On efficient equalization for OFDM/OQAM systems

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***Abstract*—The need for equalization in an OFDM/OQAM system is studied. Analytical expressions for MMSE are expressed as a function of the number of subchannels, the order of equalizer and the channel noise level.**

***Index Terms*—OFDM, Offset QAM (OQAM), equalizer, MSE**

I. Introduction

ORTHOGONAL frequency division multiplexing (OFDM) is most often designed using QAM modulation of the subchannels, rectangular pulses with a guard interval to avoid intersymbol interference (ISI) and interchannel interference (ICI).

This scheme has a couple of drawbacks. First the insertion of guard interval reduces spectral efficiency since less time is available for transmission of useful information. This also leads to a lower power efficiency since the receiver filter is not matched to the transmitted pulseshape. Furthermore, the large sidelobe level makes the system spectrally incompact. To counteract these drawbacks, OFDM with band-limited shaping pulses was first suggested by Chang [1]. To satisfy orthogonality, offset QAM (OQAM) is used as modulation in the subchannels.

The lack of guard interval makes OFDM/OQAM more spectrally efficient, but multipath effects must be eliminated by an equalizer. Hirosaki [2] has proved that a single branch fractionally spaced equalizer is sufficient to eliminate ISI and ICI simultaneously. Tu [3] independently explored the MMSE equalization problem for single carrier OQAM transmission systems, which can be regarded as a special case.

In this paper, we first present the discrete baseband model for OFDM/OQAM systems with a single branch equalizer in section II. Then, in section III, we derive the real-valued objective function of a single branch equalizer for general OFDM/OQAM systems. Initially, we treat the in-phase and quadrature components separately, resulting in real-valued coefficients similar to Hirosaki's approach [2]. Next, we find that for non-weighting OFDM/OQAM systems, the received $T/2$ spaced sequence is wide sense stationary. This is presented in section IV.

In section V, we explore the relationship of minimum mean square error (MMSE) versus equalizer length. Using the stationarity result above we derive a normal equation similar to the one for a single carrier QAM transmission system. Some earlier results have been published on this problem [4][5], but only for special cases where the correlation matrix can be easily inverted. Some authors use gradient methods to find the optimal equalizer length for LMS equalizers dynamically [6][7]. To our knowledge, the explicit closed-form expression of MMSE versus equalizer length for general cases has not earlier been published. Since the implementation complexity and system latency are directly related to equalizer length, this result has both theoretical and practical value.

The general expressions found involve the frequency responses of both transmitter and receiver filters, as well as the channel, and will be extremely difficult to establish on closed form for actual cases. Instead, we propose an approximation which agrees well with numerical results.

II. Baseband model for OFDM/OQAM system with single branch equalizer

The time discrete baseband model for an OFDM/OQAM system with N subchannels is shown in Figure 1.

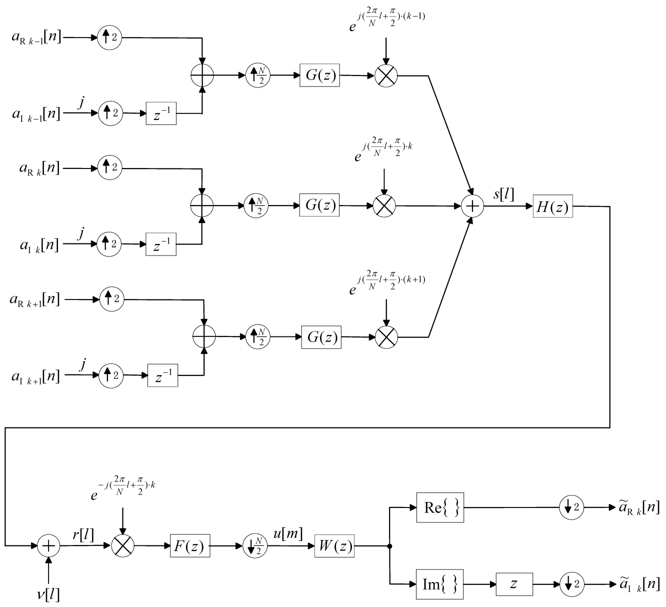

Fig. 1. Baseband model for OFDM/OQAM with equalizer

Usually the shaping pulses are band-limited to $[-1/T, 1/T]$. Then in the absence of carrier frequency offset, there exists overlap only between adjacent subchannels. Thus it is sufficient to consider one subchannel k and its adjacent subchannels $k \pm 1$. Each T seconds, the transmitter takes N complex

