Computing the Losses in a Sinusoidal Controlled PWM Inverter

The basic structure of a sinusoidal pulse width modulation (PWM) inverter driving a motor is shown in Fig. 1. The inverter produces the three voltages V_A, V_B, and V_C relative to the negative side of the DC bus which is taken as ground in Fig. 1. The motor's line to line voltage is just the difference between the inverter output voltages. Thus the peak line to line voltage is V_{DC} and the peak to peak line to line voltage is $2V_{DC}$.

I Semiconductor losses

The semiconductor losses are the sum of the individual IGBT and diode losses. Averaged over one cycle, the losses in each of the IGBTs are equal and averaged over one cycle the losses in each of the diodes are equal. It typically is the device loss averaged over one cycle that is required for the thermal calculations since the thermal system is a low pass filter with a very long time constant. Using the average power in the thermal calculations is valid as long as the motor is turning fast enough that the period of one electrical cycle is much less than each thermal time constant. Ultimately the thermal time constants must be calculated to determine the motor speeds for which using the average power is valid.

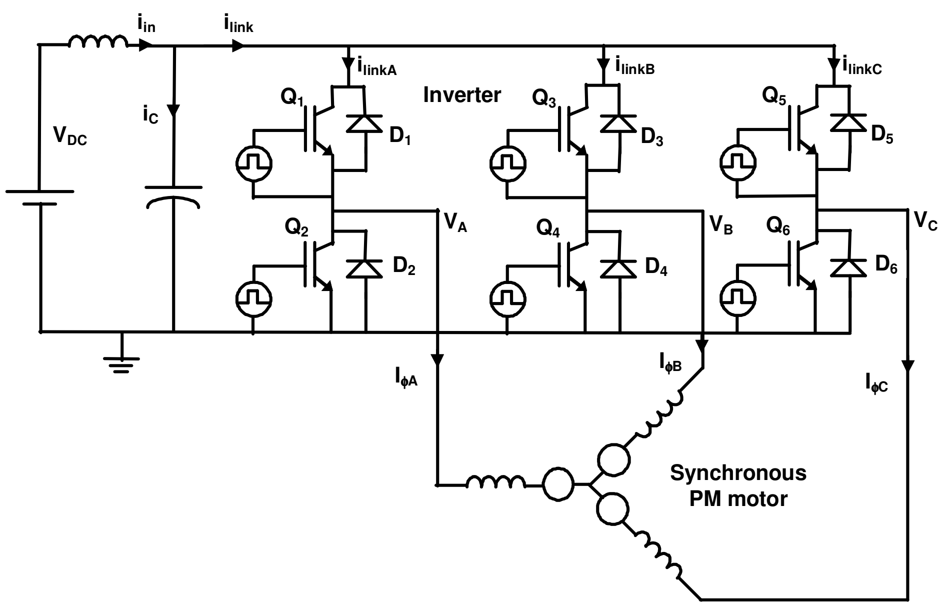

Figure 1 IGBT inverter driving a PM synchronous motor

The diode losses have predominately conduction losses. The average diode losses are computed by averaging the average diode losses in each switching cycle.

$$\langle P_{diode} \rangle = \frac{1}{N} \sum_n \frac{1}{T} \int_{(n-1)T}^{nT} I_f(t) V_f(t)\, dt \qquad (1.1)$$

where here N is the number of switching cycles in one motor electrical cycle, T is the switching period, $I_f(t)$ is the diodes forward current which is equal to the phase current when the diode is conducting, and $V_f(t)$ if the diodes forward current when it is conducting. The diode's forward voltage when it is conducting can approximated very well as

$$V_f = V_{fo} + R_f \bullet I_f \tag{1.2}$$

A particular diode is on when its opposite IGBT is off. Thus if Q_1 conducts, D_2 conducts when Q_1 turns off. Also, if Q_1 conducts, the diode D_1 does not conduct. Vice versa, if D_1 conducts Q_1, does not conduct even if it is on. Thus Q_1 and D_2 take turns conducting for one motor half cycle while D_1 and Q_2 take turns conducting for the other motor half cycle. Substituting this expression for the forward voltage in Eq. 1.2 into Eq. 1.1 with the above considerations gives

$$\left\langle P_{diode} \right\rangle = \frac{1}{N}\sum_n \frac{1}{T}\int_{(n-1+D_n)T}^{nT} I_f(t)\left(V_{fo} + R_f I_f(t)\right)dt \tag{1.3}$$

Here the IGBT is on for D_nT of the nth switching cycle and its opposite diode is on for $(1-D_n)T$ of the nth switching cycle. The integral in Eq. 1.3 can be evaluated if the inverter's switching frequency is high enough that the motors phase current and thus the diode's current is constant during a switching cycle.

$$\left\langle P_{diode} \right\rangle = \frac{1}{N}\sum_n V_{fo}\left(1 - D_n\right)I_f(nT) + \frac{1}{N}\sum_n R_f\left(1 - D_n\right)I_f(nT)^2 \tag{1.4}$$

Thus the average power has two parts that can be evaluated separately. One is due to the diode's offset voltage V_{fo} and the other is due to the diode's resistance R_f. Again if the switching frequency is high enough T small enough) the sums in Eq. 1.4 can be approximated with integrals

$$\left\langle P_{fo} \right\rangle = \frac{1}{NT}\sum_n V_{fo}\left(1 - D_n\right)I_f(nT)T \approx \frac{1}{T_m}\int_0^{T_m/2} V_{fo}\left(1 - D_\phi(t)\right)I_\phi(t)dt \tag{1.5a}$$

$$\left\langle P_{Rf} \right\rangle = \frac{1}{NT}\sum_n R_f\left(1 - D_n\right)I_f(nT)^2 T \approx \frac{1}{T_m}\int_0^{T_m/2} R_f\left(1 - D_\phi(t)\right)I_\phi(t)^2 dt \tag{1.5b}$$

