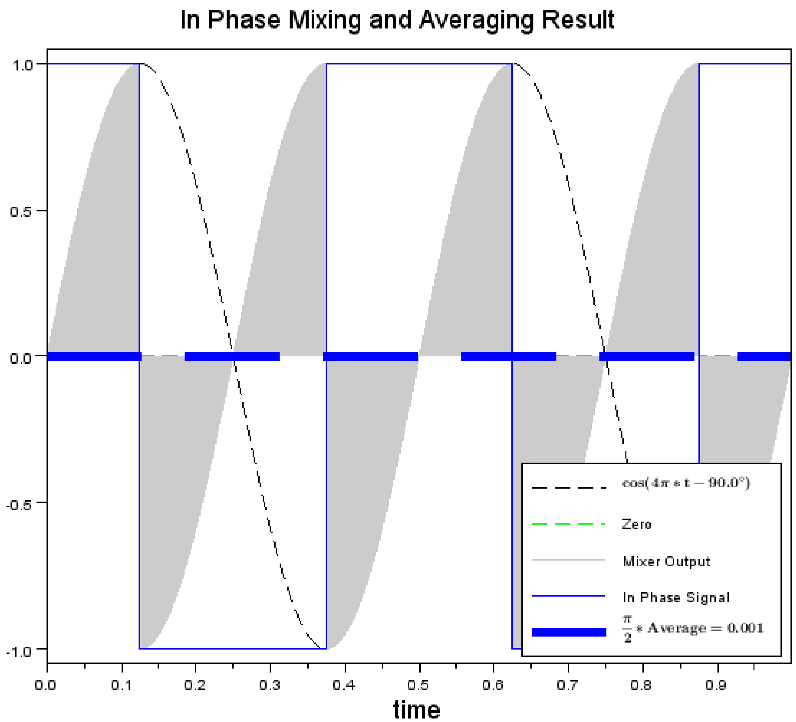


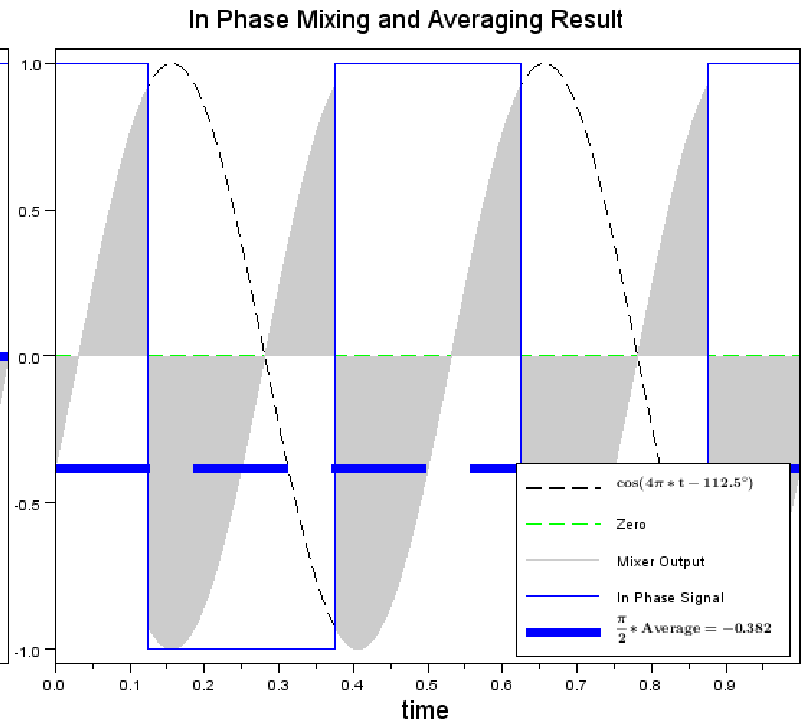
In Phase Mixing and Averaging Result
1.0
0.5
0.0
-0.5
-1.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
time
$\cos(4\pi * t - 0.0°)$
Zero
Mixer Output
In Phase Signal
$\frac{\pi}{2} * \text{Average} = 1.000$