QAM symbols

$$a_k[n] = a_{\mathrm{R}\,k}[n] + j\, a_{\mathrm{I}\,k}[n],\ k = 0, 1, \cdots, N-1,$$

and generates an OFDM/OQAM waveform

$$\begin{aligned} s[l] = \sum_{m=0}^{N-1} \sum_{n=-\infty}^{\infty} \big(& a_{\mathrm{R}\,m}[n]\, g[l-nN] \\ &+ j\, a_{\mathrm{I}\,m}[n]\, g[l-nN-N/2]\big)\, e^{j(\frac{2\pi}{N}l+\frac{\pi}{2})m} \end{aligned}$$

that is input to the channel. The transmitter filer $g[l]$ operates with a sampling interval T/N, which is also the sampling interval of the receiver filter $f[l]$. Assuming a linear time invariant channel, this can thus be modelled as a discrete LTI system with impulse response $h[l]$ with the same sampling interval. We have also included an additive noise $\nu[l]$ in the channel. Thus the received signal can be written as

$$r[l] = \sum_{m=0}^{N-1} \sum_{n=-\infty}^{\infty} y_{m,n}[l]\, e^{j(\frac{2\pi}{N}l+\frac{\pi}{2})m} * h[l] + \nu[l],$$

where $y_{m,n}[l] = a_{\mathrm{R}\,m}[n]\, g[l-nN] + j\, a_{\mathrm{I}\,m}[n]\, g[l-nN-N/2]$.

In the receiver, for subchannel k, the signal is demodulated, filtered by the receiver filter $f[l]$ and down-sampled to yield a sequence with a sampling interval $T/2$:

$$\begin{aligned} u[m] =\ & r[l]\, e^{-j(\frac{2\pi}{N}l+\frac{\pi}{2})k} * f[l]\Big|_{l=m\frac{N}{2}} \\ =\ & \left(\sum_{s=k-1}^{k+1} j^{(s-k)} \sum_{n=-\infty}^{\infty} y_{s,n}[l]\, e^{j\frac{2\pi}{N}l(s-k)} \right) \\ & * \ h[l]\, e^{-j\frac{2\pi}{N}kl} * f[l] + \nu[l]\, e^{-j(\frac{2\pi}{N}l+\frac{\pi}{2})k} * f[l]\Big|_{l=m\frac{N}{2}} . \end{aligned} \tag{1}$$

Here, and in the rest of this paper, we omit the subscript k, since we only need to analyze one subchannel.

III. Optimal equalizer

For the equalizer $W(z)$ in Figure 1, we will assume a single branch, two-sided transversal filter with coefficients $w_k^*,\ k = -K, \cdots, K$. We will refer to the constant K as the *equalizer order*. Then the received symbols before the detector can be written as

$$\begin{aligned} \tilde{a}[n] &= \mathrm{Re}\left\{\mathbf{w}^H \mathbf{u}_{2n}\right\} + j\, \mathrm{Im}\left\{\mathbf{w}^H \mathbf{u}_{2n+1}\right\} \\ &= \mathbf{w}_r^T \mathbf{u}_{2n,r} + \mathbf{w}_i^T \mathbf{u}_{2n,i} + j\, \left(\mathbf{w}_r^T \mathbf{u}_{2n+1,i} - \mathbf{w}_i^T \mathbf{u}_{2n+1,r}\right), \end{aligned} \tag{2}$$