In Eq. 1.5 the fact that the diode's forward current is equal to the motor's phase current while it is conducting has been used. The integrals in Eq. 1.5 are over only one half of a cycle because during the other half cycle the current flows in the IGBT in parallel with the diode rather than the diode. Thus the above result requires that the current be the reference waveform since the sign of the phase current determines if the current flows in the diode or the IGBT.

$$I_{\phi}(t) = I_{\phi p}\sin(\omega_m t) \quad (1.6)$$

The phase voltage averaged over one switching cycle is

$$V_{\phi g}(t) = D_{\phi}(t)V_{DC} \quad (1.7)$$

The duty cycle is constrained to between zero and one so that to obtain a sinusoidal voltage across the motor from line to line the duty cycle for a phase must be

$$D_{\phi}(t) = \frac{1}{2} + D_p \sin(\omega_m t + \varphi_{\phi}) \quad (1.8)$$

Here 0 < Dp < ½ and φ_{ϕ} is either 0, -120, or 120 degrees depending on the phase ϕ. The line to line voltage for phases a and b of the motor is

$$V_{ab} = V_{DC}\left(\frac{1}{2} + D_p \sin(\omega_m t)\right) - V_{DC}\left(\frac{1}{2} + D_p \sin(\omega_m t - 120)\right) = V_{DC}D_p(\sin(\omega_m t) - \sin(\omega_m t - 120))$$
(1.9)

which is a sine wave that can be written in phasor form as

$$V_{ab} = V_{DC}D_p - V_{DC}D_p \exp(-j2\pi/3) = V_{anp}\exp(j\alpha) - V_{anp}\exp(j(\alpha - 2\pi/3)) \quad (1.10)$$

In Eq. 1.10 the line to line voltage for phases a and b of the motor has been written in terms of the AC part of the inverter voltage line to ground and in terms of the motor voltage line to neutral. The voltages are assumed to be balanced three phase and (to be general) the motor's line to neutral voltage is assumed to be out of phase with the AC part of the inverter voltage line to ground. Simplifying Eq. 1.10 gives

$$V_{ab} = V_{DC}D_p(1 - \exp(-j2\pi/3)) = V_{anp}(1 - \exp(-j2\pi/3))\exp(j\alpha)$$

$$V_{DC}D_p = V_{anp}\exp(j\alpha) \quad (1.11)$$

Since the left side of Eq. 1.11 is real α must be zero and $V_{anp} = V_{DC}D_p$. The AC part of the inverter's line to ground voltage is equal to the motor's line to neutral voltage. From Eq. 1.7 the AC part of a given phase of the inverter has the same phase as the motor's line to neutral voltage which is the reference waveform. Thus Eq. 1.5 can be written as

$$\langle P_{fo}\rangle = \frac{V_{fo}}{T_m}\int_0^{T_m/2}\left(\frac{1}{2} - D_p \sin(\omega_m t + \theta)\right) I_{\phi p}\sin(\omega_m t)dt \quad (1.12a)$$

$$\langle P_{Rf}\rangle = \frac{R_f}{T_m}\int_0^{T_m/2}\left(\frac{1}{2} - D_p \sin(\omega_m t + \theta))\right) I_{\phi p}{}^2\sin^2(\omega_m t)dt \quad (1.12b)$$

In Eq. 1.12 the line to neutral voltage leads the phase current by the positive angle θ or equivalently the phase current lags the line to neutral voltage by the angle positive θ. Thus θ is the motor's power factor angle and a plus angle is a lagging power factor (inductive load). Doing the integrals in Eq. 1.12 gives

$$\langle P_{fo}\rangle = I_{\phi p} V_{fo}\left(\frac{1}{2\pi} - \frac{D_p}{4}\cos(\theta)\right) \quad (1.13a)$$

$$\langle P_{Rf}\rangle = I_{\phi p}{}^2 R_f\left(\frac{1}{8} - \frac{2D_p}{3\pi}\cos(\theta)\right) \quad (1.13b)$$

Thus the average loss in one diode in the sinusoidal PWM inverter is

$$\langle P_{diode}\rangle = I_{\phi p} V_{fo}\left(\frac{1}{2\pi} - \frac{D_p}{4}\cos(\theta)\right) + I_{\phi p}{}^2 R_f\left(\frac{1}{8} - \frac{2D_p}{3\pi}\cos(\theta)\right) =$$

$$I_{\phi p}\left(\frac{V_{fo}}{2\pi} + \frac{I_{\phi p}R_f}{8}\right) - I_{\phi p}D_p\left(\frac{V_{fo}}{4} + \frac{I_{\phi p}R_f}{3\pi/2}\right)\cos(\theta) \quad (1.14)$$

Since the cos(θ) is the motor's power factor (PF) Eq. 1.14 can be written as

$$\langle P_{diode}\rangle = I_{\phi p}\left(\frac{V_{fo}}{2\pi} + \frac{I_{\phi p}R_f}{8}\right) - I_{\phi p}D_p\left(\frac{V_{fo}}{4} + \frac{I_{\phi p}R_f}{3\pi/2}\right)PF \quad (1.15)$$

A similar procedure can be followed to find the losses in the IGBTs. Starting with the IGBT's conduction losses

$$\langle P_{cond}\rangle = \frac{1}{N}\sum_n \frac{1}{T}\int_{(n-1)T}^{nD_nT} I_C(t)\left(V_{Co} + R_C I_C(t)\right)dt = \frac{1}{N}\sum_n D_n I_C(nT)\left(V_{Co} + R_C I_C(nT)\right) \quad (1.16)$$

Turning the sum into an integral and substituting for the current and duty cycle gives

$$\langle P_{Co}\rangle = \frac{V_{Co}}{T_m}\int_0^{T_m/2}\left(\frac{1}{2} + D_p\sin(\omega_m t + \theta)\right)I_{\phi p}\sin(\omega_m t)dt \quad (1.17a)$$

$$\langle P_{RC}\rangle = \frac{R_C}{T_m}\int_0^{T_m/2}\left(\frac{1}{2} + D_p\sin(\omega_m t + \theta))\right)I_{\phi p}{}^2\sin^2(\omega_m t)dt \quad (1.17b)$$