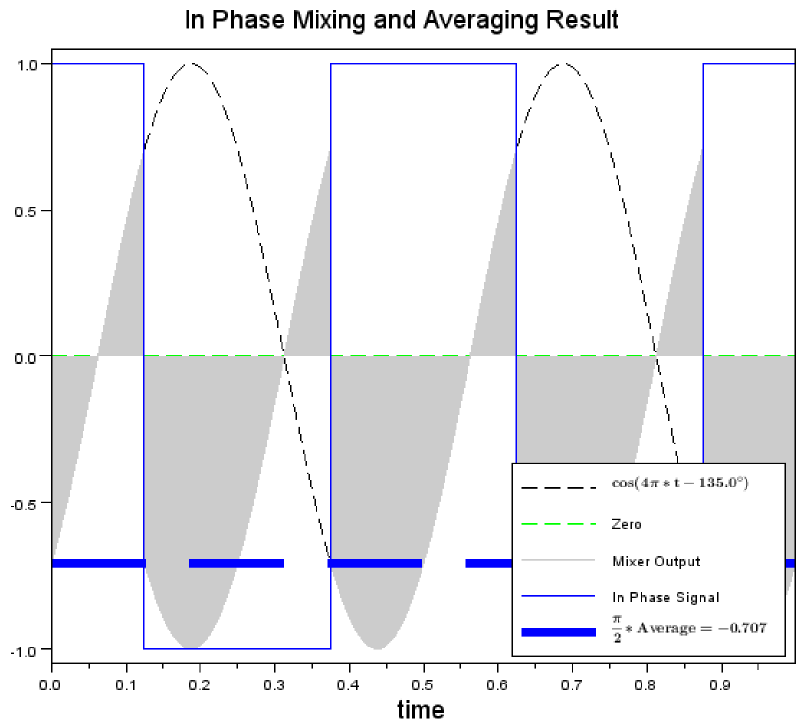


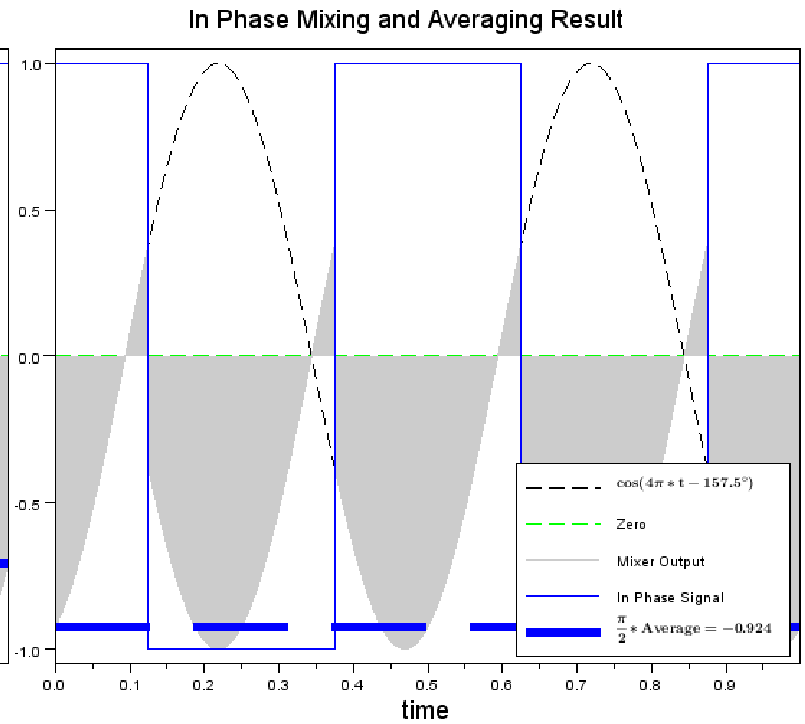
In Phase Mixing and Averaging Result
1.0
0.5
0.0
-0.5
-1.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
time
$\cos(4\pi * t - 22.5°)$
Zero
Mixer Output
In Phase Signal
$\frac{\pi}{2} * \text{Average} = 0.924$

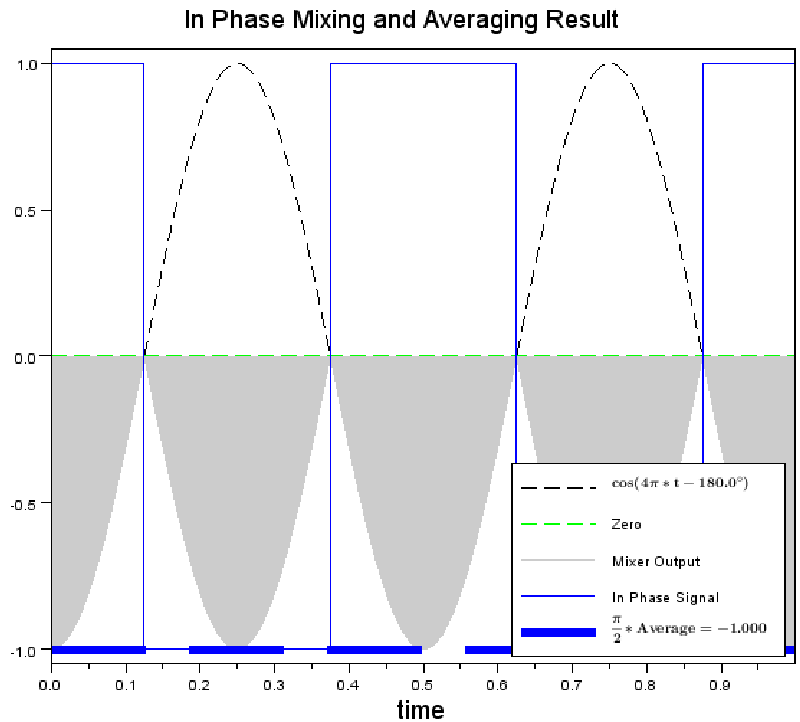

In Phase Mixing and Averaging Result
1.0
0.5
0.0
-0.5
-1.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
time
$\cos(4\pi * t - 45.0°)$
Zero
Mixer Output
In Phase Signal
$\frac{\pi}{2} * \text{Average} = 0.708$
In Phase Mixing and Averaging Result
$\cos(4\pi * t - 67.5°)$
Zero
Mixer Output
In Phase Signal
$\frac{\pi}{2} * \text{Average} = 0.383$


In Phase Mixing and Averaging Result
1.0
0.5
0.0
-0.5
-1.0
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
time
$\cos(4\pi * t - 90.0°)$
Zero
Mixer Output
In Phase Signal
$\frac{\pi}{2} * \text{Average} = 0.001$


In Phase Mixing and Averaging Result
1.0
0.5
0.0
-0.5
-1.0
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
time
$\cos(4\pi * t - 112.5°)$
Zero
Mixer Output
In Phase Signal
$\frac{\pi}{2} * \text{Average} = -0.382$


In Phase Mixing and Averaging Result
1.0
0.5
0.0
-0.5
-1.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
time
$\cos(4\pi * t - 135.0°)$
Zero
Mixer Output
In Phase Signal
$\frac{\pi}{2} * \text{Average} = -0.707$


In Phase Mixing and Averaging Result
1.0
0.5
0.0
-0.5
-1.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
time
$\cos(4\pi * t - 157.5°)$
Zero
Mixer Output
In Phase Signal
$\frac{\pi}{2} * \text{Average} = -0.924$



In Phase Mixing and Averaging Result
1.0
0.5
0.0
-0.5
-1.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
time
$\cos(4\pi * t - 180.0°)$
Zero
Mixer Output
In Phase Signal
$\frac{\pi}{2} * \text{Average} = -1.000$

A summary table of the scaled average output is shown below:

Signal Phase Delay(θ)*	In Phase Result	$\cos(\theta)$
0.0	1.000	1.000
22.5	0.924	0.924
45.0	0.708	0.707
67.5	0.383	0.383
90.0	0.001	0.000
112.5	-0.382	-0.383
135.0	-0.707	-0.707
157.5	-0.924	-0.924
180.0	-1.000	-1.000

**signal phase delay between our in phase signal and our input signal.*

At this point, you may be excited and think that we can extract the phase directly from the output of the in phase result. However, I have set up the conditions in such a way that makes the problem appear simpler than it is. If you ventured down to the appendix, you saw that the result of the scaled in phase average for an arbitrary input of $a\cos(\omega t-\theta)$ is $a\cos(\theta)$, with emphasis added to the 'a'. In the examples given above, 'a' is set '1', meaning the output of the scaled average is $\cos(\theta)$. In a real world situation, 'a' can be anything.

