



**Universitat Autònoma  
de Barcelona**

**Advanced Design of Communication Systems**

**LAB 2: Slow wave structure and application to a  
miniaturized power divider**

Eduard Ametller Navas

David Eslava Sabaté

Master in Telecommunication Engineering

# Contents

<b>I</b>	<b>Introduction</b>	<b>1</b>
1.1	Inverter . . . . .	1
1.2	Power divider . . . . .	1
1.3	Realization with $\lambda/4$ transmission lines . . . . .	2
1.3.1	Case 1 . . . . .	2
1.3.2	Case 2 . . . . .	3
1.4	Resistive power divider . . . . .	3
1.4.1	Case 1 . . . . .	3
1.4.2	Case 2 . . . . .	4
1.5	Wilkinson Divider . . . . .	5
<b>II</b>	<b>Objectives and preliminary considerations</b>	<b>7</b>
2.1	Objective . . . . .	7
2.2	Theory . . . . .	7
2.3	Technology . . . . .	8
2.4	Specifications . . . . .	8
<b>III</b>	<b>Procedure</b>	<b>10</b>
3.1	Question 1 . . . . .	10
3.2	Question 2 . . . . .	10
3.3	Question 3 . . . . .	15
3.4	Question 4 . . . . .	19
3.5	Question 5 . . . . .	22

<b>IV</b>	<b>Comparisons</b>	<b>24</b>
3.5.1	$\lambda/4$ conventional line with $\lambda/4$ slow wave structure . . . . .	25
3.5.2	Visual comparison between electrical model and EM simulation . . . . .	26
3.5.3	The designed splitter can be used as a combiner to DC until 3GHz? . . . . .	27

## Part I

# Introduction

### 1.1 Inverter

Is a two port network, reciprocal, symmetrical and lossless which has a load impedance  $Z_L$ ,

$$Z_i = \frac{K^2}{Z_L}; Y_i = \frac{J^2}{Y_L} \quad (1)$$

where  $K^2 = J^2 = -Z_{12}^2$

we can implement an inverter with a  $\lambda/4$  transmission line as we see in Figure 1.

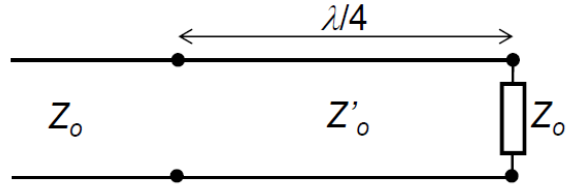


Figure 1

where  $K = Z'_o$  so

$$Z_{in} = \frac{Z_o'^2}{Z_o} \quad (2)$$

### 1.2 Power divider

A reciprocal 3 port device that if it has no losses and is symmetrical it has the following S matrix:

$$\mathbf{S} = \begin{pmatrix} 0 & \alpha & \alpha \\ \alpha & \gamma & -\gamma \\ \alpha & -\gamma & \gamma \end{pmatrix}$$

Figure 2

where  $|\alpha| = \frac{1}{\sqrt{2}}$  and  $|\gamma| = \frac{1}{2}$ . We can see that  $S_{11} = 0$ , so the input port is matched. When ports 2 and 3 are terminated all power returns to 1.

### 1.3 Realization with $\lambda/4$ transmission lines

This implementations are lossless but are monoband.

#### 1.3.1 Case 1

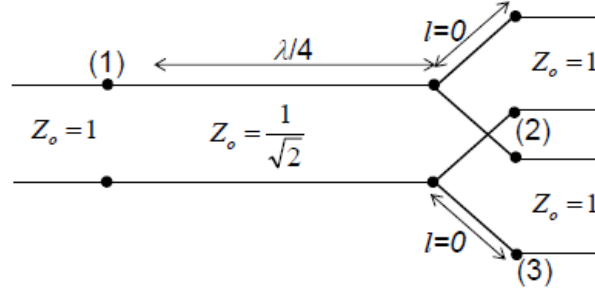


Figure 3

In order to port 1 to be matched we need  $Z_{in} = 50\Omega$  and we have  $Z_L = 25\Omega$  ( $50//50$ ) at the end of the  $\lambda/4$  transmission line. Following equation 2,  $Z'_0 = 35.35\Omega$ .

### 1.3.2 Case 2

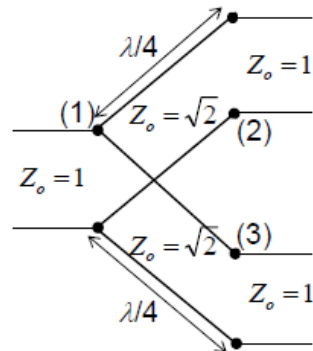


Figure 4

With this configuration  $Z_{in}$  of each line is  $\frac{Z_o'^2}{50}$  which are in parallel in port 1 where I have my  $Z_{in}$  that has to be  $50\Omega$ , so  $Z_o' = 70.71\Omega$ , easier to implement.

## 1.4 Resistive power divider

They work with independence of frequency but they have losses.

### 1.4.1 Case 1

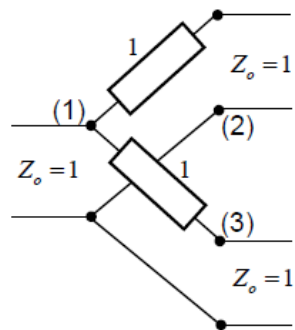


Figure 5

With S matrix:

$$\mathbf{S} = \frac{1}{2} \begin{pmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 1/4 & 1/4 \\ 1/2 & 1/4 & 1/4 \end{pmatrix}$$

Figure 6

#### 1.4.2 Case 2

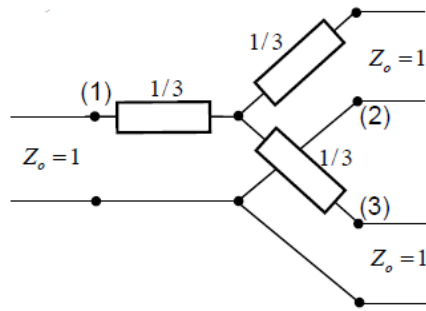


Figure 7

With S matrix:

$$\mathbf{S} = \frac{1}{2} \begin{pmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix}$$

Figure 8

Physical implementation with SMD resistances:

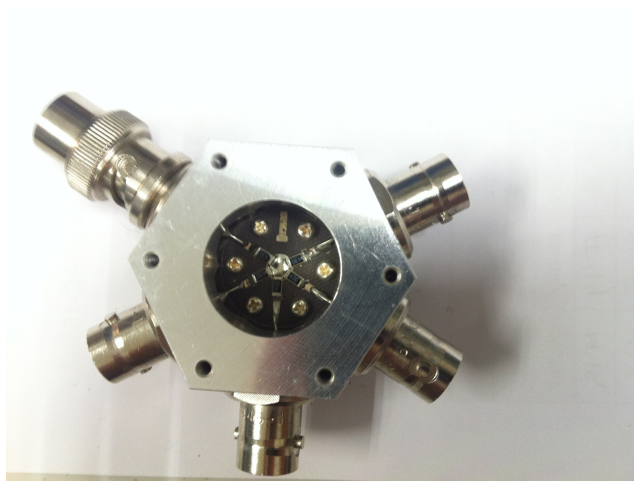


Figure 9

## 1.5 Wilkinson Divider

Previous dividers had some handicaps as:

Output ports (2) and (3) weren't isolated,  $S_{23} \neq 0$ .