The average conduction losses in the IGBT are found from doing the integrals in Eq. 1.17

$$\langle P_{cond}\rangle = I_{\phi p}\left(\frac{V_{Co}}{2\pi}+\frac{I_{\phi p}R_C}{8}\right)+I_{\phi p}D_p\left(\frac{V_{Co}}{4}+\frac{I_{\phi p}R_C}{3\pi/2}\right)PF \tag{1.18}$$

The average switching losses in the IGBT are computed using

$$\langle P_{sw}\rangle = \frac{1}{N}\sum_n\left(\frac{E_{on}(nT)}{T}+\frac{E_{off}(nT)}{T}\right) \tag{1.19}$$

The switching energy as a function of time is found assuming

$$E_{on}(nT) = E_{on\_test}\frac{V_{DC}}{V_{DCteston}}\frac{I_{Con}(nT)}{I_{Cteston}} \tag{1.20a}$$

$$E_{off}(nT) = E_{off\_test}\frac{V_{DC}}{V_{DCtestoff}}\frac{I_{Coff}(nT)}{I_{Ctestoff}} \tag{1.20b}$$

The switching energy is assumed proportional to the DC voltage and the IGBT current. This assumes the switching time is independent of the current or voltage. This assumption is reasonable put not exact. Substituting Eq. 1.20 into Eq. 1.19 gives

$$\langle P_{sw}\rangle = \frac{1}{NT}\sum_n\left(E_{on\_test}\frac{V_{DC}}{V_{DCteston}}\frac{I_{Con}(nT)}{I_{Cteston}}+E_{off\_test}\frac{V_{DC}}{V_{DCtestoff}}\frac{I_{Coff}(nT)}{I_{Ctestoff}}\right) \tag{1.21}$$

If the switching frequency is high enough the collector turn on and turn off currents can be approximated by the fundamental of the motor's phase current during the switching interval. Further if the switching frequency is high enough (the switching period T small enough) Eq. 1.21 can be approximated with an integral. Multiply the numerator and denominator by T, the switching period, to approximate the sum in Eq. 1.21 with the integral

$$\langle P_{sw}\rangle = \left(E_{on\_test}\frac{V_{DC}}{V_{DCteston}}\frac{I_{\phi p}}{I_{Cteston}}+E_{off\_test}\frac{V_{DC}}{V_{DCtestoff}}\frac{I_{\phi p})}{I_{Ctestoff}}\right)\frac{F_{sw}}{T_m}\int_0^{T_m/2}\sin(\omega_m t)dt \tag{1.22}$$

Again the integral is only done for the half of the motor's period when the phase current is positive because the current is only carried by the IGBTs for the half of the motor's period when the current is positive. For the other half of the motor's period when the current is negative it flows the diodes in parallel with the IGBTs. Doing the integral in Eq. 1.22 gives

$$\langle P_{sw}\rangle = \frac{Fsw}{\pi}\left(E_{on\_test}\frac{V_{DC}}{V_{DCteston}}\frac{I_{\phi p}}{I_{Cteston}}+E_{off\_test}\frac{V_{DC}}{V_{DCtestoff}}\frac{I_{\phi p})}{I_{Ctestoff}}\right) \tag{1.23}$$

II DC Link Capacitor RMS Current

The link capacitor filters the ripple current generated by the inverter. The ac ripple current flows through the capacitor's equivalent series resistance (ESR) producing losses. The capacitors cannot get too hot with the losses it experiences. Typically the manufacturer specifies a maximum ripple current rating for its capacitor, which if the user stays below this rating the capacitor does not get too hot. Thus the rms value of the current in the link capacitor must be computed.

The link current is the sum of the current in the three upper switched and diodes. For any switch the current is in the IGBT for the first half cycle and in the diode for the second half cycle. The rms ripple current in the link capacitor is equal to the rms value of the AC part of the link current. The rms value of the AC part of the link current is related to the total rms value of the link current and the average value of the link current by Parsaval's theorem.

$$I_{linkrms}^{\ 2} = I_{linkdc}^{\ 2} + I_{linkac}^{\ 2}$$

$$I_{linkac} = \sqrt{I_{linkrms}^{\ 2} - I_{linkdc}^{\ 2}} \qquad (2.1)$$

The link current is given by

$$i_{link}(t) = i_{linkA}(t) + i_{linkB}(t) + i_{linkC}(t)$$

The average value of the link current is

$$I_{linkdc} = \langle i_{link} \rangle = \frac{1}{N}\sum_n \frac{1}{T}\int_{(n-1)T}^{nT} i_{linkA}(t)dt + \frac{1}{N}\sum_n \frac{1}{T}\int_{(n-1)T}^{nT} i_{linkB}(t)dt + \frac{1}{N}\sum_n \frac{1}{T}\int_{(n-1)T}^{nT} i_{linkC}(t)dt \qquad (2.2)$$

Now both the current in the IGBT and the diode are included in the sum so that both half cycles of the phase current sine wave are included in the sum. The average link current in each phase is the same so only one integral in Eq. 2.2 must be done. Further, during the interval an IGBT or diode is on, the individual phase link currents are equal to the motor phase current otherwise they are zero. Thus, with the assumption that the phase current does not change much during a switching cycle

$$I_{linkdc} = \frac{3}{N}\sum_n \frac{1}{T}\int_{(n-1)T}^{(n-1+D_{An})TT} i_{\phi A}(t)dt = \frac{3}{N}\sum_n D_{An} \cdot i_{linkA}(nT) \qquad (2.3)$$

Again the sum can be approximated with an integral to obtain

$$I_{linkdc} = \frac{3}{NT}\sum_n D_{An} \cdot i_{linkA}(nT)T = \frac{3}{T_m}\int_0^{T_m}\left(\frac{1}{2} + D_p \sin(\omega_m t + \theta)\right) I_{\phi p}\sin(\omega_m t)dt$$

$$I_{linkdc} = \frac{3}{2} D_p I_{\phi p} \cos(\theta) \tag{2.4}$$

This result could be found more simply by setting the average DC link power to the average motor power but the procedure used to obtain the average link current is the same procedure used to obtain the rms value of the link current.