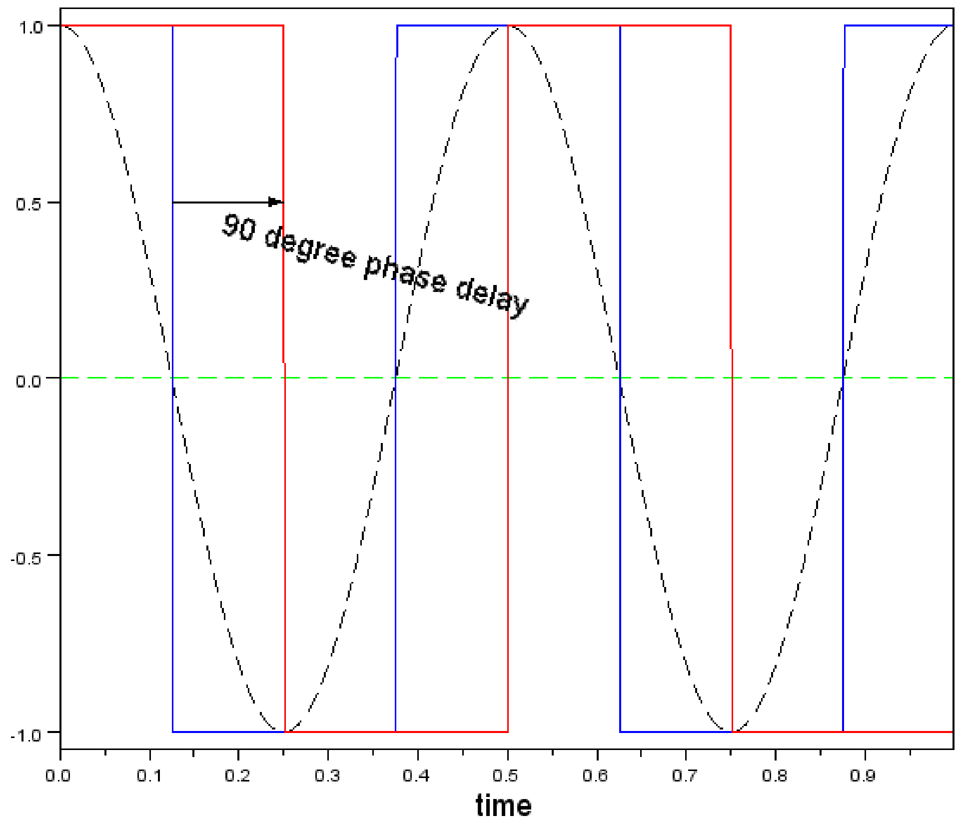
To illustrate this point, let's say that 'a' was 7. With a 60 degree phase shift on our input relative to our in phase signal, the output of the scaled average would be $7\cos(\theta)=7\cos(60^{\circ})=3.5$, which is obviously not something we can convert directly into a phase, *if* we only had the in phase scaled average to work with.

Basically, we have two unknowns ('a' and θ) and only one equation:

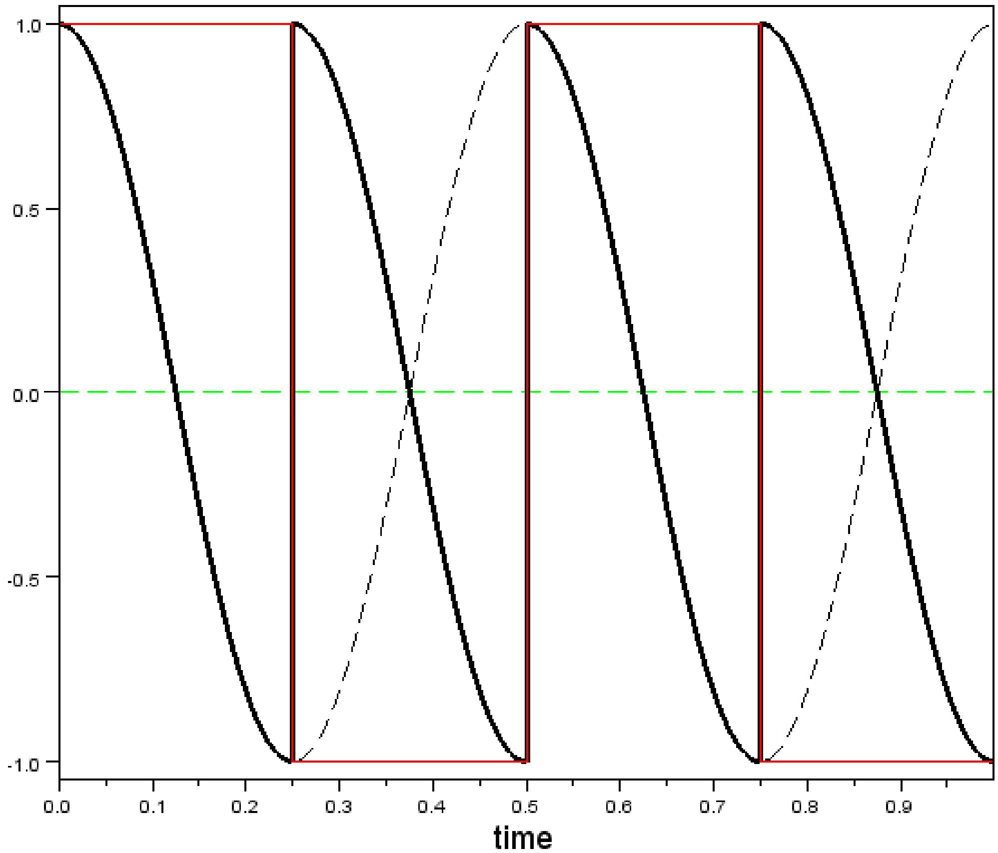
$a\cos(\theta)=$ In phase scaled average.