where $\{\cdot\}^H$ represents the conjugate-transpose, and

$$\begin{aligned} \mathbf{u}_n &= \left[u[n+K] \quad \cdots \quad u[n-K]\right]^T \\ \mathbf{u}_{n,r} &= \mathrm{Re}\left\{\mathbf{u}_n\right\},\ \mathbf{u}_{n,i} = \mathrm{Im}\left\{\mathbf{u}_n\right\} \\ \mathbf{w} &= \left[w_{-K} \quad \cdots \quad w_K\right]^T \\ \mathbf{w}_r &= \mathrm{Re}\left\{\mathbf{w}\right\},\ \mathbf{w}_i = \mathrm{Im}\left\{\mathbf{w}\right\}. \end{aligned}$$

The target of the equalizer will be to reduce disturbances to a minimum. This requirement can be formulated as a mean square error (MSE) minimization problem with objective function

$$J(\mathbf{w}) = E\left[\left|e[n]\right|^2\right] = E\left[\left|a[n] - \tilde{a}[n]\right|^2\right]. \tag{3}$$

Then substituting (2) into (3), we can rewrite the objective function as

$$\begin{aligned} J(\mathbf{w}) = &\begin{bmatrix}\mathbf{w}_r^T & \mathbf{w}_i^T\end{bmatrix} \begin{bmatrix}\mathbf{A}_1 & -\mathbf{B} \\ -\mathbf{B}^T & \mathbf{A}_2\end{bmatrix} \begin{bmatrix}\mathbf{w}_r \\ \mathbf{w}_i\end{bmatrix} \\ &- 2\begin{bmatrix}\mathbf{p}_1^T & \mathbf{p}_2^T\end{bmatrix} \begin{bmatrix}\mathbf{w}_r \\ \mathbf{w}_i\end{bmatrix} + \sigma_a^2, \end{aligned} \tag{4}$$

where

$$\begin{aligned} \mathbf{A}_1 &= E\left[\mathbf{u}_{2n,r}\mathbf{u}_{2n,r}^T + \mathbf{u}_{2n+1,i}\mathbf{u}_{2n+1,i}^T\right] \\ \mathbf{A}_2 &= E\left[\mathbf{u}_{2n,i}\mathbf{u}_{2n,i}^T + \mathbf{u}_{2n+1,r}\mathbf{u}_{2n+1,r}^T\right] \\ \mathbf{B} &= -E\left[\mathbf{u}_{2n,r}\mathbf{u}_{2n,i}^T - \mathbf{u}_{2n+1,i}\mathbf{u}_{2n+1,r}^T\right] \\ \mathbf{p}_1 &= E\left[\mathbf{u}_{2n,r}\, \mathrm{Re}\left\{a[n]\right\} + \mathbf{u}_{2n+1,i}\, \mathrm{Im}\left\{a[n]\right\}\right] \\ \mathbf{p}_2 &= E\left[\mathbf{u}_{2n,i}\, \mathrm{Re}\left\{a[n]\right\} - \mathbf{u}_{2n+1,r}\, \mathrm{Im}\left\{a[n]\right\}\right] \\ \sigma_a^2 &= E\left[\left|a[n]\right|^2\right]. \end{aligned} \tag{5}$$

These expressions are valid both for single carrier [3] and multicarrier [2] systems. The latter ones can even have frequency weighting, i.e. different transmitted signal power in each subchannel. For the rest of this paper we will assume *non-weighting* systems, i.e. each subchannel has the same signal power.

IV. Complex-valued objective function

In the following we assume (as is common for OFDM/OQAM systems) that the shaping filter $g[l]$ and receiver filter $f[l]$ are band-limited to $[-1/T, 1/T]$, and defined by identical real-valued symmetric pulses, i.e. $f[l] = g[l] = f[-l]$. Thus ICI comes only from adjacent subchannels. We further assume that

$$\begin{aligned} E\left[a_{\mathrm{R}\,m}[n_1]\, a_{\mathrm{R}\,k}[n_2]\right] &= E\left[a_{\mathrm{I}\,m}[n_1]\, a_{\mathrm{I}\,k}[n_2]\right] \\ &= \begin{cases}\sigma_a^2/2, & \text{if } m = k \text{ and } n_1 = n_2 \\ 0, & \text{otherwise,}\end{cases} \\ E\left[a_{\mathrm{R}\,m}[n_1]\, a_{\mathrm{I}\,k}[n_2]\right] &= 0,\ \forall\ m, k, n_1, n_2, \end{aligned} \tag{6}$$

where σ_a^2 is the average power of the sent QAM symbols.