Symmetry in the power splitting is destroyed if ports (2) and (3) are loaded asymmetrically.

The solution is the Wilkinson Divider who has the following S matrix:

$$\mathbf{S} = -\frac{j}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Figure 10

And is implemented like shown in Figures 11 and 12 which is similar to Figure4 but adding a resistor of  $100\Omega$  .



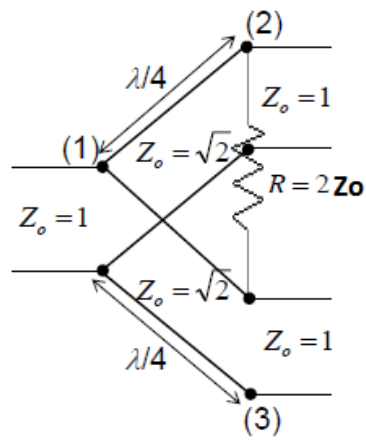


Figure 11

Physical microstrip implementation



Figure 12

## Part II

# Objectives and preliminary considerations

### 2.1 Objective

To design a  $35.35 \Omega$  transmission line with an electrical length of  $90^\circ$  (impedance inverter) based on a capacitively loaded transmission line with a slow wave factor of  $\frac{1}{2}$ , and to use it for the design of a power divider.

### 2.2 Theory

A power divider with an input and two output ports can be implemented by using an impedance inverter with  $K = 35.35\Omega$ , namely, a  $90^\circ$  line with a  $35.35\Omega$  characteristic impedance. In order to reduce the size of the device, we can replace the ordinary  $90^\circ$  line with a capacitively loaded line.

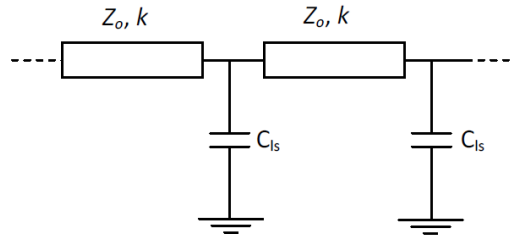


Figure 13

And, to a first order approximation, this structure can be modelled as:

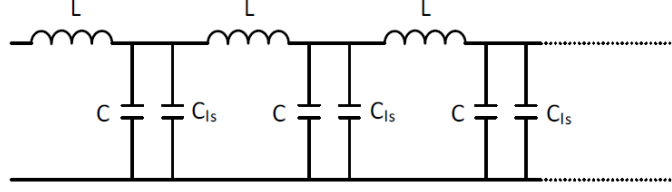


Figure 14

Relevant formulas for design purposes are:

Bragg frequency:

$$f_c = \frac{1}{\pi \sqrt{L(C + C_{ls})}} \quad (3)$$

Characteristic impedance:

$$Z_B = \sqrt{\frac{L}{C + C_{ls}}} \quad (4)$$

Phase shift per cell<sup>1</sup>:

$$\phi_{Cell} = \omega \sqrt{L(C + C_{ls})} \quad (5)$$

Standing wave ratio:

$$SWR = \frac{v_p}{v_{po}} = \frac{l/\sqrt{LC}}{l/\sqrt{L(C + C_{ls})}} = \frac{1}{\sqrt{1 + \frac{C_{ls}}{C}}} \quad (6)$$

## 2.3 Technology

We work with microstrip technology, on a h=50mils substrate with  $\varepsilon_r = 10.2$ .

## 2.4 Specifications

The following set of specifications applies:

- Operating frequency: 1GHz
- $Z_B = 35.35 \, \Omega$

---

<sup>1</sup>In our case it will be 30° and 10°.

- $\text{SWR} = 1/2$
- Consider two cases:
  - 3 cells each with a phase shift of  $30^\circ$ .
  - 9 cells each with a phase shift of  $10^\circ$ .

## Part III

# Procedure

### 3.1 Question 1

**Solve the previous equations with the above specifications for the two cases.**

We have made a program that using equations 3, 4, 5, 6, 7 and 8 and giving the specifications we obtain the desired values needed for the design. See Annex 1 Calculations.

>> For the unit cell of  $30^\circ$  :

$C=2.945833\text{nF}$  ;  $C_{ls}=0.5893446\text{pF}$  ;  $L=1.768034\text{pH}$

$\beta l= 25.981^\circ$  ;  $f_c= 4.4102\text{GHz}$  ;  $Z_0= 0.024499\Omega$

>> For the unit cell of  $10^\circ$  :

$C=0.9819444\text{nF}$  ;  $C_{ls}=0.1964482\text{pF}$  ;  $L=0.5893446\text{pH}$

$\beta l= 8.6603^\circ$  ;  $f_c= 13.231\text{GHz}$  ;  $Z_0= 0.024499\Omega$

$$\beta l = \omega \sqrt{LC} \quad (7)$$

$$Z_0 = \sqrt{\frac{L}{C}} \quad (8)$$

Where L and C are showed in Figure 14 .

### 3.2 Question 2

**Simulate with ADS the resulting circuit model and verify the characteristics of the inverter.**

With the values obtained in Question 1 and de model in 14 we proceed to simulate electrically this model for 3 and 9 cells:

>> **For the unit cell of 30° :**

Figure 15 shows 3 cells electric model in ADS

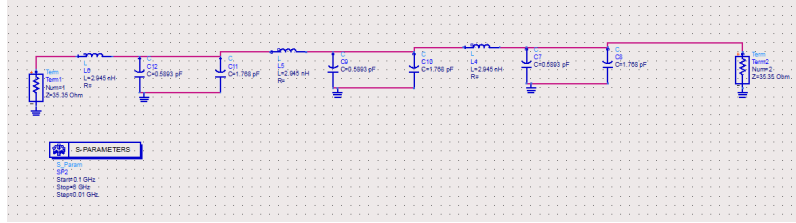


Figure 15

In Figure16 we can observe a good transmission until 3.3GHz but  $f_c$  should be 4.4102GHz. We can also see that the transmission pole is displaced from 1GHz to 1.919GHz.