Previously, I had mentioned generating two signals, and in phase signal and a quadrature phase signal. Now we are going to generate our second signal, the quadrature phase signal. To solve for the 2 unknowns, we need two equations. We can create another equation by generating another signal, which we will call the quadrature phase signal. A quadrature phase signal is generated by delaying the in phase signal by 90 degrees.

In the figure below, the quadrature phase signal (red) is displayed on top of the cosine (dashed black line) and the in phase signal (blue) for reference:



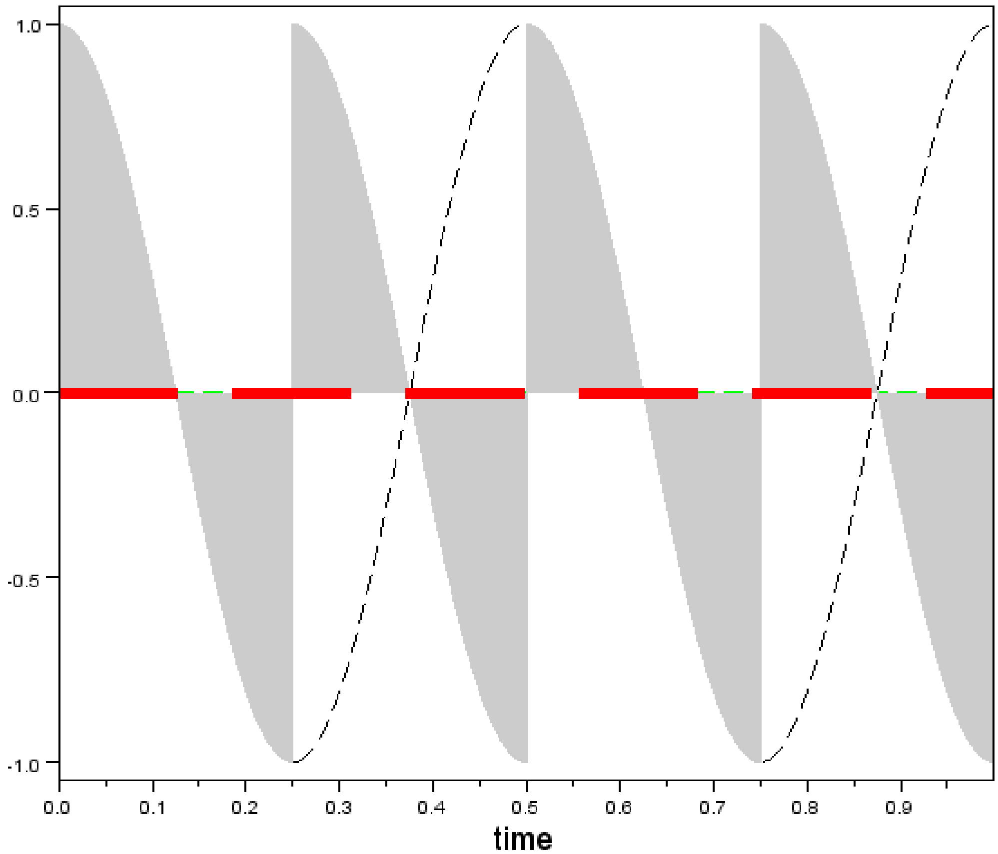
We will perform the same mixing (multiplying) with the new quadrature phase signal and the cosine.



The dashed line represents the original signal, and the solid line represents the mixer output.

The next step is to low pass filter the output. We will accomplish this in the example by taking the average of the mixed outputs. Remember, To recover something meaningful from the average we multiply the average by $\pi/2$. We will call this scaled average the 'quadrature phase output'.

