sribalaji asked on 24-12-11, 03:03: "*I learnt that maximum power transfer occurs when load impedance is equal to source impedance. Can anyone explain the reason why?*"

I'll try to explain it to you.
First I have to correct your statement slightly (**pico** also did it before):

The *load impedance* must be **conjugate** (not equal) with the *source impedance* (at used frequency), i.e. the *real components* of $\boldsymbol{Z_I}$ (=R_I) and of $\boldsymbol{Z_{LOAD}}$ (=R_{LOAD}) must be equal ($\boldsymbol{Z_I}$ and R_I stand for the internal impedance and resistance of the source, respectively) and the *imaginary components* must be equal as to their absolute value but they must have opposite signs (it results in nullifying the reactive part of the total impedance $\boldsymbol{Z}$ within the circuit).

Note: In general both the impedances are functions of frequency (the same holds for their real/imaginary components), so the condition can usually meet at only one (used) frequency (I suppose that a harmonic (=sine) waveform is used).

I'll offer you below the proof of the above statement:
Let's consider the simplest case first, i.e. when $\boldsymbol{Z_I} = R_I$ and $\boldsymbol{Z_{LOAD}} = R_{LOAD}$ (the imaginary components X_I and X_{LOAD} of both impedances are zero; see Fig.1):

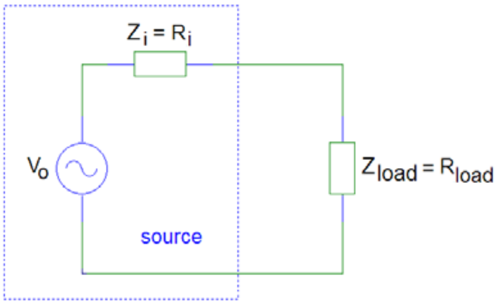

Fig. 1: Both internal and load impedances are purely active (real component ≠ 0; imaginary component = 0)

The current flowing through R_I and R_{LOAD} is as follows:

$$I = \frac{V_0}{R_I + R_{LOAD}} \tag{1}$$

(the voltage drop over R_{LOAD} equals $V = V_0 \cdot \frac{R_{LOAD}}{R_I + R_{LOAD}}$) (2)

The active power P_{LOAD} dissipated in R_{LOAD} ('*transferred*' from source to load) is given as:

$$P_{LOAD} = R_{LOAD} \cdot I^2 = V_0^2 \cdot \frac{R_{LOAD}}{(R_I + R_{LOAD})^2} \tag{3}$$

As P_{LOAD} is the simple function of R_{LOAD} shown above, it is easy to find its local extremes by

a) setting its first derivative to zero ($P_{LOAD}' = dP_{LOAD} / dR_{LOAD} = 0$), and

b) inspecting the polarity of its second derivative to find out if the extreme found is maximum ($P_{LOAD}'' < 0$) or minimum ($P_{LOAD}'' > 0$):

ad a) $$\frac{dP_{LOAD}}{dR_{LOAD}} = V_0^2 \cdot \frac{(R_I + R_{LOAD})^2 - 2R_{LOAD} \cdot (R_I + R_{LOAD})}{(R_I + R_{LOAD})^4} = V_0^2 \cdot \frac{R_I^2 - R_{LOAD}^2}{(R_I + R_{LOAD})^4} = 0 \qquad (4)$$

Because $R_I^2 - R_{LOAD}^2$ equals $(R_I - R_{LOAD}) \cdot (R_I + R_{LOAD})$, the expression can be simplified as follows (both R_I and R_{LOAD} are **always positive**):

$$\frac{dP_{LOAD}}{dR_{LOAD}} = P_{LOAD}{}' = V_0^2 \cdot \frac{R_I - R_{LOAD}}{(R_I + R_{LOAD})^3} = 0 \qquad (5)$$

The equation holds true if the numerator is zero, then its solution is:

$$\boldsymbol{R_{LOAD}} = \boldsymbol{R_I} \qquad (6)$$

ad b) $$\frac{d^2P_{LOAD}}{dR_{LOAD}^2} = P_{LOAD}{}'' = V_0^2 \cdot \frac{-(R_I + R_{LOAD})^3 - 3 \cdot (R_I - R_{LOAD}) \cdot (R_I + R_{LOAD})^2}{(R_I + R_{LOAD})^6} = \qquad (7)$$

$$= V_0^2 \cdot \frac{-(R_I + R_{LOAD}) - 3 \cdot (R_I - R_{LOAD})}{(R_I + R_{LOAD})^4} = V_0^2 \cdot \frac{2R_{LOAD} - 4R_I}{(R_I + R_{LOAD})^4} \qquad (8)$$

Substituting $\boldsymbol{R_{LOAD} = R_I}$, we obtain:

$$P_{LOAD}{}''_{(@ R_{LOAD} = R_I)} = V_0^2 \cdot \frac{-2R_I}{(2R_I)^4} = -\frac{V_0^2}{8R_I^3} < 0 \qquad (9)$$