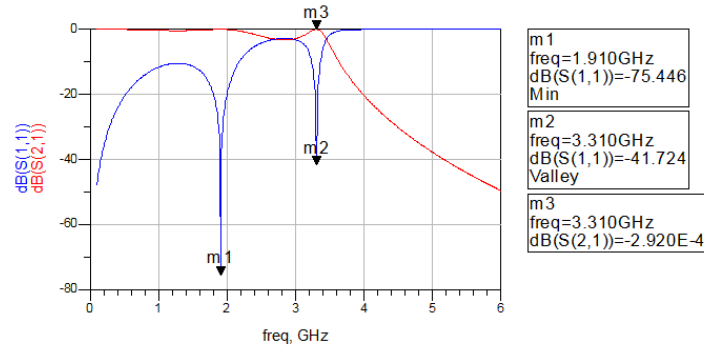


Figure 16

we also see in Figure17 that the total  $\beta l$  is  $90^\circ = \frac{\pi}{2} = \frac{\lambda}{4}$  as we wanted.

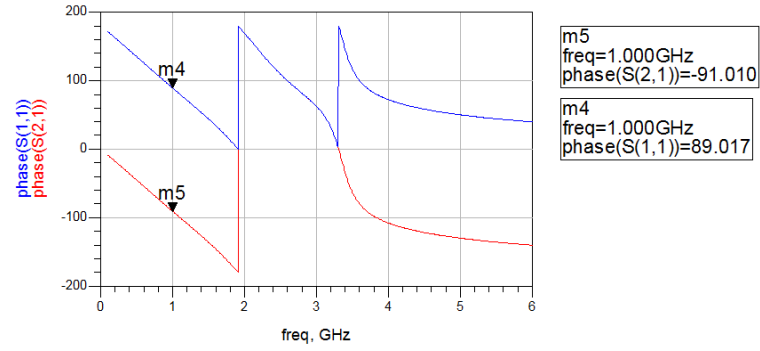


Figure 17

Now that we have this  $\lambda/4$  inverter implemented with periodic structures in order to decrease its physical dimensions we use it as part of a power divider as the one seen in Figure 3. simulated in Figure 18.

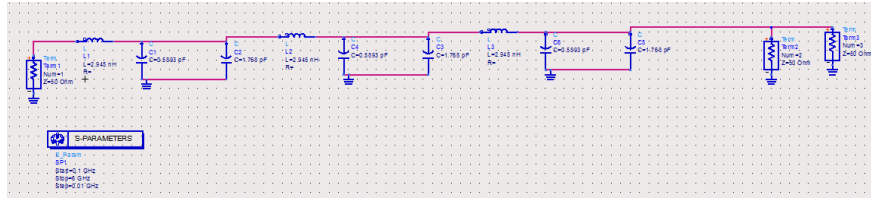


Figure 18

As we can see in Figure 19 insertion losses from port 1 to 2 is 3dB, the same from port 1 to 3. This shows us how half the power is going from 1 to 2 and the other half from 1 to 3. So its perfectly working as a power divider.

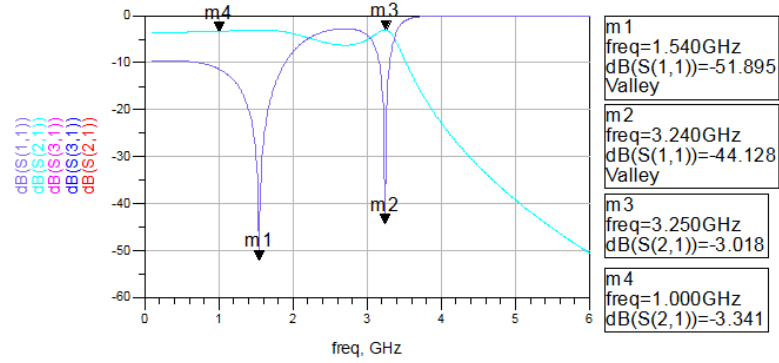


Figure 19

>> For the unit cell of  $10^0$  :

Figure 20 shows 9 cells electric model in ADS



Figure 20

In Figure21 we can observe a good transmission until 11.28GHz but  $f_c$  should be 13.231GHz. We can also see that the transmission pole is displaced from 1GHz to 2GHz but we have 20dB of return losses in 1GHz which is very good.

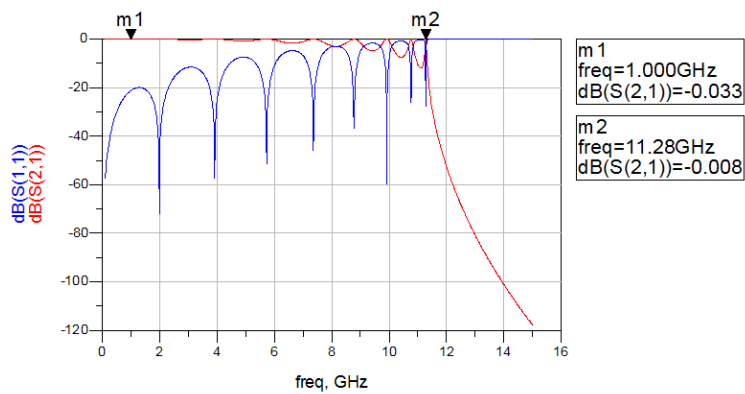


Figure 21



We also see in Figure22 that the total  $\beta l$  is  $90^\circ = \frac{\pi}{2} = \frac{\lambda}{4}$  as we wanted.

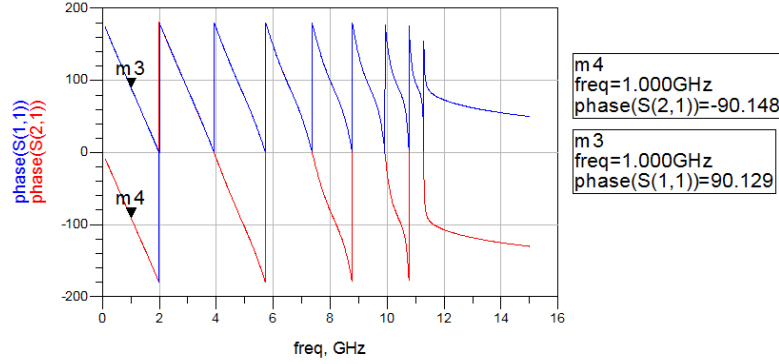


Figure 22

Now that we have this  $\lambda/4$  inverter implemented with periodic structures in order to decrease its physical dimensions we use it as part of a power divider as the one seen in Figure 3. simulated in Figure 23.