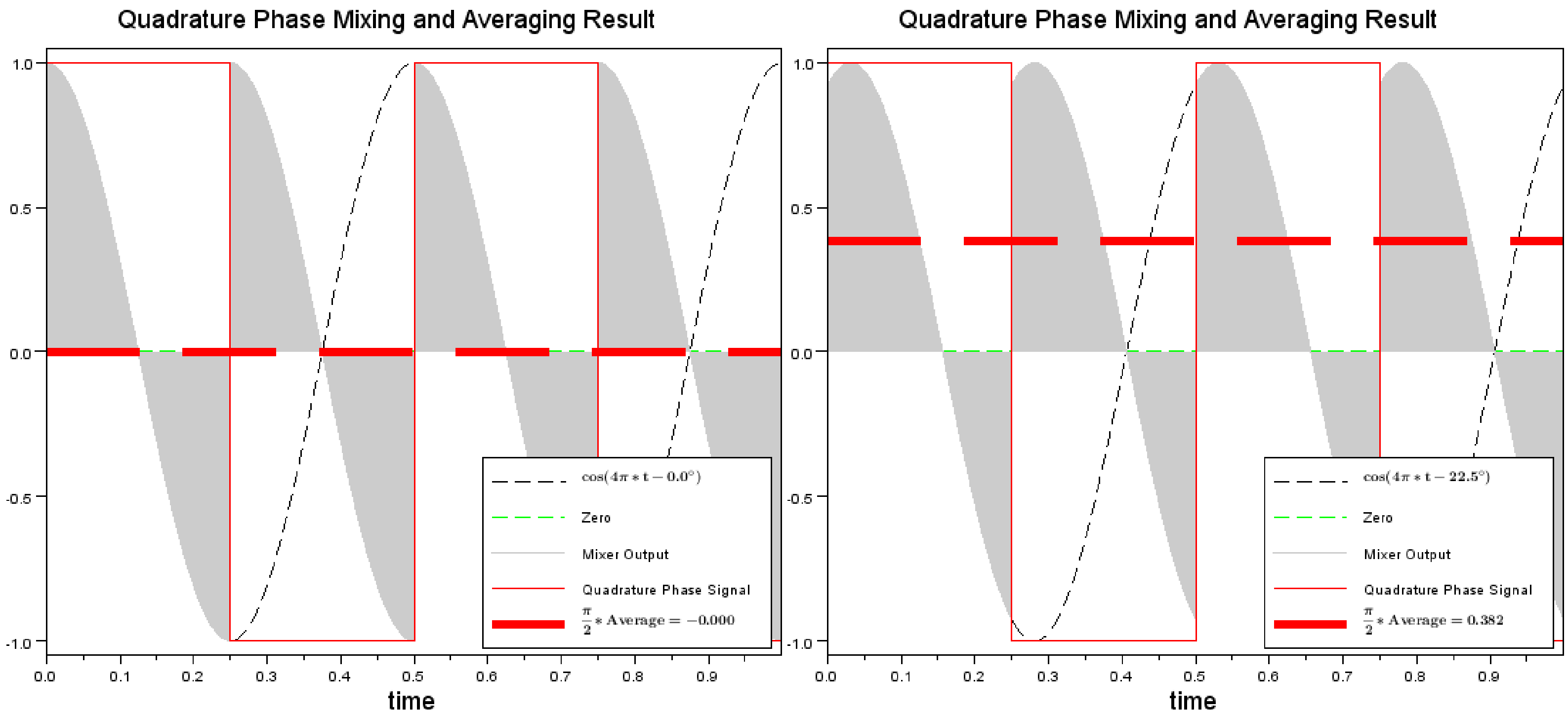
The figure below shows the original signal (thin dashed black line) the mixer output (grey signal with the area under the curve shaded to emphasize what will be averaged) and the result of the scaled average (thick dashed red line):

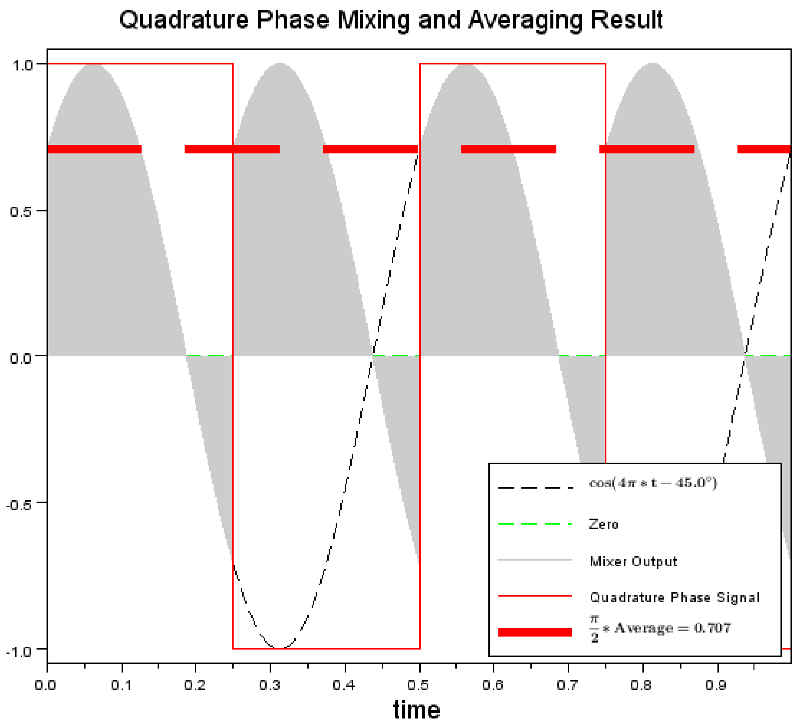


In this case, the output of the scaled average is 0.000. Same input signal, different phase signal, and different output. Lets explore more of the story.

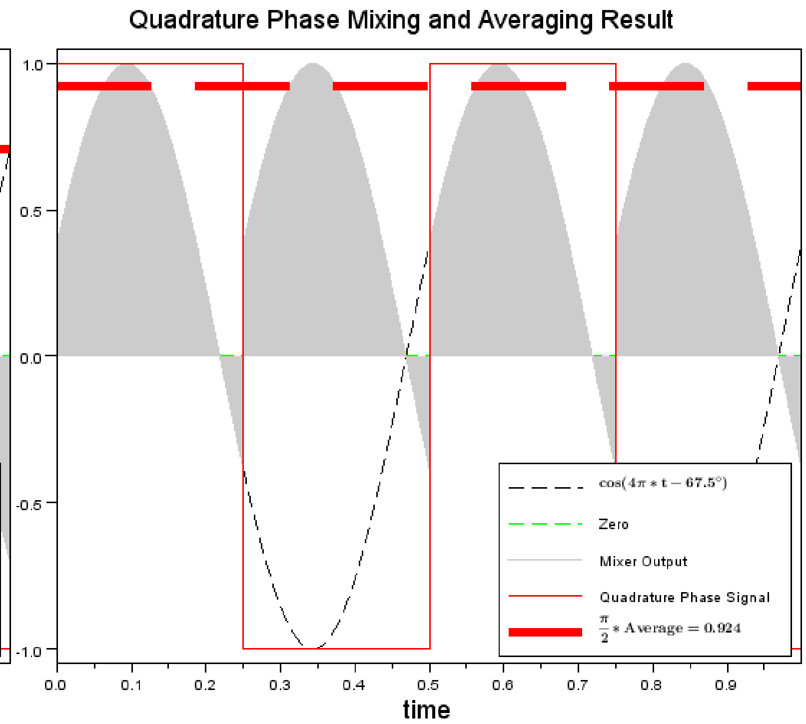
The following graphics illustrate an input signal $\cos(4\pi t - \theta)$ being phase delayed from 0 to 180 degrees. The gray shaded area represents the area under the curve that is being averaged. The result of the averaging, scaled by $\pi/2$, is displayed by the thick red dashed line.