Without loss of generality, we may assume that $\sigma_a^2 = 1$. The additive noise is assumed white, zero-mean with variance σ_ν^2. Note that we don't make any assumption about the distribution of additive noise and sent QAM symbols.

Then based on (1) and (5), after some derivation, it can be proved mathematically that $\mathbf{A}_1 = \mathbf{A}_2$ and $\mathbf{B}^T = -\mathbf{B}$ (cf. [8] for more details), and we can rewrite the objective function (4) in a complex-valued form as

$$J(\mathbf{w}) = \mathbf{w}^H \mathbf{R} \mathbf{w} - 2\mathrm{Re}\left\{\mathbf{p}^H \mathbf{w}\right\} + 1. \tag{7}$$

The correlation matrix $\mathbf{R}$ is Toeplitz-shaped and given by

$$\mathbf{R} = \begin{bmatrix} R[0] & \cdots & R[-2K] \\ \vdots & \ddots & \vdots \\ R[2K] & \cdots & R[0] \end{bmatrix}.$$

Here

$$R[\tau] = E\left[u[m]\, u^*[m+\tau]\right]$$
$$= \frac{1}{2}\int_{-1}^{1} G^2(f)\,\left|H_k(f)\right|^2\, e^{-j\pi f\tau}\, df + \sigma_\nu^2\, p_t\,[\frac{N}{2}\tau], \quad (8)$$

and $\mathbf{p} = \left[p[K]\cdots p[-K]\right]^T$, where

$$p\,[\tau] = E\left[u[2n+\tau]\,\mathrm{Re}\left\{a[n]\right\} - j\, u[2n+1+\tau]\,\mathrm{Im}\left\{a[n]\right\}\right]$$
$$= \frac{1}{2}\int_{-1}^{1} G^2(f)\, H_k(f)\, e^{j\pi f\tau}\, df. \quad (9)$$

We have also defined $p_t\,[l]$ as the overall response of the cascade of $g[l]$ and $f[l]$, i.e. $p_t[l] = g[l] * f[l]$, while $G(f)$ is just the frequency response of the down sampled shaping filter, i.e.

$$G(f) = \sum_{s=-\infty}^{\infty} g[s\frac{N}{2}]\, e^{-j\pi f s}, \quad (10)$$

and $H_k(f)$ is the equivalent channel response of subchannel k, which can be formulated as

$$H_k(f) = \sum_{l=-\infty}^{\infty} h[l]\, e^{-j\frac{2\pi}{N}(f+k)l}. \quad (11)$$

We note that the objective function (7) now has a form similar to the single carrier QAM case.

V. MMSE VERSUS EQUALIZER LENGTH

In order to minimize the implementation complexity and system latency, the equalizer order should not be larger than necessary. Thus it is important to know the minimum equalizer length for a given interference level.

By setting $\nabla J(\mathbf{w}) = 0$, we obtain the normal equation

$$\mathbf{R}\mathbf{w} = \mathbf{p}. \quad (12)$$

For $\mathbf{R}$ nonsingular, the optimal coefficient vector can be expressed as $\mathbf{w}_o = \mathbf{R}^{-1}\mathbf{p}$ and the corresponding minimum mean square error (MMSE) is

$$J_{min} = 1 - \mathbf{p}^H\mathbf{w}_o. \quad (13)$$

The further analysis is more conveniently performed in frequency domain. We note that $\mathbf{R}\mathbf{w}$ is a column vector with entries that can be viewed as the inner products between the rows of $\mathbf{R}$ and $\mathbf{w}$. Then by using Parseval's relation to rewrite these inner products in frequency domain, and taking DTFT of both sides of (12), we have