This result (9) determines that the extreme found at $R_{LOAD} = R_I$ (6) is the **maximum of $\boldsymbol{P_{LOAD}}$ with respect to $\boldsymbol{R_{LOAD}}$** and the maximum itself equals:

$$P_{LOAD\_MAX} = \frac{V_0^2}{4R_I} = \frac{V_0^2}{4R_{LOAD}} = \frac{\left(\frac{V_0}{2}\right)^2}{R_{LOAD}} \qquad (10)$$

A slightly more complicated situation takes place if the imaginary components of impedances are not zero – the current is determined by the absolute value of the total impedance $\boldsymbol{Z}$ within the circuit:

$$\boldsymbol{Z} = \boldsymbol{Z_I} + \boldsymbol{Z_{LOAD}} = (R_I + R_{LOAD}) + \mathbf{j}\,(X_I + X_{LOAD}), \qquad (11)$$

where **j** stands for the imaginary unit ($\sqrt{(-1)}$)

The absolute value of the complex quantity $\boldsymbol{Z}$: $|\boldsymbol{Z}| = \sqrt{(R_I + R_{LOAD})^2 + (X_I + X_{LOAD})^2}$ (12)

So the current equals:

$$I = \frac{V_0}{|Z|} = \frac{V_0}{\sqrt{(R_I + R_{LOAD})^2 + (X_I + X_{LOAD})^2}} \tag{13}$$

The active power P_{LOAD} dissipated in R_{LOAD} ('*transferred*' from source to load) will be:

$$P_{LOAD} = R_{LOAD} \cdot I^2 = V_0^2 \cdot \frac{R_{LOAD}}{(R_I + R_{LOAD})^2 + (X_I + X_{LOAD})^2} \tag{14}$$

We have a function of **two variables** R_{LOAD} and X_{LOAD} now. Calculating its first partial derivative with respect to R_{LOAD} and setting it to zero we obtain:

$$\frac{\partial P_{LOAD}}{\partial R_{LOAD}} = V_0^2 \cdot \frac{(R_I + R_{LOAD})^2 + (X_I + X_{LOAD})^2 - 2R_{LOAD} \cdot (R_I + R_{LOAD})}{\left((R_I + R_{LOAD})^2 + (X_I + X_{LOAD})^2\right)^2} = 0 \tag{15}$$

Again, the equation holds true if the numerator is zero:

$$(R_I + R_{LOAD})^2 + (X_I + X_{LOAD})^2 - 2R_{LOAD} \cdot (R_I + R_{LOAD}) = 0, \quad \text{simplifying it:}$$

$$R_{LOAD}^2 = R_I^2 + (X_I + X_{LOAD})^2 \tag{16}$$

Similarly, calculating the first partial derivative with respect to X_{LOAD} now and setting it to zero we obtain:

$$\frac{\partial P_{LOAD}}{\partial X_{LOAD}} = V_0^2 \cdot \frac{-2R_{LOAD} \cdot (X_I + X_{LOAD})}{\left((R_I + R_{LOAD})^2 + (X_I + X_{LOAD})^2\right)^2} = 0 \tag{17}$$

It means, that $(X_I + X_{LOAD})$ must be zero (for the whole expression to be zero), therefore:

$$X_{LOAD} = -X_I \tag{18}$$

Substituting this result (18) in the above (16) we obtain the **final conditions for max. power transfer**:

$$\boldsymbol{R_{LOAD} = R_I} \quad \text{and} \tag{19}$$

$$\boldsymbol{X_{LOAD} = -X_I} \tag{20}$$

Let's believe that the second derivatives are less than zero ;-) (you can rely on it)
Well, that's it...

I believe it's helped you to understand "*the reason why*" (what you have asked).

Best Regards,
Eric

Appendix

Rules needed to calculate derivatives in the proof:

(independent variable here ... x)

a) $(\mathbf{constant})' = \frac{d\,\mathbf{constant}}{dx} = 0$

b) $(\mathbf{constant} \cdot x^n)' = \frac{d\left(\mathbf{constant} \cdot x^n\right)}{dx} = \mathbf{constant} \cdot n \cdot x^{n-1}$

c) $(u(x) \cdot v(x))' = \frac{d\,(u(x) \cdot v(x))}{dx} = u'(x) \cdot v(x) + u(x) \cdot v'(x) = v(x) \cdot \frac{du(x)}{dx} + u(x) \cdot \frac{dv(x)}{dx}$

d) $\left(\frac{u(x)}{v(x)}\right)' = \frac{d\left(\frac{u(x)}{v(x)}\right)}{dx} = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v^2} = \frac{v(x) \cdot \frac{du(x)}{dx} - u(x) \cdot \frac{dv(x)}{dx}}{v^2}$

e) $u''(x) = \frac{d}{dx} u'(x)$

Conditions for local extremes of a function u(x):

Maximum:

1. $u'(x) = \frac{du(x)}{dx} = 0$
2. $u''(x) = \frac{d^2u(x)}{dx^2} = \frac{du'(x)}{dx} < 0$

Minimum:

1. $u'(x) = \frac{du(x)}{dx} = 0$
2. $u''(x) = \frac{d^2u(x)}{dx^2} = \frac{du'(x)}{dx} > 0$