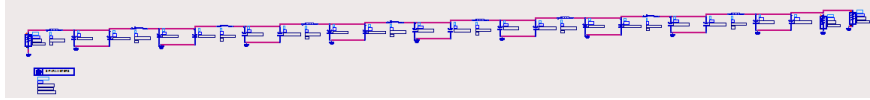


Figure 23

As we can see in Figure 24 insertion losses from port 1 to 2 is 3dB, the same from port 1 to 3. This shows us how half the power is going from 1 to 2 and the other half from 1 to 3. So its perfectly working as a power divider.

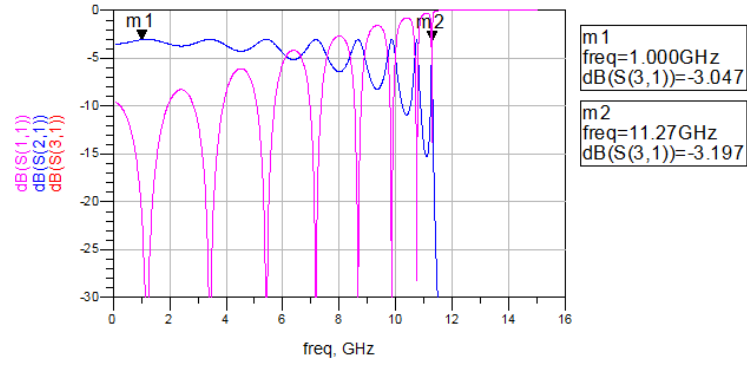


Figure 24

### 3.3 Question 3

Simulate with ADS by considering transmission line sections.

With the values obtained in Question 1 and de model in 13 we proceed to simulate electrically this model for 3 and 9 cells:

>> For the unit cell of 30° :

Figure 25 shows 3 cells electric model in ADS

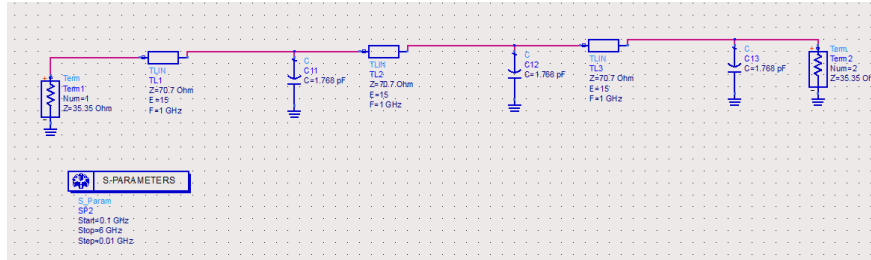


Figure 25

In Figure 26 we can observe a good transmission until 3.52 GHz but  $f_c$  should be 4.4102 GHz. We can also see that the transmission pole is displaced from 1 GHz to 1.950 GHz.

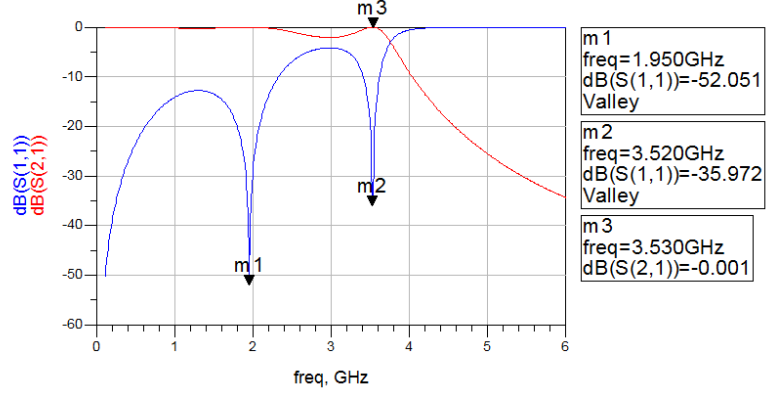


Figure 26

we also see in Figure27that the total  $\beta l$  is  $90^\circ = \frac{\pi}{2} = \frac{\lambda}{4}$  as we wanted.

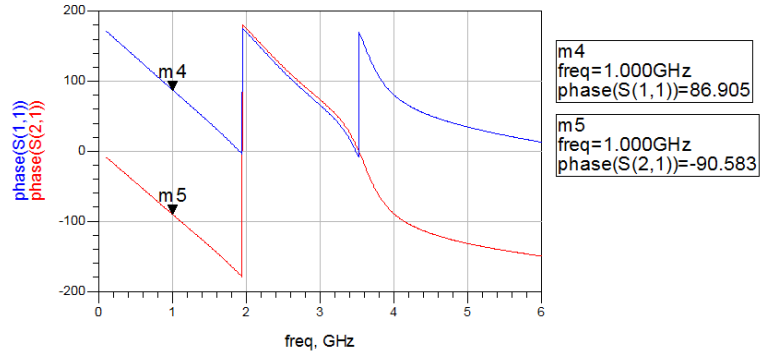


Figure 27

Now that we have this  $\lambda/4$  inverter implemented with periodic structures in order to decrease its physical dimensions we use it as part of a power divider as the one seen in Figure 3.simulated in Figure 28.

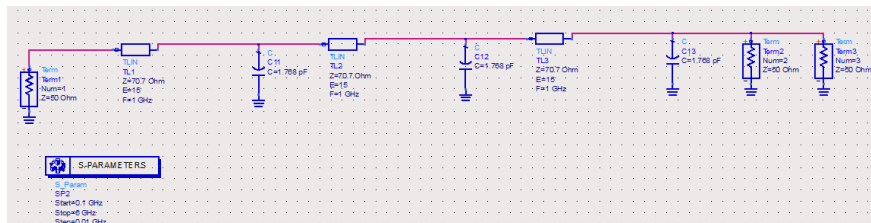


Figure 28

As we can see in Figure 29 insertion losses from port 1 to 2 is 3dB, the same from port 1 to 3. This shows us how half the power is going from 1 to 2 and the other half from 1 to 3. So its perfectly working as a power divider.