$$P_K(f) = \frac{1}{2}\int_{-1}^{1} R_K(f,f')\, W_K^*(f')\, df', \quad (14)$$

where

$$P_K(f) = \sum_{m=-K}^{K} p\,[-m]\, e^{-j\pi f m}$$
$$R_K(f,f') = \sum_{m=-K}^{K}\sum_{n=-K}^{K} R[m-n]\, e^{-j\pi f' n}\, e^{-j\pi f m}$$
$$W_K(f) = \sum_{n=-K}^{K} w_n^*\, e^{-j\pi f n}. \quad (15)$$

Note that $R_K(f,f')$ is actually the two-dimensional DTFT of the correlation matrix $\mathbf{R}$, and (14) is a Fredholm integral equation of the first kind.

Based on Parseval's relation, we can also rewrite (13) in frequency domain as

$$J_{min}(K) = 1 - \frac{1}{2}\int_{-1}^{1} P_K(-f)\, W_K(f)\, df. \quad (16)$$

A. MMSE for one-tap equalizer

The simplest possible equalizer has only a single tap. This corresponds to setting $K = 0$. In this case, $\mathbf{R} = R[0]$ is a scalar and can thus be easily inverted. Then based on (8) and (9), the optimal coefficient of one-tap equalizer can then be written as

$$w_0 = \frac{p[0]}{r[0]} = \frac{\int_{-1}^{1} G^2(f)\, H_k(f)\, df}{\int_{-1}^{1} G^2(f)\, |H_k(f)|^2\, df + 2\,\sigma_\nu^2}, \quad (17)$$

and using (13), we get

$$J_{min}(0) = 1 - \frac{\left|\int_{-1}^{1} G^2(f)\, H_k(f)\, df\right|^2}{2\,\int_{-1}^{1} G^2(f)\,\left|H_k(f)\right|^2\, df + 4\,\sigma_\nu^2}. \quad (18)$$

B. MMSE for infinite-tap equalizer

At the other extreme, we now consider the case of an infinite-tap equalizer. Substituting (9) into the expression for $P_K(f)$ in (15) and taking the limit, we have

$$P_\infty(f) = \frac{1}{2}\int_{-1}^{1} G^2(f')\, H_k(f')$$
$$\times\left(\lim_{K\to\infty}\sum_{m=-K}^{K} e^{-j\pi(f'+f)m}\right) df'$$
$$= G^2(f)\, H_k(-f). \quad (19)$$

Similarly by using (8), we write the two-dimensional DTFT of $\mathbf{R}$ for $K \to \infty$ as

$$R_\infty(f,f') = 2\, G^2(f')\,\left(|H_k(f')|^2 + \sigma_\nu^2\right)\,\delta(f+f'). \quad (20)$$

Then substituting (19) and (20) into (14), we have

$$W_\infty(f) = \frac{H_k^*(f)}{\left|H_k(f)\right|^2 + \sigma_\nu^2}. \quad (21)$$

At last by substituting (19) and (21) into (16), we obtain

$$J_{min}(\infty) = 1 - \frac{1}{2}\int_{-1}^{1}\frac{G^2(f)\,\left|H_k(f)\right|^2}{\left|H_k(f)\right|^2 + \sigma_\nu^2}\, df. \quad (22)$$

We can see that for $K \to \infty$, the optimal equalizer is not related to the shaping pulse $G(f)$, but the MMSE is still affected by different shaping pulses.