To find the rms value of the link current, the mean value of the square of the link current must be computed. Again both half cycles of the phase current are included in the sum.

$$I_{linkrms}{}^2 = \left\langle i_{link}{}^2 \right\rangle = \frac{1}{N} \sum_n \frac{1}{T} \int_{(n-1)T}^{nT} \left(i_{linkA}(t) + i_{linkB}(t) + i_{linkC}(t) \right)^2 dt$$

During the interval the IGBT or diode are on the individual phase link currents are equal to the motor phase current.

$$I_{linkrms}{}^2 = \frac{1}{N} \sum_n \frac{1}{T} \int_{(n-1)T}^{nT} \Big(i_{linkA}(t)^2 + i_{linkB}(t)^2 + i_{linkC}(t)^2 + 2 i_{linkA}(t) i_{linkB}(t) + 2 i_{linkA}(t) i_{linkC}(t) + 2 i_{linkB} i_{linkC} \Big) dt \tag{2.5}$$

Due to the symmetry of the currents (they are equal except for their phase) Eq. 2.5 can be simplified to

$$I_{linkrms}{}^2 = \frac{1}{N} \sum_n \frac{3}{T} \int_{(n-1)T}^{nT} i_{linkA}(t)^2 dt + \frac{1}{N} \sum_n \frac{6}{T} \int_{(n-1)T}^{nT} i_{linkA}(t) i_{linkB}(t) dt \tag{2.6}$$

The first term in Eq. 2.6 can be simplified since the phase A link current is equal to the phase A current while the switch is on so that with the assumption that the phase current does not change much during a switching cycle, the first integral in Eq. 2.6 becomes

$$Term_1 = \frac{1}{N} \sum_n \frac{3}{T} \int_{(n-1)T}^{(n-1+D_{An})T} i_{\phi A}(t)^2 dt = \frac{1}{N} \sum_n 3 D_{An} \cdot i_{\phi A}(nT)^2 \tag{2.7}$$

Following the usual procedure for approximating the sums with integrals

$$Term_1 = \frac{1}{NT} \sum_n 3 D_n \cdot i_{\phi A}(nT)^2 T = \frac{3}{T_m} \int_0^{T_m} D(t) \cdot i_{\phi A}(t)^2 dt$$

$$Term_1 = \frac{3}{T_m}\int_0^{T_m}\left(\frac{1}{2} + D_p \sin\left(\omega_m t + \theta\right)\right) I_{\phi p}{}^2 \sin^2(\omega_m t)dt \tag{2.8}$$

It can be recognized that the first term in the integral in Eq. 2.8 is just 3/2 time the rms phase current squared so

$$Term_1 = \frac{3}{2} I_{\phi rms}{}^2 + \frac{3}{T_m}\int_0^{T_m} D_p I_{\phi p}{}^2 \sin\left(\omega_m t + \theta\right)\sin^2(\omega_m t)dt \tag{2.9}$$

Using some trigonometric identities the last term in Eq. 2.9 integrates to zero so

$$Term_1 = \frac{3}{2} I_{\phi rms}{}^2 \tag{2.10}$$

When computing the second term in Eq. 2.6

$$Term_2 = \frac{1}{N}\sum_n \frac{6}{T}\int_{(n-1)T}^{nT} i_{linkA}(t) i_{linkB}(t)dt \tag{2.11}$$

the contribution will be zero if either $i_{linkA}(t)$ or $i_{linkB}(t)$ are equal to zero or equivalently if either of the switches (IGBT and diode) in phases A or B are off. Assuming that the IGBTs in phases A and B are synchronized so they turn on at the same time but turn off at different times, Eq. 2.11 can be written as

$$Term_2 = \frac{1}{N}\sum_n \frac{6}{T}\int_{(n-1)T}^{(n-1+\min(D_{An}, D_{Bn}))T} i_{\phi A}(t) i_{\phi B}(t)dt \tag{2.12}$$

Now the integral runs to the minimum of D_{An} and D_{Bn} since the corresponding switch turns off first making its corresponding link current zero. For times less than this, both switches are on and thus the link currents are equal to their corresponding phase currents. With the assumption that the phase current does not change much during a switching cycle, the first integral in Eq. 2.12 becomes

$$Term_2 = \frac{1}{N}\sum_n 6\min\left(D_{An}, D_{Bn}\right) i_{\phi A}(nT) i_{\phi B}(nT) \tag{2.13}$$

Following the usual procedure for approximating the sums with integrals

$$Term_2 = \frac{1}{NT}\sum_n 6\min\left(D_{An}, D_{Bn}\right) i_{\phi A}(nT) i_{\phi B}(nT)T = \frac{6}{T_m}\int_0^{T_m} \min\left(D_A(t), D_B(t)\right) i_{\phi A}(t) i_{\phi B}(t)dt \tag{2.14}$$

To do the integral in Equation 14 the minimum of the phase A and phase B duty cycle functions must be determined at each instant of time during the cycle. These duty cycle functions are plotted in Fig. 1 for D_p = 0.25. The minimum of these two duty cycle functions is plotted in Fig. 2 for D_p = 0.25 and for D_p = 0.125. From Fig. 2

$$\min\left(D_{An}, D_{Bn}\right) = \begin{cases} D_{Bn} = \dfrac{1}{2} + D_p \sin(\omega t + \theta - 2\pi/3) & 0 \le \omega t + \theta \le 150^o \\ D_{An} = \dfrac{1}{2} + D_p \sin(\omega t + \theta) & 150^o \le \omega t + \theta \le 330^o \\ D_{Bn} = \dfrac{1}{2} + D_p \sin(\omega t + \theta - 2\pi/3) & 330^o \le \omega t + \theta \le 360^o \end{cases} \tag{2.15}$$