Observe the change in the averaged output as the input signals phase changes relative to the quadrature phase signal:

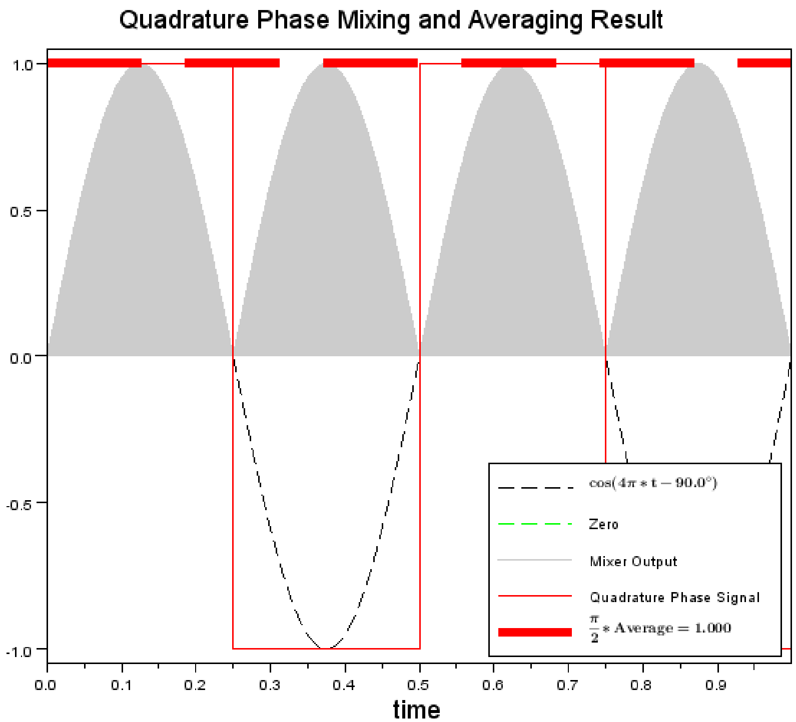

Quadrature Phase Mixing and Averaging Result
Quadrature Phase Mixing and Averaging Result
1.0
0.5
0.0
-0.5
-1.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
time
$\cos(4\pi * t - 0.0°)$
Zero
Mixer Output
Quadrature Phase Signal
$\frac{\pi}{2} * \text{Average} = -0.000$
$\cos(4\pi * t - 22.5°)$
$\frac{\pi}{2} * \text{Average} = 0.382$

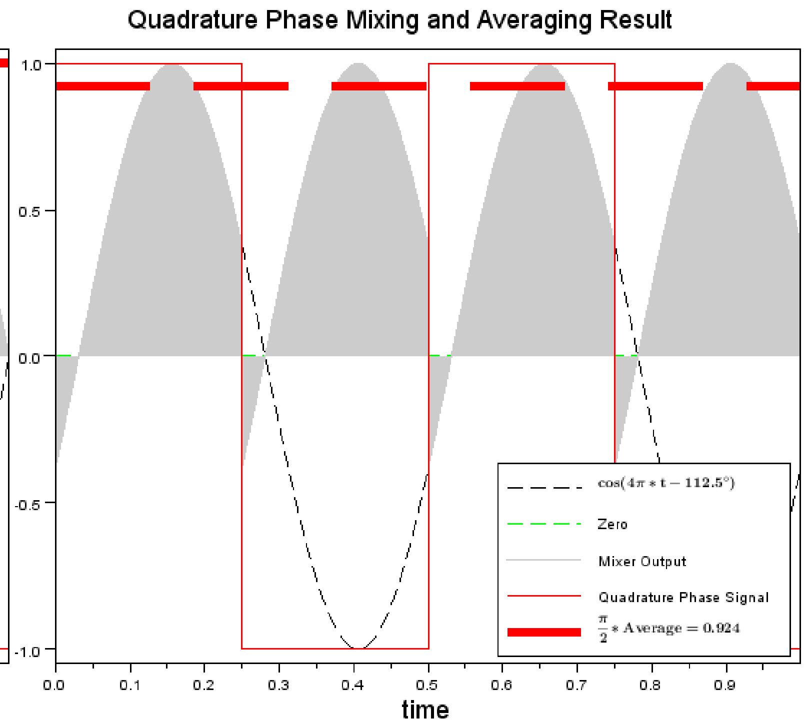


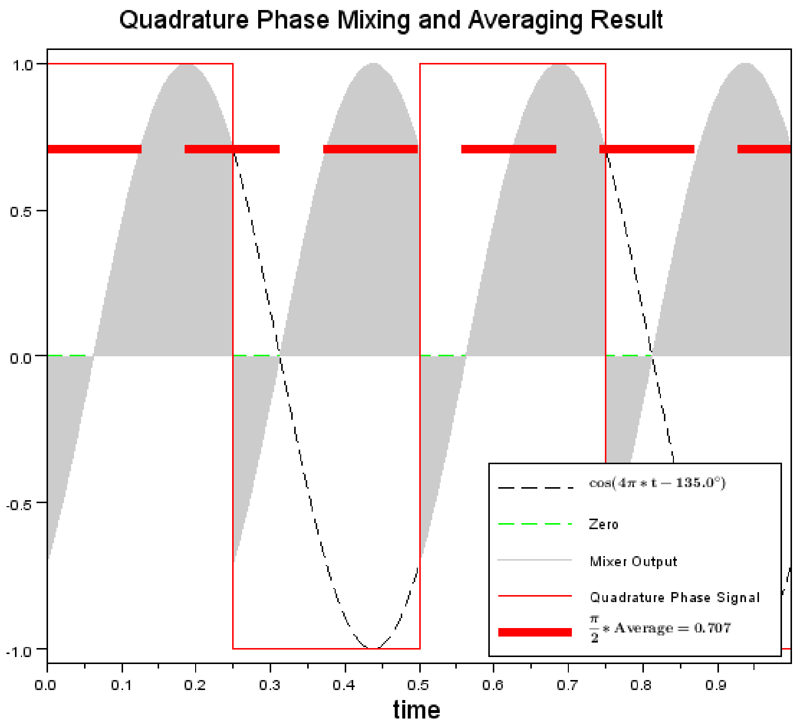
Quadrature Phase Mixing and Averaging Result
1.0
0.5
0.0
-0.5
-1.0
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
time
$\cos(4\pi * t - 45.0°)$
Zero
Mixer Output
Quadrature Phase Signal
$\frac{\pi}{2} * \text{Average} = 0.707$