C. MMSE for finite-tap equalizer

Having found closed-form expression for the two extreme cases $K = 0$ and $K = \infty$, we will now attack the more difficult problem of finding a general expression for $J_{min}(K)$. Intuitively, $W_K(f)$ should be close to $W_\infty(f)$, thus we define

$$\Delta W_K(f) = W_K(f) - W_\infty(f). \tag{23}$$

Similarly, we further define

$$\begin{aligned}\Delta P_K(f) &= P_K(f) - P_\infty(f)\\ \Delta R_K(f,f') &= R_K(f,f') - R_\infty(f,f').\end{aligned} \tag{24}$$

Then based on the definitions in (23) and (24), the expression of MMSE in (16), and omitting the second order small term $\Delta P_K(-f)\,\Delta W_K(f)$, we have

$$\begin{aligned}J_{min}(K) \simeq J_{min}(\infty) - \frac{1}{2}\int_{-1}^{1}\big(&\Delta W_K(f)\,P_\infty(-f)\\ &+ \Delta P_K(-f)\,W_\infty(f)\big)\,df.\end{aligned} \tag{25}$$

Note that in (25), only $\Delta W_K(f)$ is unknown. Substituting (23) and (24) into (14), then subtracting $P_\infty(f)$ from both sides, and using (20) and (21), we have

$$\Delta W_K(f) = \frac{\Delta P_K^*(-f) - \frac{1}{2}\int_{-1}^{1}\frac{\Delta R_K^*(-f,f')\,H_k^*(f')}{|H_k(f')|^2+\sigma_\nu^2}\,df'}{G^2(f)\;(|H_k(f)|^2+\sigma_\nu^2)}. \tag{26}$$

Then substituting (19), (21) and (26) into (25), after some tedious but straightforward derivation, we find

$$J_{min}(K) \simeq J_{min}(\infty) + J_1 + J_2, \tag{27}$$

where

$$\begin{aligned}J_1 &= \mathrm{Re}\left\{\sum_{|m|=K+1}^{\infty} p^*[m]\int_{-1}^{1}\frac{H_k(f)\,e^{j\pi f m}}{|H_k(f)|^2+\sigma_\nu^2}\,df\right\}\\
J_2 &= -\frac{1}{4}\,\mathrm{Re}\left\{\sum_{m=-K}^{K}\sum_{|n-m|=K+1}^{\infty} R^*[n]\right.\\
&\quad\times\left(\int_{-1}^{1}\frac{H_k(f)\,e^{-j\pi f m}}{|H_k(f)|^2+\sigma_\nu^2}\,df\right)\\
&\quad\left.\times\left(\int_{-1}^{1}\frac{H_k^*(f)\,e^{-j\pi f(n-m)}}{|H_k(f)|^2+\sigma_\nu^2}\,df\right)\right\}\\
&\quad-\frac{1}{2}\sum_{m=K+1}^{\infty}\mathrm{Re}\left\{\left(\int_{-1}^{1}\frac{H_k(f)\,\cos(m\pi f)}{|H_k(f)|^2+\sigma_\nu^2}\,df\right)\right.\\
&\quad\left.\times\left(\int_{-1}^{1}G^2(f)\,H_k^*(f)\,\cos(m\pi f)\,df\right)\right\}\\
&\quad+\frac{1}{2}\sum_{m=K+1}^{\infty}\mathrm{Re}\left\{\left(\int_{-1}^{1}\frac{H_k(f)\,\sin(m\pi f)}{|H_k(f)|^2+\sigma_\nu^2}\,df\right)\right.\\
&\quad\left.\times\left(\int_{-1}^{1}G^2(f)\,H_k^*(f)\,\sin(m\pi f)\,df\right)\right\}.\end{aligned} \tag{28}$$

We have now got an approximate formula of MMSE for a finite-tap optimal equalizer. We can see that the MMSE for $1 \le K < \infty$ is composed of three terms. The first term is the MMSE of the infinite-tap equalizer. The second and third terms are related to $p[\tau]$ and $R[\tau]$ respectively.