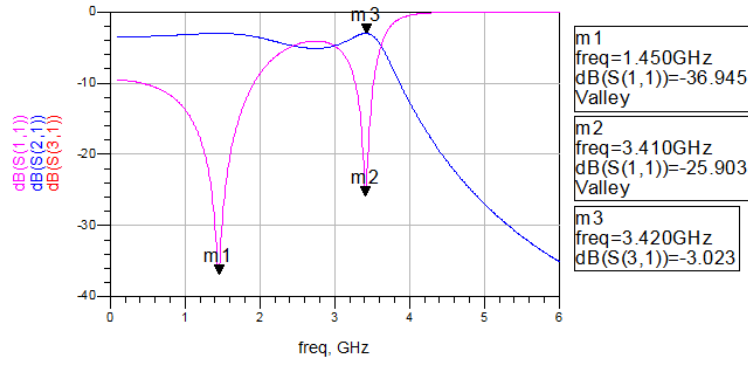


Figure 29

>> For the unit cell of  $10^\circ$  :

Figure 30 shows 9 cells electric model in ADS

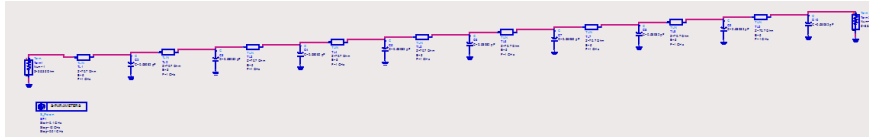


Figure 30

In Figure31 we can observe a good transmission until 12.3GHz but  $f_c$  should be 13.231GHz. We can also see that the transmission pole is displaced from 1GHz to 2GHz.

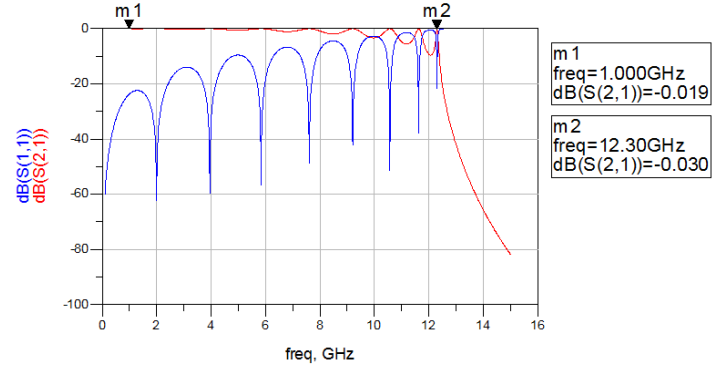


Figure 31

we also see in Figure32 that the total  $\beta l$  is  $90^\circ = \frac{\pi}{2} = \frac{\lambda}{4}$  as we wanted.

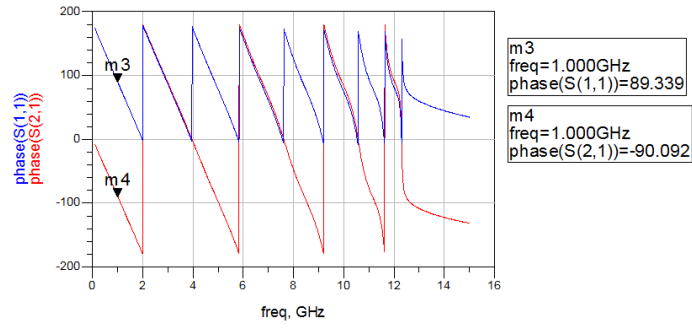


Figure 32

Now that we have this  $\lambda/4$  inverter implemented with periodic structures in order to decrease its physical dimensions we use it as part of a power divider as the one seen in Figure 3. simulated in Figure 33.

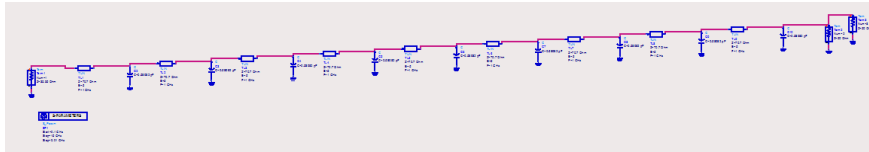


Figure 33

As we can see in Figure 34 insertion losses from port 1 to 2 is 3dB, the same from

port 1 to 3. This shows us how half the power is going from 1 to 2 and the other half from 1 to 3. So its perfectly working as a power divider.

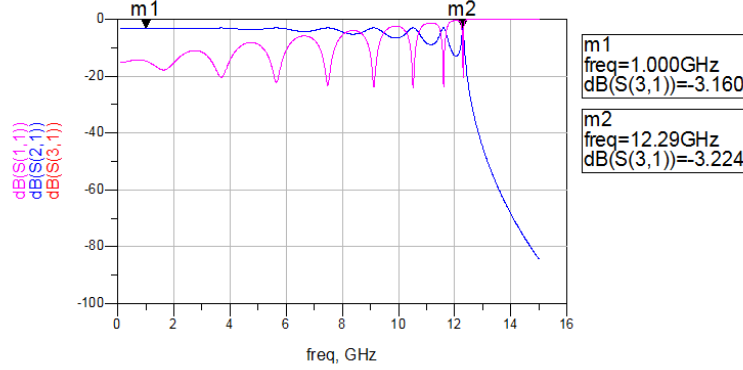


Figure 34

### 3.4 Question 4

**Simulate with Momentum by implementing the capacitors with parallel plate capacitances.**

We need to implement plate capacitances for the values of  $C_{ls}=0.5893446\text{pF}$ ,  $-1.8j^2$  in Smith chart, for the unit cell of  $30^\circ$  and  $C_{ls}=0.1964482\text{pF}$ ,  $-5.3967j$  in Smith chart, for the  $10^\circ$  unit cell.

We start with a width of 3.5mm and a height of 3.7mm, we continue varying the values of height and width until achieving a good result.

In order to help us reduce the time of interactions and have a guide of how to vary the parameters we create a program in Matlab which given at least 3 points of table in Figure 35 can predict and tell us the W and L needed to implement the desired capacitance. This program can be seen in Annex 2 Prediction algorithm.

---

<sup>2</sup>This conversion is made by  $Z_{in-smith} = \frac{1}{jC\omega 50}$  where  $\omega = 2\pi \cdot 10^9$ .