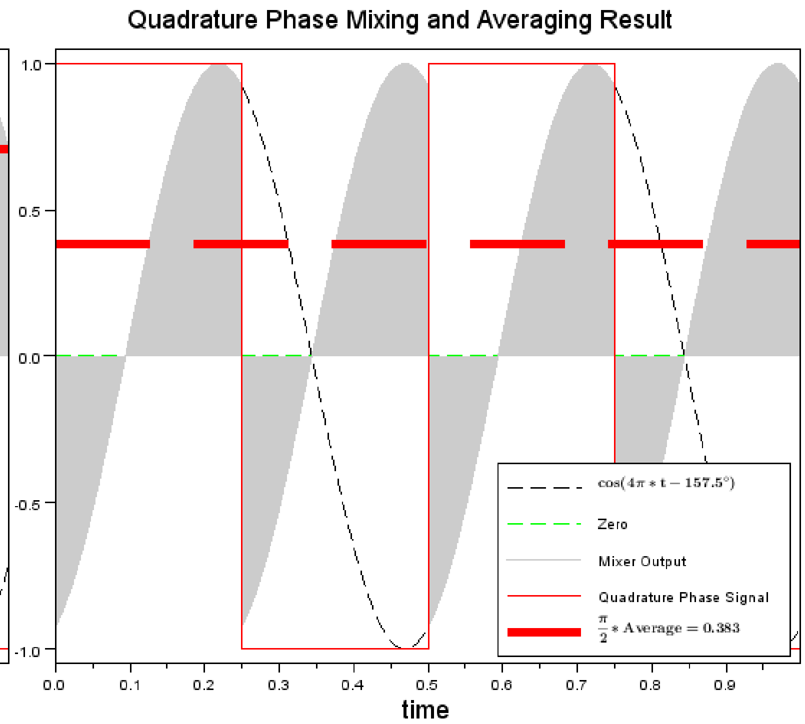


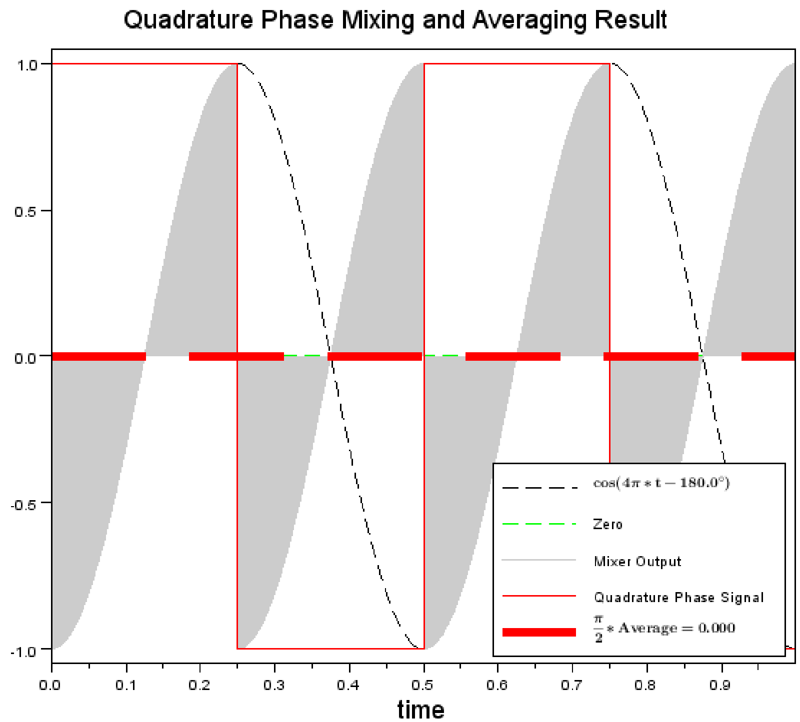
Quadrature Phase Mixing and Averaging Result
1.0
0.5
0.0
-0.5
-1.0
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
time
$\cos(4\pi * t - 67.5°)$
Zero
Mixer Output
Quadrature Phase Signal
$\frac{\pi}{2} * \text{Average} = 0.924$


Quadrature Phase Mixing and Averaging Result
$\cos(4\pi * t - 90.0°)$
Zero
Mixer Output
Quadrature Phase Signal
$\frac{\pi}{2} * \mathrm{Average} = 1.000$
time


Quadrature Phase Mixing and Averaging Result
$\cos(4\pi * t - 112.5°)$
Zero
Mixer Output
Quadrature Phase Signal
$\frac{\pi}{2} * \mathrm{Average} = 0.924$
time


Quadrature Phase Mixing and Averaging Result
1.0
0.5
0.0
-0.5
-1.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
time
$\cos(4\pi * t - 135.0°)$
Zero
Mixer Output
Quadrature Phase Signal
$\frac{\pi}{2} * \text{Average} = 0.707$


Quadrature Phase Mixing and Averaging Result
1.0
0.5
0.0
-0.5
-1.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
time
$\cos(4\pi * t - 157.5°)$
Zero
Mixer Output
Quadrature Phase Signal
$\frac{\pi}{2} * \text{Average} = 0.383$