D. Example: MMSE for two-path transmitting channel

As an example, we will consider a two-path transmitting channel with impulse response $h[l] = \delta[l] + \alpha\,e^{-j\varphi}\,\delta[l-\epsilon]$, where α is the amplitude attenuation factor, φ is the phase shift, and ϵ is the delay of the second path respectively. Here we assume $0 \le \alpha < 1$, and ϵ is a positive integer much smaller than N. The equivalent frequency response of subchannel k can thus be written as

$$H_k(f) = 1 + \alpha\,e^{-j(\frac{2\pi}{N}\,\epsilon f + \varphi_k)}, \tag{29}$$

where

$$\varphi_k = \frac{2\pi}{N}\,\epsilon\,k + \varphi. \tag{30}$$

The transmitter and receiver filters $g[l]$ and $f[l]$ are square root raised cosine pulses with a roll-off factor equal to one, i.e. $G(f) = \sqrt{2}\;\cos\left(\frac{\pi f}{2}\right)$. By substituting (29) into (18), we have

$$J_{min}(0) = \frac{\sigma_\nu^2\,\left[1 + (\epsilon/N)^2\;C_1\; + O\left((\epsilon/N)^4\right)\right]}{1+\alpha^2+2\,\alpha\,\cos(\varphi_k)+\sigma_\nu^2}, \tag{31}$$

where

$$\begin{aligned}C_1 &= \alpha\left(4\,\pi^2/3 - 8\right)\\ &\quad\times\left(\alpha/\sigma_\nu^2 - \frac{\cos(\varphi_k)}{1+\alpha^2+2\,\alpha\,\cos(\varphi_k)+\sigma_\nu^2}\right).\end{aligned} \tag{32}$$

Then substituting (29) into (22), we have

$$J_{min}(\infty) = \frac{\sigma_\nu^2\,\left[1 + (\epsilon/N)^2\;C_2 + O\left((\epsilon/N)^4\right)\right]}{1+\alpha^2+2\,\alpha\,\cos(\varphi_k)+\sigma_\nu^2}, \tag{33}$$

where

$$\begin{aligned}C_2 &= \alpha\left(4\,\pi^2/3 - 8\right)/\left(1+\alpha^2+2\,\alpha\,\cos(\varphi_k)+\sigma_\nu^2\right)^2\\ &\quad\times\left[\cos(\varphi_k)\left(1+\alpha^2+2\,\alpha\,\cos(\varphi_k)+\sigma_\nu^2\right)\right.\\ &\quad\left.+\;4\,\alpha\,\sin^2(\varphi_k)\right].\end{aligned} \tag{34}$$

By omitting the high order term $O\left((\epsilon/N)^4\right)$, we can write the maximum gain that can be acquired by increasing the number of equalizer taps as

$$G_{max} \stackrel{\text{def}}{=} \frac{J_{min}(0)}{J_{min}(\infty)} \simeq 1 + \frac{\epsilon^2\;(C_1 - C_2)}{N^2 + \epsilon^2\,C_2}. \tag{35}$$

First we note that G_{max} is decreasing with N, approaching 1 (0 dB) as $N \to \infty$. This is expected since a large N implies almost flat response in each subchannel, requiring only a one-tap equalizer. We also find that the numerator of (35) is increasing with decreasing noise power σ_ν^2, whereas the denominator is almost constant. This means that the convergence towards 0dB with increasing N is slower for lower σ_ν^2. In other words, the gain of using a multi-tap equalizer decreases with increasing σ_ν^2.

An example will illustrate this. In Figure 2, we show the behavior of G_{max} for subchannel $k = \frac{N}{2}$ for a case with $\alpha = 0.5$, $\epsilon = 1$, and $\varphi = 0$. Three different noise levels

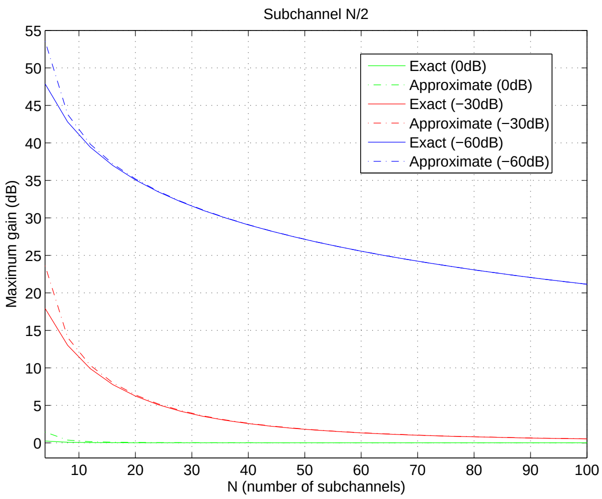

Fig. 2. The maximum gain acquired by multi-tap equalizer.

