Test questions

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A system is transmitting a sequence $I(k)$ of QAM-4 symbols, with symbol duration T, modulated around carrier frequency ω_0. The designer is using an ideal rectangular filter as shaping filter $g(t)$ for this modulation, but made a mistake and, instead of having the total bandwidth $1/T$, it only occupies half of the band. The shaping filter is thus given in frequency as

$$G(f) = \begin{cases} \sqrt{2T} & \text{for } \frac{-1}{4T} \leq f \leq \frac{1}{4T} \\ 0 & \text{elsewhere} \end{cases}$$

Note that

$$\mathcal{E}_g = \int_{-\infty}^{\infty} g^2(t)\, dt = 1$$

and that the autocorrelation of this filter is given, in the time domain, by

$$h(t) = g(t) \otimes g(-t) = \frac{\sin\left(\frac{\pi t}{2T}\right)}{\frac{\pi t}{2T}}.$$

The signal is amplified and transmitted with a maximum PSD γ_M. It is then transmitted through an AWGN channel with (double-side band) spectral density $N_0/2$. We assume coherent demodulation at the receiver. The receiver applies matched filtering followed by sampling at the symbol rate. Even though there is intersymbol interference, no equalization is performed and the samples are directly used as decision variables.

1. Draw the block diagram of the equivalent baseband transmission.
2. Write the complex baseband expressions of the transmitted and received signals, as well as the decision variable.
3. Compute the PSD of the transmitted signal.
4. Compute the average transmitted energy per symbol E_s.
5. Characterize the decision variable (expression and probability distribution) by treating the ISI (intersymbol interference) as a Gaussian noise which is independent from the additive noise and from the symbol of interest.

   NB: $1 + \left(\frac{1}{3}\right)^2 + \left(\frac{1}{5}\right)^2 + \left(\frac{1}{7}\right)^2 + \ldots = \frac{\pi^2}{8}$
6. Compute the error probability as a function of E_s/N_0.
7. Compare steps 3-6 above, for a better designed system that uses exactly the same shaping filter, has exactly the same transmitted PSD on the same bandwidth $1/2T$ but that works with double symbol duration $2T$. Which system has the best bitrate and which system has the best error probability?

8. In order to compare the 2 systems more precisely, an independent consultant wants to compare them using the formula of capacity. He is using the following model. In both cases, he reuses the characterization of the decision variable $r(k) = I(k) + \nu(k)$ obtained in step 5. The noise $\nu(k)$ is assumed to be Gaussian with the same variance as in step 5 (which takes into account the possible influence of ISI). The symbols are no longer fixed to a QAM-4 constellation but can be any continuous random variable with a fixed variance, equal to the variance of the QAM-4 constellation. Compute the capacity for this discrete-time channel model in both cases. Express them in bits/sec.

9. For γ_M=-60 dBm/Hz and $N_0/2 = -82$ dBm/Hz, which one of the two systems exhibits the best capacity?