Quadrature Phase Mixing and Averaging Result
1.0
0.5
0.0
-0.5
-1.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
time
$\cos(4\pi * t - 180.0°)$
Zero
Mixer Output
Quadrature Phase Signal
$\frac{\pi}{2} * \text{Average} = 0.000$

A summary table of the scaled average output is shown below:

Signal Phase Delay(θ)*	Quadrature Phase Result	$\sin(\theta)$
0.0	0.000	0.000
22.5	0.382	0.383
45.0	0.707	0.707
67.5	0.924	0.924
90.0	1.000	1.000
112.5	0.924	0.924
135.0	0.707	0.707
157.5	0.383	0.383
180.0	0.000	0.000

**signal phase delay between our quadrature phase signal and our input signal.*

Combining the results from both tests, we get the following table:

Signal Phase Delay(θ)	In Phase Result	$\cos(\theta)$	Quadrature Phase Result	$\sin(\theta)$
0.0	1.000	1.000	0.000	0.000
22.5	0.924	0.924	0.382	0.383
45.0	0.708	0.707	0.707	0.707
67.5	0.383	0.383	0.924	0.924
90.0	0.001	0.000	1.000	1.000
112.5	-0.382	-0.383	0.924	0.924
135.0	-0.707	-0.707	0.707	0.707
157.5	-0.924	-0.924	0.383	0.383
180.0	-1.000	-1.000	0.000	0.000

And with our arbitrary input of $a\cos(\omega t - \theta)$, we have 2 unknowns ('a' and θ), and 2 equations:

$a\cos(\theta) =$ In phase scaled average = in phase result.

$a\sin(\theta) =$ Quadrature phase scaled average = quadrature phase result.

From here, we can determine both of the unknowns with the following equations*:

Equation 1: $\theta = \tan^{-1}\left(\frac{Quadrature\_phase\_result}{In\_phase\_result}\right)$

Equation 2: $a = \frac{\pi}{2}\sqrt{(In\_phase\_result)^2 + (Quadrature\_phase\_result)^2}$

**There is an explanation in the appendix for both of these equations and how they produce the desired result.*

Now that we know 'a' and θ , we know the input signal $a\cos(\omega t - \theta)$. Remember, θ is the phase difference between the input signal and our in phase signal. If we know the magnitude (a) and the phase (θ) of a signal, we can determine its complex phasor form, and with a little math and the right signals, we can determine a complex impedance.

In summary, these are the steps needed to perform synchronous detection:

1.) Generate an "in phase" and "quadrature phase" signal. These signals then become the reference signals by which you will define your unknown input.
2.) Multiply your input signal by these two reference signals
3.) Low pass filter the output of the multiplication and sample it
4.) Use the following equations to determine the magnitude and phase of the input signals

Equation 3: $\theta = \tan^{-1}\left(\frac{Quadrature\_phase\_result}{In\_phase\_result}\right)$

Equation 4: $a = \frac{\pi}{2}\sqrt{(In\_phase\_result)^2 + (Quadrature\_phase\_result)^2}$

5.) **Appendix:**

When multiplying by a +1 or -1 (up mixing with a clock), the result is periodic. We can then take one period and observe the behavior since the result will be the same for 1 cycle, 2 cycles and so on …

The mathematical expression for averaging over one cycle of the in-phase signal multiplied by the cosine with an arbitrary phase shift is:

$$\frac{\omega}{2\pi}\left(\int_{-\frac{\pi}{2\omega}}^{\frac{\pi}{2\omega}} a\cos(\omega t-\phi)dt+\int_{\frac{\pi}{2\omega}}^{\frac{3\pi}{2\omega}} -a\cos(\omega t-\phi)dt\right)=\frac{2}{\pi}a\cos(\phi)$$

The mathematical expression for averaging over one cycle with the 90 degree phase shifted (in-quadrature) signal multiplied by the cosine with an arbitrary phase shift is:

$$\frac{\omega}{2\pi}\left(\int_{0}^{\frac{\pi}{\omega}} a\cos(\omega t-\phi)dt+\int_{\frac{\pi}{\omega}}^{\frac{2\pi}{\omega}} -a\cos(\omega t-\phi)dt\right)=\frac{2}{\pi}a\sin(\phi)$$

**Note: to recover* $a*\cos(\phi)$ *(as needed earlier in the memo), we need to multiply by* $\pi/2$ *to cancel the* $2/\pi$ *present in the result. And remember, the result of *just* the in-phase or in-quadrature part is* $a*\cos(\phi)$ *or* $a*\sin(\phi)$.

We can now recover the angle by:

$$\tan^{-1}\left(\frac{\frac{2}{\pi}a\sin(\phi)}{\frac{2}{\pi}a\cos(\phi)}\right)=\tan^{-1}\left(\frac{\sin(\phi)}{\cos(\phi)}\right)=\phi$$

And ‘a’ can be calculated by either (using knowledge of in-phase result and calculated angle):

$$\frac{\pi}{2\cos(\phi)}*\frac{2}{\pi}a\cos(\phi)=a$$

Or using the result from the in-phase and in-quadrature results:

$$\frac{\pi}{2}\sqrt{\left(\frac{2}{\pi}a\cos(\phi)\right)^2+\left(\frac{2}{\pi}a\sin(\phi)\right)^2}$$

$$\frac{\pi}{2}\sqrt{\frac{4}{\pi^2}a^2\cos^2(\phi)+\frac{4}{\pi^2}a^2\sin^2(\phi)}$$

$$\frac{\pi}{2}\sqrt{\frac{4}{\pi^2}a^2\left(\cos^2(\phi)+\sin^2(\phi)\right)}$$

$$\frac{\pi}{2}\frac{2}{\pi}a\sqrt{\cos^2(\phi)+\sin^2(\phi)}$$

$$\sqrt{\cos^2(\phi)+\sin^2(\phi)}=1$$

$$\frac{\pi}{2}\frac{2}{\pi}a*1=a$$