Thus Eq.14 can be written as

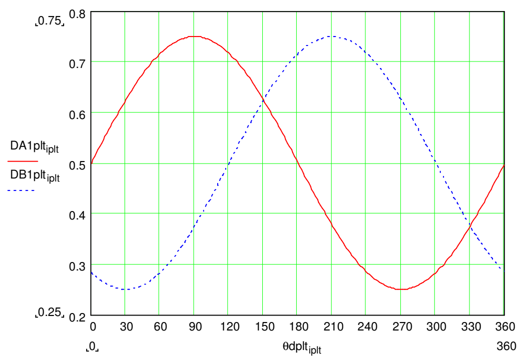

Fig. 1 Plot of the phase A and phase B duty cycle versus time for D_p = 0.25.

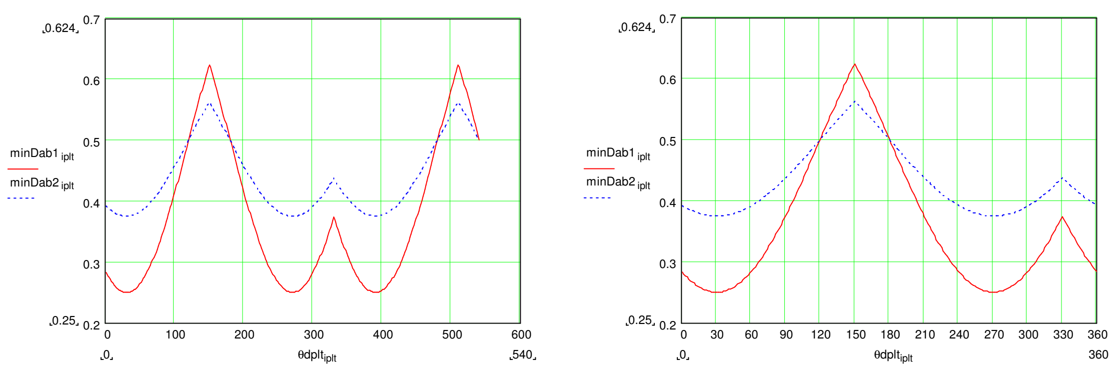

Fig. 2 Plot of minimum of the phase A and phase B duty cycles versus time for D_p = 0.25 and D_p = 0.125.

$$Term_2 = \frac{6 I_{\phi p}{}^2}{T_m} \int_{\omega t + \theta = 0^o}^{150^o} \left(\frac{1}{2} + D_p \sin(\omega t + \theta - 2\pi/3 \right) \sin(\omega t) \sin(\omega t - 2\pi/3) dt +$$

$$\frac{6 I_{\phi p}{}^2}{T_m} \int_{\omega t + \theta = 150^o}^{330^o} \left(\frac{1}{2} + D_p \sin(\omega t + \theta \right) \sin(\omega t) \sin(\omega t - 2\pi/3) dt +$$

$$\frac{6I_{\phi p}{}^2}{T_m}\int_{\omega t+\theta=330^o}^{360^o}\left(\frac{1}{2}+D_p\sin(\omega t+\theta-2\pi/3)\right)\sin(\omega t)\sin(\omega t-2\pi/3)dt$$

(2.16)

In Eq. 2.16 use was made of the fact that the integral is over one cycle and its value does not depend on where the integral starts. Doing the above integral gives

$$Term_2=6I_{\phi p}{}^2\left\{-\frac{1}{8}+\frac{\sqrt{3}}{4\pi}D_p+\frac{\sqrt{3}}{6\pi}D_p\cos(2\theta)\right\}$$

The rms link ripple current is thus equal to

$$I_{linkrms}=\sqrt{\frac{3}{4}I_{\phi p}{}^2-\frac{6I_{\phi p}{}^2}{8}+6I_{\phi p}{}^2D_p\left\{\frac{\sqrt{3}}{4\pi}+\frac{\sqrt{3}}{6\pi}\cos(2\theta)\right\}}$$

$$I_{linkrms}=\sqrt{\frac{3\sqrt{3}}{\pi}I_{\phi p}{}^2D_p\left\{\frac{1}{2}+\frac{1}{3}\cos(2\theta)\right\}} \qquad (2.17)$$

Equation 2.17 and Eq. 2.1 can be combined to find the rms value of the ac part of the link current, which is assumed to be equal to the rms ripple current in the link capacitor.

$$I_{linkac}=\sqrt{\frac{3\sqrt{3}}{\pi}I_{\phi p}{}^2D_p\left\{\frac{1}{2}+\frac{1}{3}\cos(2\theta)\right\}-\frac{9}{4}D_p{}^2I_{\phi p}{}^2\cos^2(\theta)} \qquad (2.18)$$

Note that the rms value of the ripple current (ac part of the link current) is zero if $D_p = 0$. This occurs because if this case the IGBTs and diodes are on for half the time in each phase. At any instant of time the link current is either equal to

$$I_{link}=I_{\phi A}+I_{\phi B}+I_{\phi C}=0$$

if the currents are flowing through the IGBTs or

$$I_{link}=-(I_{\phi A}+I_{\phi B}+I_{\phi C})=0$$

if the current is flowing through the diodes. In either case the link current is zero at each instant of time if all of the devices switch at the same time.