	Width(mm)	Height(mm)	Imaginary Part
	3,5	3,7	2,073
	3,5	2	3,389
	3,5	4	1,935
	3,5	6,3	1,242
	3,5	5,6	1,4
CLS 30	3,5	4,35	1,812
	3,5	3	2,482
	3,5	1,8	3,66
	2	2	5,187
CLS 10	1,95	1,95	5,374

Figure 35

So for the  $30^0$  cells we have:



Figure 36

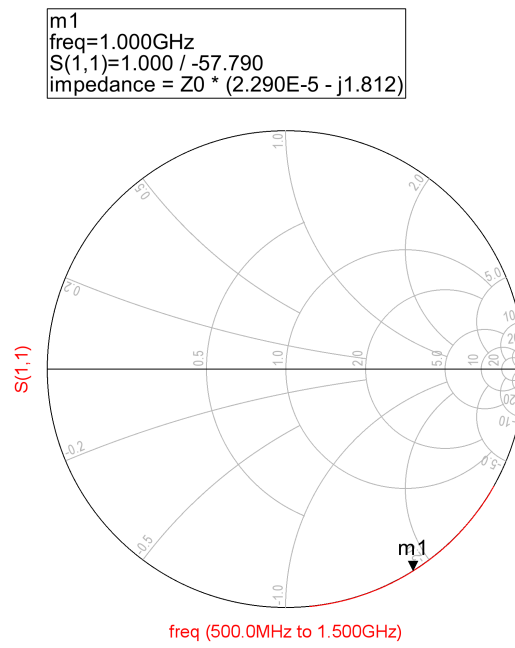


Figure 37

And for the  $10^0$  cells we have:

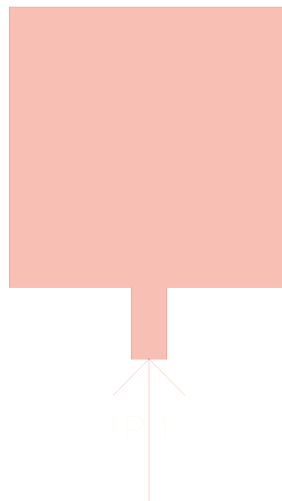


Figure 38



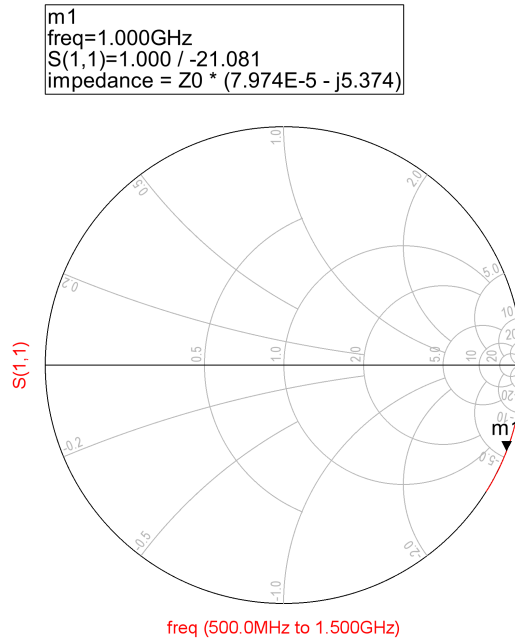


Figure 39

### 3.5 Question 5

Simulate with Momentum the power divider and determine its performance.

>> For the unit cell of 30° :

With the values of question 1 and Linecalc we exactly reproduce Figure28 in a layout with the given technology in Figure 40. Where (3) and (4) are the transmission lines and  $C_{ls}$  respectively. (1) are the access/feeding lines, usually of 7mm and (2) is a little line of  $25\Omega$  that is the impedance we see from the line of  $35.35\Omega$  to the left because of the parallel of both of  $50\Omega$ .

We decide to put capacitances up and down alternatively to avoid coupling between them.

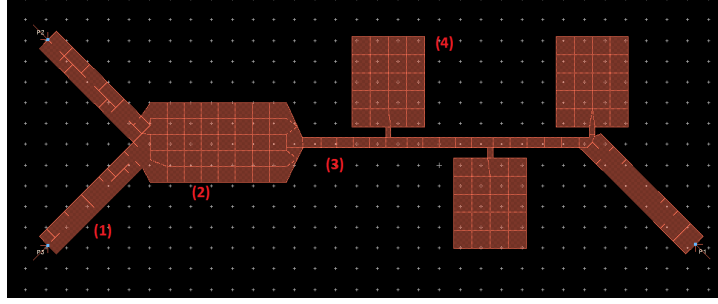


Figure 40

In Figure 41 we can observe a good transmission until 2.882GHz but  $f_c$  should be 4.4102GHz. We can also see that the transmission pole is displaced from 1GHz to 759.4MHz.

As we can see, insertion losses from port 1 to 2 is around 3dB, the same from port 1 to 3. This shows us that more or less half the power is going from 1 to 2 and the other half from 1 to 3. So its perfectly working as a power divider. Probably it's not exactly 3dB because of asymmetry in the layout and coupling between capacitances and access lines etc.

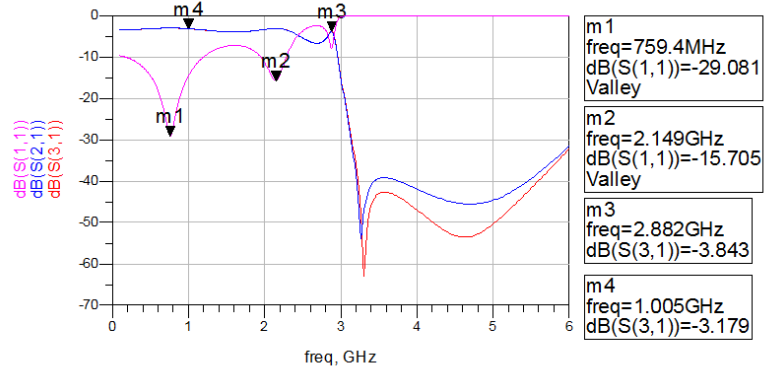


Figure 41

>> For the unit cell of  $10^\circ$  :

In Figure 42 we do the same with the values of the 9 cells case.

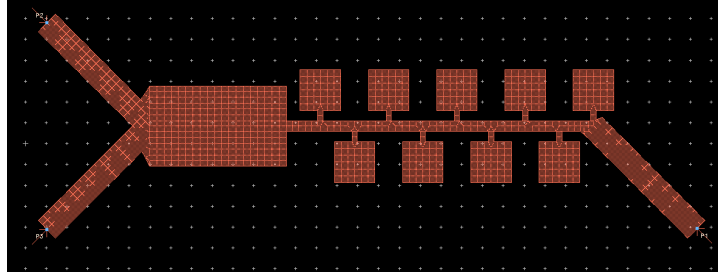


Figure 42

In Figure 43 we can observe a good transmission until 6.12GHz but  $f_c$  should be 13.231GHz. We can also see that the transmission pole is in 1GHz as is expected to be.