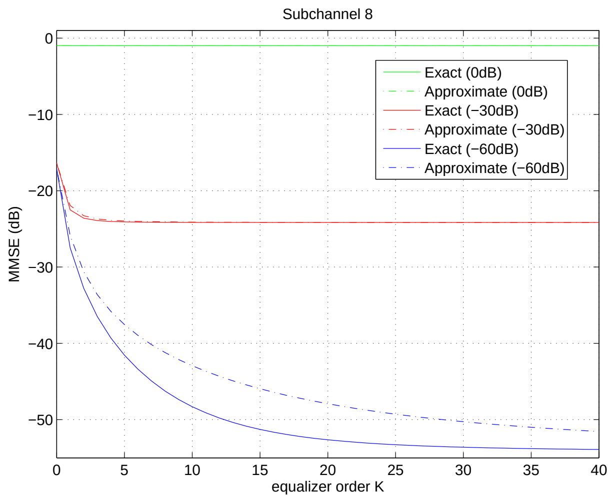

Fig. 3. MMSE versus equalizer order K (the curves for 0dB are overlapped).

are used, $\sigma_\nu^2 = 0$, -30 and -60dB. In the figure, the curves obtained by (35) are plotted together with exact curves found by numerical evaluation of the integrals in (18) and (22). We see that (35) gives a good approximation to G_{max}, especially for large N.

The quantity G_{max} is useful for determining when it is worthwhile to use multi-tap equalizer. To assess how large equalizer order is needed in a given situation, an expression for the MMSE vs. K is needed. An approximate value for this quantity can be found by first finding expressions for $R[\tau]$ and $p[\tau]$ by substituting (29) into (8) and (9) respectively. Then substituting this result into (27), and after some approximation (cf. [8] for more details), we get

$$J_{min}(K) \simeq J_{min}(\infty) + \frac{B}{K\,(K+1)\,N^2}, \tag{36}$$

where

$$\begin{aligned} B = &\frac{4\,\epsilon^2\,\alpha^2}{\pi\,\left(1+\alpha^2+2\,\alpha\,\cos\left(\varphi_k\right)+\sigma_\nu^2\right)^4} \\ &\times\left[1+\alpha^2+2\,\alpha\,\cos(\varphi_k)\,\left(1+\frac{2\,\left(2-\pi^2/3\right)\epsilon^2}{N^2}\right)\right. \\ &\left.+\sigma_\nu^2\right]\left[\left(1+\alpha^2+2\,\alpha\,\cos\left(\varphi_k\right)+\sigma_\nu^2\right)^2\right. \\ &\left.-\,4\,\sin^2(\varphi_k)\,\sigma_\nu^2\right]. \end{aligned} \tag{37}$$

The constant B is independent of K, giving an inverse quadratic convergence towards $J_{min}(\infty)$ with increasing K. This behavior is illustrated in Figure 3 where the number of subchannels is set to 16 and the other conditions are identical to the ones used in Figure 2. In the figure, the approximation (36) is shown together with exact curves obtained by numerical inversion of the correlation matrix $\mathbf{R}$. Note that for $K = 0$, the MMSE is calculated by formula (31).

We can see that for $\sigma_\nu^2 = 0$ and -30 dB, the approximate MMSE matches well with the exact value. For $\sigma_\nu^2 = -60$ dB, the approximation can be used as an upper bound on $J_{min}(K)$.

VI. Conclusion

We have shown how to formulate the equalization problem for OFDM/OQAM in a way that gives expressions similar to single carrier QAM. This enables us to obtain expressions for MMSE as a function of the number of subchannels N, equalizer order K and channel noise level σ_ν^2. These expressions are useful for determining how complicated equalizers are necessary in a given case.

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