Using a trigonometric identity to simplify the $\cos^2(\theta)$ term gives the final result

$$I_{linkac}=\frac{3}{2}\frac{I_{\phi p}}{\sqrt{2}}\sqrt{D_p\left\{\frac{4}{\sqrt{3}\pi}-D_p+\left(\frac{8}{3\sqrt{3}\pi}-D_p\right)\cos(2\theta)\right\}}=$$

$$\frac{3}{2} I_{\phi rms} \sqrt{D_p \left\{ \frac{4}{\sqrt{3}\pi} - D_p + \left(\frac{8}{3\sqrt{3}\pi} - D_p \right) \cos(2\theta) \right\}} \tag{2.19}$$

Evaluating the constants gives

$$I_{linkac} = \frac{3}{2} I_{\phi rms} \sqrt{D_p \{0.7351 - D_p + (0.4901 - D_p)\cos(2\theta)\}} \tag{2.20}$$

Because $D_p \leq 0.5$, the rms value of the ripple current (ac part of the link current) must be positive for any angle θ and thus any power factor. The current in Eqs 19 and 20 have a maximum when the peak of the ac part of the duty cycle is equal to

$$D_p = \frac{2}{\sqrt{3}\pi} \frac{1 + \frac{2}{3}\cos(2\theta)}{1 + \cos(2\theta)} = 0.3676 \frac{1 + \frac{2}{3}\cos(2\theta)}{1 + \cos(2\theta)} \tag{2.21}$$

At unity power factor Eq. 2.21 says that the maximum ripple current occurs at D_p = 0.3063. Since this is larger than the physical maximum value of Dp = ½, the maximum ripple current occurs at a duty cycle of ½. Thus for unity power factor (θ = 0) the rms value of the ripple current is

$$I_{linkac} = \frac{3}{2} I_{\phi rms} \sqrt{\frac{1}{2} \left\{ \frac{20}{3\sqrt{3}\pi} - 1 \right\}} = 0.335 \frac{3}{2} I_{\phi rms} = 0.5033 I_{\phi rms} \tag{2.22}$$

III Estimating the Value of the DC Link Capacitor

The minimum value of the link capacitor depends on its ripple current at the PWM switching frequency and at the fundamental frequency. The minimum required C imposed by the PWM switching frequency can be estimated from the capacitor's rms ripple current. Though this current is distributed over a spectrum of frequencies, assume it is all at the PWM switching frequency to obtain a worst case constraint.

$$C_{link} \geq \frac{I_{linkac}}{V_{linkac} 2\pi F_{sw}} \tag{3.1}$$

Here I_{linkac} is given by Eqs. 2.19 and 2.20, V_{linkac} is the specified rms ripple voltage across the capacitor, and F_{sw} is the switching frequency.

The capacitor's ripple current at the fundamental frequency can be found by taking the cycle average of the link current

$$I_{linkfund}(nT) = \langle i_{link} \rangle_{cyc} = \sum_n \frac{1}{T} \int_{(n-1)T}^{nT} i_{linkA}(t)dt + \sum_n \frac{1}{T} \int_{(n-1)T}^{nT} i_{linkB}(t)dt + \sum_n \frac{1}{T} \int_{(n-1)T}^{nT} i_{linkC}(t)dt \qquad (3.2)$$

$$I_{linkfund}(nT) = D_{An} \cdot i_{l\phi A}(nT) + D_{Bn} \cdot i_{l\phi B}(nT) + D_{Cn} \cdot i_{l\phi C}(nT)$$

$$I_{linkfund}(t) = D_A(t) \cdot i_{l\phi A}(t) + D_B(t) \cdot i_{\phi B}(t) + D_C(t) \cdot i_{l\phi C}(t)$$

$$I_{linkfund}(t) = \left(\frac{1}{2} + D_p \sin(\omega t + \theta)\right) \cdot I_{\phi p} \sin(\omega t) + \left(\frac{1}{2} + D_p \sin\left(\omega t + \theta - \frac{2\pi}{3}\right)\right) \cdot I_{\phi p} \sin\left(\omega t - \frac{2\pi}{3}\right) +$$
$$\left(\frac{1}{2} + D_p \sin\left(\omega t + \theta + \frac{2\pi}{3}\right)\right) \cdot I_{\phi p} \sin\left(\omega t + \frac{2\pi}{3}\right) \qquad (3.3)$$

The average of Eq. 3.3 has already been computed above in Eq. 2.4. It can be shown that the AC part of Eq. 3.3 is zero, that is its rms value equals its average value. Thus there is not ripple current in the link capacitor bank at the fundamental frequency.

IV Inverter's Thermal Circuit

The thermal circuit for a phase leg module such as the International Rectifier GA500TD60U mounted to the inverter's case and the inverter mounted to another surface is shown in Figs. 3 and 4.