As we can see, insertion losses from port 1 to 2 is around 3dB, the same from port 1 to 3. This shows us that more or less half the power is going from 1 to 2 and the other half from 1 to 3. So its perfectly working as a power divider. Probably it's not exactly 3dB because of asymmetry in the layout and coupling between capacitances and access lines etc.

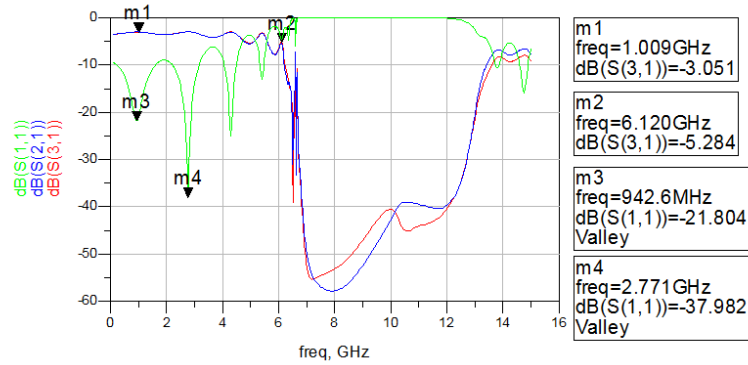


Figure 43

## Part IV

# Comparisons

### 3.5.1 $\lambda/4$ conventional line with $\lambda/4$ slow wave structure

As seen in Figure 44 we have accomplished the objective, the splitter is now 4mm smaller!

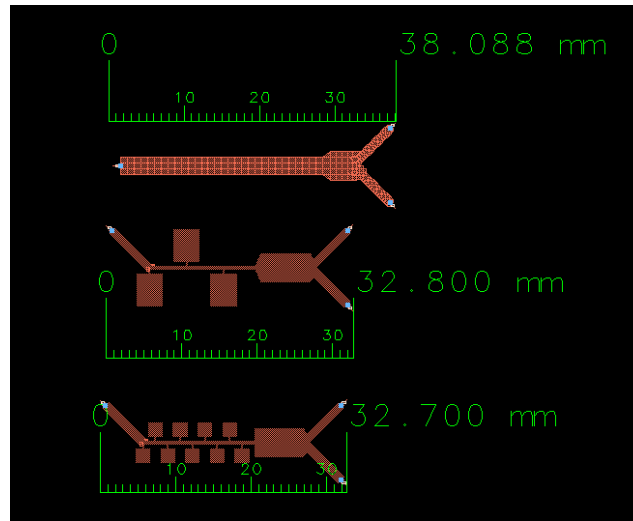


Figure 44

The comparison between 3 cells and the conventional line is shown in Figure 45 where we can see that the performance on insertion losses is very good in both cases, but conventional line has this performance for every frequency but slow wave structure just until around 3GHz.

The worst part are the return losses where for the conventional line at 1GHz we have 32dB and with slow wave structure at 1GHz we just have around 15dB. Moreover for conventional line return losses are 10dB at least until high frequencies, but with slow wave structure for a little bit more than 1GHz start to be less than 10dB and for more than 3GHz there is total reflection.

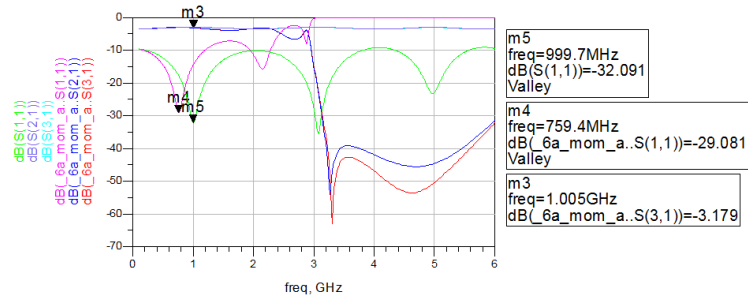


Figure 45

With 9 cells instead of 3 bandwidth seems to improve as well as return losses that now for 1GHz are 21dB and are more than 10dB until 3GHz and total reflection doesn't occur until around 6GHz.

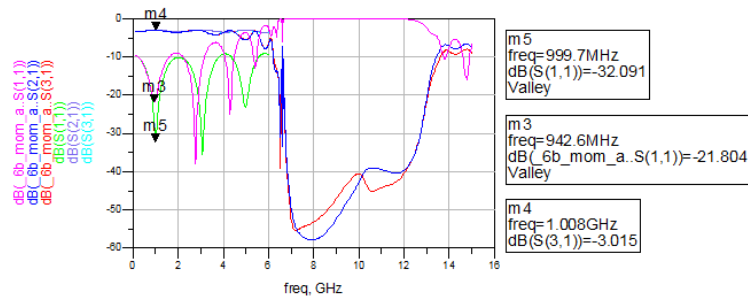


Figure 46

### 3.5.2 Visual comparison between electrical model and EM simulation

This has been commented all over this inform but here all together in a graph to see it more visual.

For 3 cells:

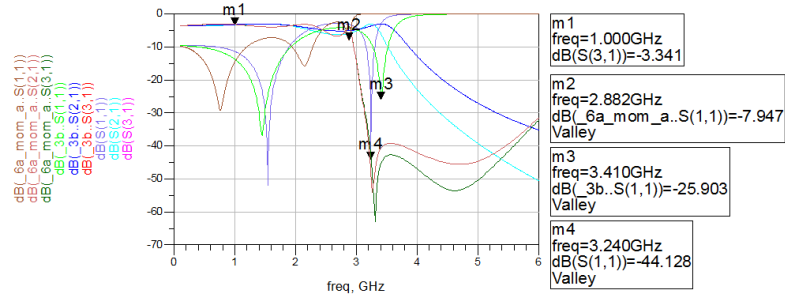


Figure 47

For 9 cells:

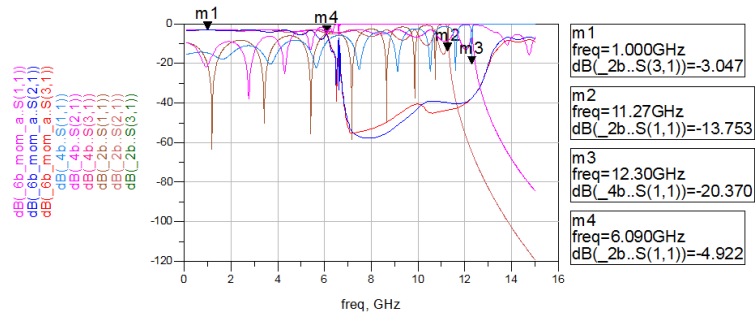


Figure 48

There are some frequency displacements and discrepancies between electric and EM simulations, that's because equations are approximations and the implementation of the layout is far from being optimal because there is coupling between capacitances and access lines, is not fully symmetrical etc.