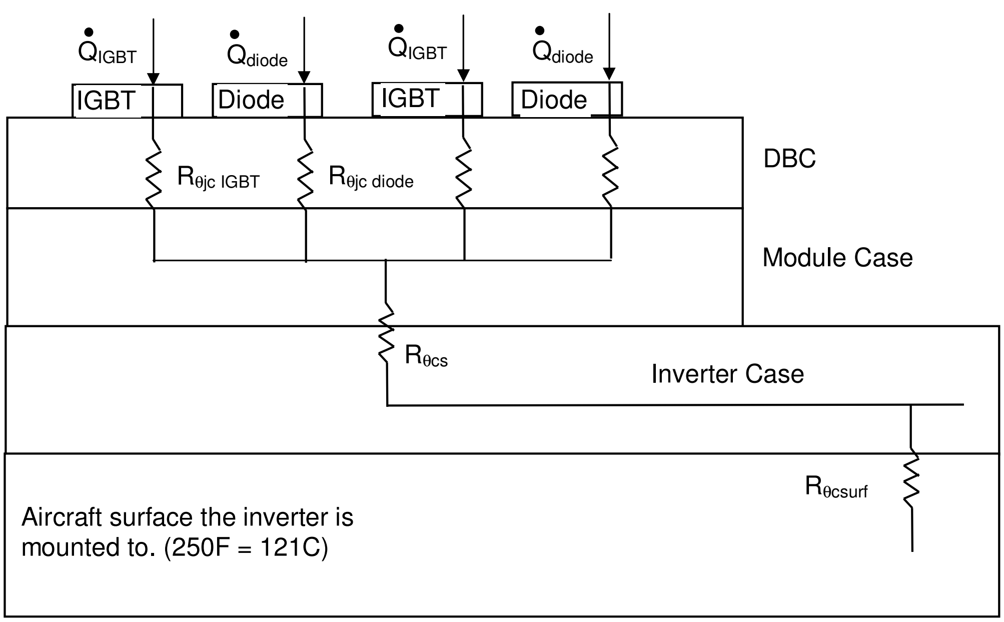

Fig. 3 Thermal circuit for a phase leg module

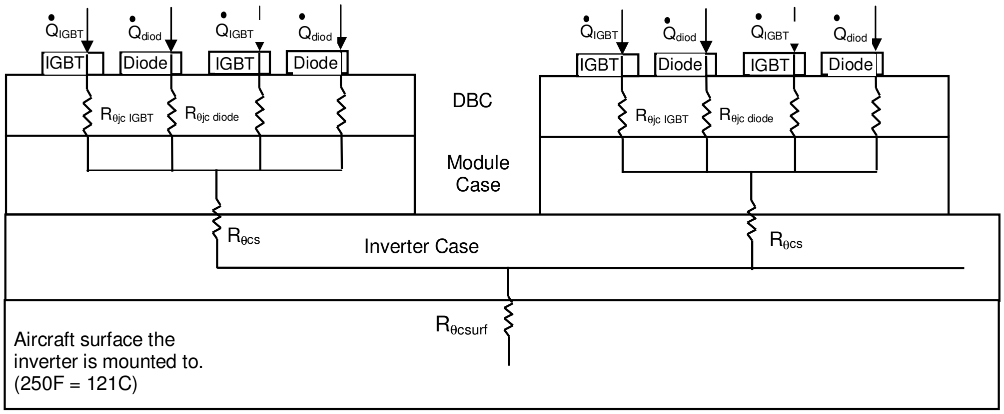

Fig. 4 Thermal circuit for two of the required three phase leg modules (International Rectifier GA500TD60U).

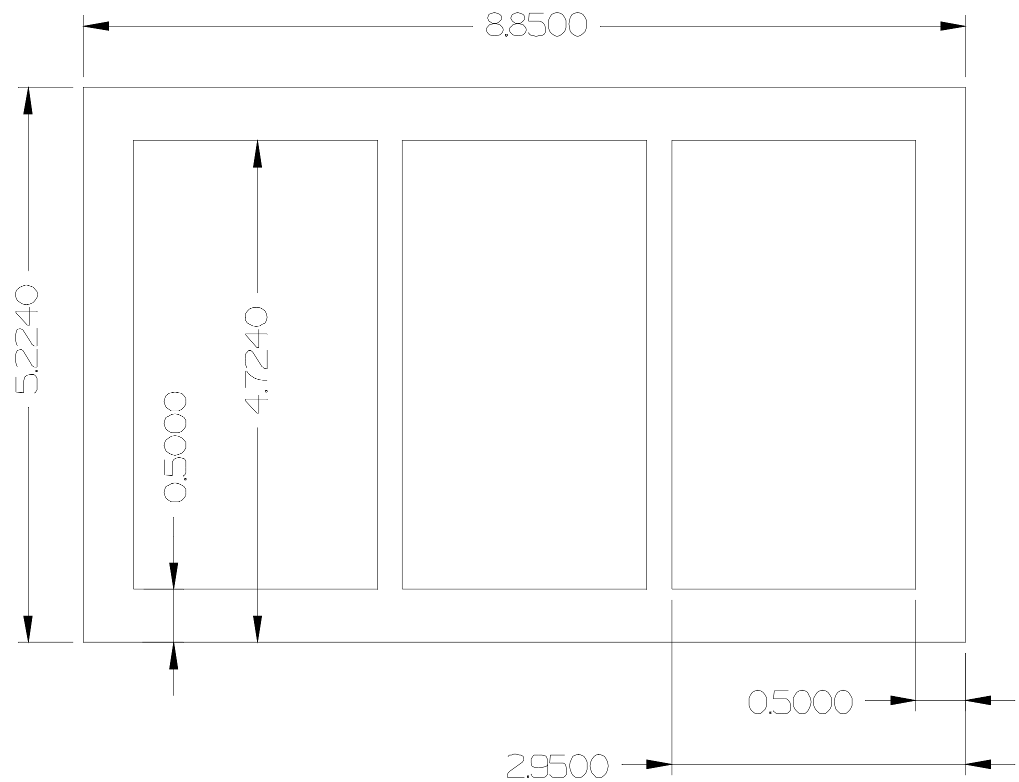

Fig. 5 Basic inverter foot print using three International Rectifier GA500TD60U phase leg modules.

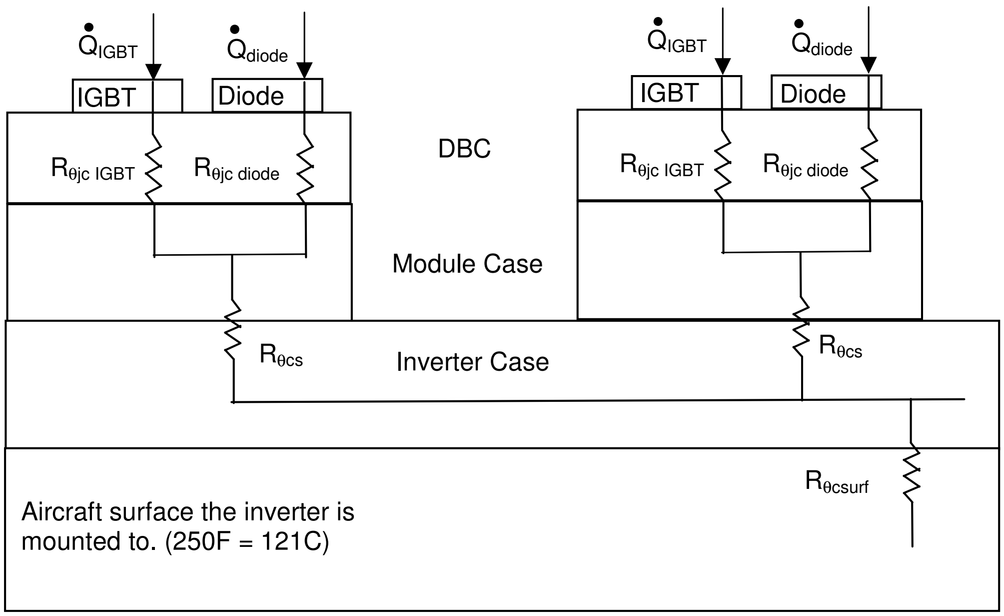

Fig. 6 Thermal circuit for two (one phase leg) of six switches (Toshiba MG800J1US51).

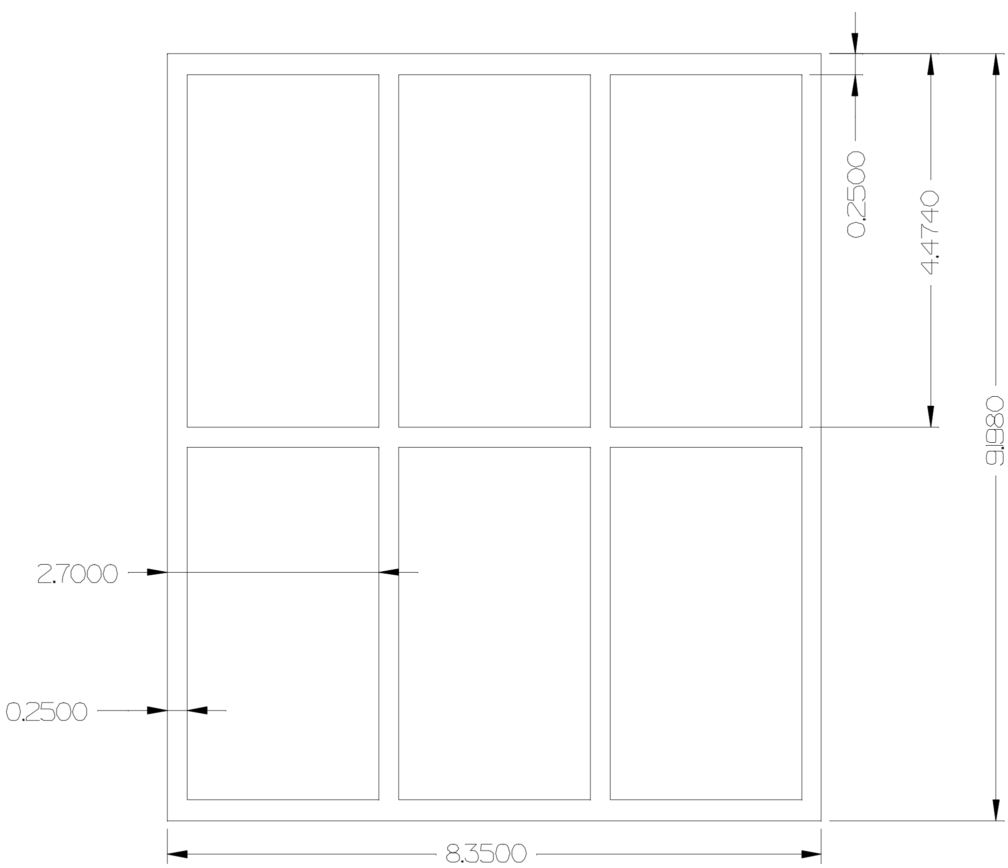

Fig. 7 Basic inverter foot print using six switches (Toshiba MG800J1US51).

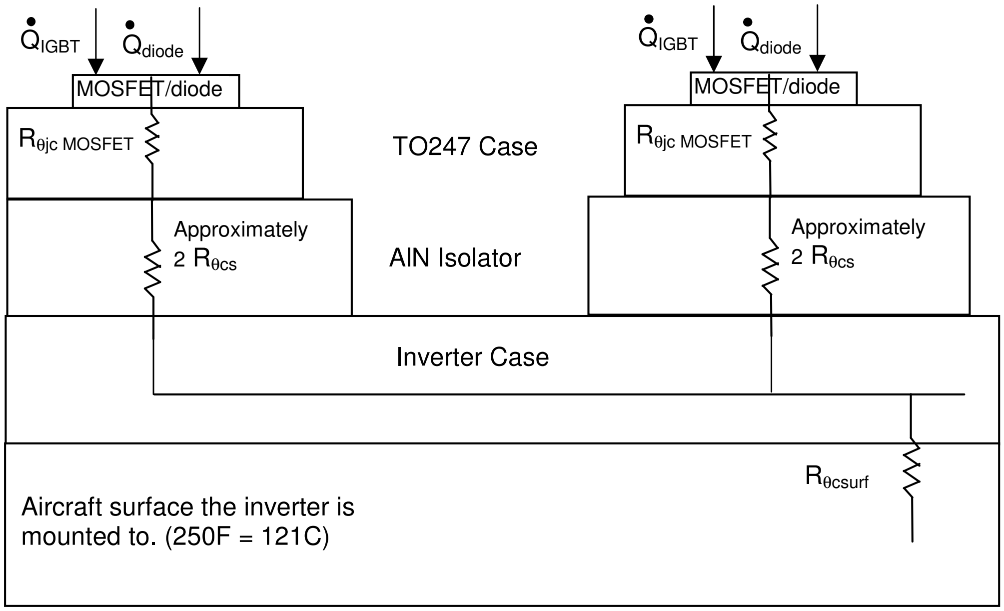

Fig. 8 Thermal circuit for two (one phase leg) of six switches (Infineon Cool MOSFET).