### 3.5.3 The designed splitter can be used as a combiner to DC until 3GHz?

This design wouldn't be the optimal one to be used as a combiner so as we have said in the introduction we would need high return losses in ports 2 and 3 so the solution has to be Wilkinson or resistive. In port one we need more than 10dB that we achieve also.

The resistive splitter/combiner would be like:

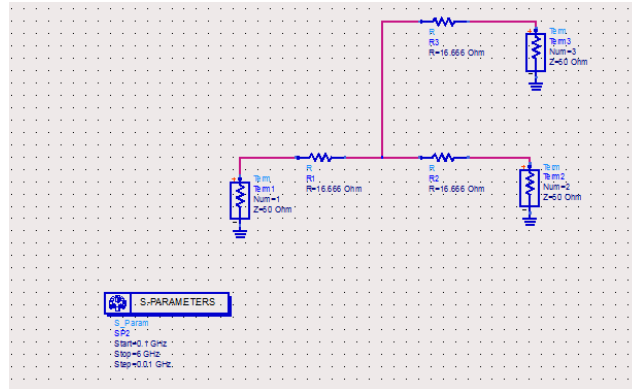


Figure 49

With this response we see we have infinite return losses but also have high insertion losses which is bad because we are losing a lot of power.

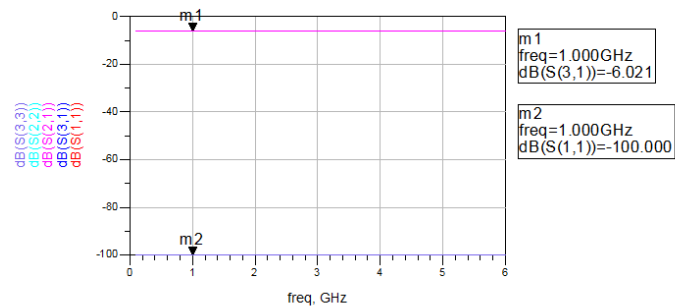


Figure 50

Comparison of return losses of conventional line with slow wave structure 3 cells:

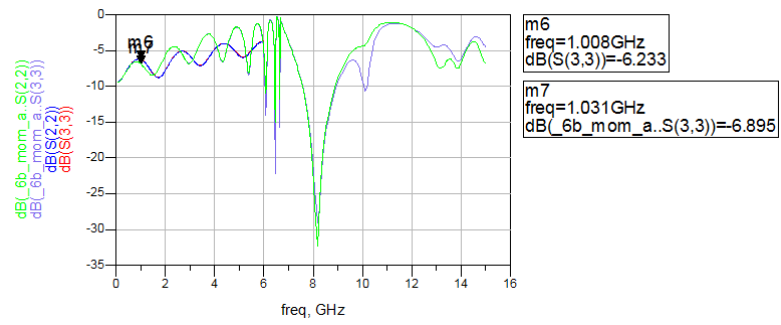


Figure 51

Comparison of return losses of conventional line with slow wave structure 9 cells:

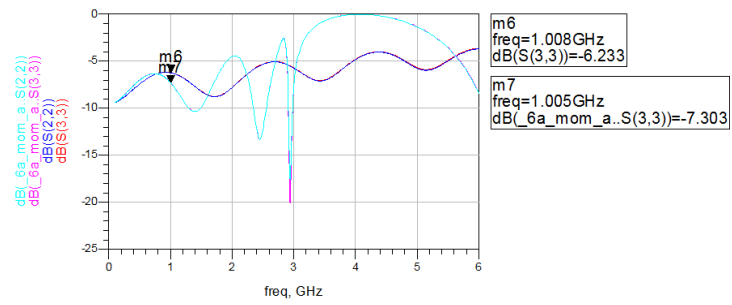


Figure 52



# Annex

## Calculations

```
1
2 function [ Sol ] = CalcParam(grades)
3
4 syms L C C_ls;
5
6 Zb = sqrt(L/(C+C_ls))-35.35;
7 phi_cell = 2*pi*(1E9)*sqrt(L*(C+C_ls))-grades*pi/180;
8 swr = 1/sqrt(1+C_ls/C)-1/2;
9
10 [SolA, SolB, SolC]=solve( Zb , phi_cell , swr , L, C, C_ls);
11
12 fprintf( '\nC: ');
13 disp(vpa(SolA(1),7))
14 fprintf( 'C_ls: ');
15 disp(vpa(SolB(1),7))
16 fprintf( 'L: ');
17 disp(vpa(SolC(1),7))
18
19 B1=2*pi*1E9*sqrt(SolC(1)*SolA(1))*180/pi;
20 fc=1/(pi*sqrt(SolC(1)*(SolA(1)+SolB(1))));
21 Zo=sqrt(SolC(1)/SolA(1));
22
23 fprintf( 'B1: ');
24 disp(vpa(B1,5))
25 fprintf( 'fc: ');
26 disp(vpa(fc,5))
27 fprintf( 'Zo: ');
28 disp(vpa(Zo,5))
29
30 end
```

## Prediction algorithm

We have developed an algorithm that predicts the value of the exact distance we need to fulfil an specification using 3 or more input data obtained empirically, the more data the more accurate the predictions will be.

```
1
2 function [ ] = Predict(Y,X,x)
3
```

```

4  for i = 1 : length(Y)
5      [YVm, ix] = max(Y);
6      YVm(i) = YVm;
7      XVm(i) = X(ix);
8      Y(ix) = -Inf;
9  end
10
11  YVm = fliplr(YVm);
12  XVm = fliplr(XVm);
13
14  plot(YVm,XVm, 'r')
15
16  if length(Y)<=3
17      Coeffs=polyfit(YVm,XVm, length(Y)-1);
18  else
19      Coeffs=polyfit(YVm,XVm,3);
20  end
21
22  Cub=0;
23  for n=1:length(Coeffs)
24      Cub=Cub+Coeffs(n)*x^(length(Coeffs)-n);
25  end
26
27  Cub
28  end

```