Nonlinear Companding Transform for Reduction of Peak-to-Average Power Ratio in OFDM Systems

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***Abstract*—High peak-to-average power ratio (PAPR) of the transmitted signal is one of the limitations to employing orthogonal frequency division multiplexing (OFDM) system. In this paper, we propose a new nonlinear companding algorithm that transforms the OFDM signals into the desirable statistics form defined by a linear piecewise function. By introducing the variable slopes and an inflexion point in the target probability density function, more flexibility in the companding form and an effective trade-off between the PAPR and bit error rate performances can be achieved. A theoretical performance study for this algorithm is presented and closed-form expressions regarding the achievable transform gain and signal attenuation factor are provided. We also investigate the selection criteria of transform parameters focusing on its robustness and overall performance aspects. The presented theoretical analyses are well verified via computer simulations.**

***Index Terms*—High power amplifier (HPA), nonlinear companding transform (NCT), orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAPR).**

I. Introduction

As a promising technique, orthogonal frequency division multiplexing (OFDM) has been widely applied in modern wireless communications due to its high spectral efficiency and low susceptibility to the multipath propagation [1]. However, a major drawback of OFDM-based transmission systems is its high instantaneous peak-to-average power ratio (PAPR), which leads to undesired in-band distortion and out-of-band radiation if the linear range of the high power amplifier (HPA) is not sufficient [2], [3]. In an OFDM system with N subcarriers, the complex baseband representation of OFDM signal is given by

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k \cdot e^{j\frac{2\pi k t}{T}}, \quad 0 \le t \le T, \tag{1}$$

where $j = \sqrt{-1}$ and the vector $\mathbf{X} = [X_0, X_1, \ldots, X_{N-1}]^T$ denotes the frequency-domain OFDM symbols and T is the symbol duration. Based on the central limit theorem, when N is large, $x(t)$ can be approximated as a complex Gaussian process; thus, it is possible that the maximum amplitude of OFDM signal may well exceed its average amplitude. To overcome this issue, various methods have been developed [4], among which, nonlinear companding transform (NCT) is an efficient solution in reducing the PAPR of OFDM signal. The concept of NCT was first introduced in [5], which used the μ-law companding and could significantly outperform the traditional clipping. Earlier NCT methods have primarily focused on designing the nonlinearity of the transfer curve [6], [7]. Later, the work of [8] first indicated the importance of exploiting the statistical characteristics of the OFDM signal. Up to now, several such NCT methods have been proposed, e.g. the exponential companding (EC) in [9], the uniform companding (UC) in [10], the piecewise companding (PC) in [11], and the trapezium or trapezoidal companding (TC) in [12] and [13], etc.

Intuitively, by compressing large signals and enlarging small ones, both the PAPR reduction and immunity of small signals from noise can be achieved [6]. However, it is worth noting that NCT is an extra pre-distortion operation applied to transmitted signal, which results in performance degradation and increased sensitivity to the HPA. It was pointed in [8] that, due to the disadvantages of nonlinear distortion, such transform should be designed cautiously so that the amount of clipped signal is as little as possible. For this reason, how to reallocate the power as well as the statistics of OFDM signal more reasonably to reduce the impact of companding distortion is the key challenge for a well-designed NCT method. Moreover, a flexible and effective trade-off among the overall performance of OFDM system with respect to the reduction in PAPR (power efficiency), bit error rate (BER), spectral regrowth (bandwidth efficiency), and the implementation complexity also should be considered.

In this paper, further motivated by the observation above, we propose a new NCT algorithm which transforms the Gaussian-distributed signal into a desirable distribution form defined by a linear piecewise function with an inflexion point. Compared to the previous methods, this algorithm can significantly reduce the impact of companding distortion on the BER performance by choosing proper transform parameters. In addition, it also allows more flexibility and freedom in the companding form to satisfy various design requirements. The analytical expressions regarding the achievable reduction in PAPR, signal attenuation factor, and the selection criteria of transform parameters are derived and verified through computer simulations.

The rest of this paper is outlined as follows. Section II briefly describes the statistical characteristic of OFDM signal, while

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Section III shows the derivation of the generic formulas for the proposed algorithm. The theoretical performance analyses are presented in Section IV. Simulation results are discussed in Section V, followed by the conclusions in Section VI.

Notation: The expectation and maximal element operator are denoted by $E\{\cdot\}$ and $\max\{\cdot\}$. We use $[\cdot]^T$, $(\cdot)^{-1}$ and $|\cdot|$ to denote the transpose, inverse and modulus operation, respectively. $\mathrm{sgn}(\cdot)$ stands for the sign function. $\mathrm{IFFT}_N\{\cdot\}$ represents the N-point inverse fast Fourier transform (IFFT) operation. $\mathrm{Prob}\{A\}$ is the probability of the event A. Bold letters denote the vectors.

II. Characterization of OFDM Signal

Generally, an OFDM signal is the sum of N independent data symbols modulated by phase-shift keying (PSK) or quadrature amplitude modulation (QAM). In discrete-time domain, since the Nyquist rate samples might not represent the peaks of the continuous-time signal, it is preferable to approximate the true PAPR on an oversampled signal. The oversampled time-domain OFDM symbols $\mathbf{x} = [x_0, x_1, \ldots, x_{JN-1}]^T$ can be calculated as

$$x_n = \frac{1}{\sqrt{JN}} \sum_{k=0}^{N-1} X_k \cdot \mathrm{e}^{j\frac{2\pi nk}{JN}}, \quad 0 \le n \le JN-1, \tag{2}$$

where $n = 0, 1, \ldots, JN-1$ is time index and J is the oversampling ratio. Usually, $J \ge 4$ is used to accurately describe the PAPR of the continuous-time signal. This oversampling process can be achieved by performing a JN-point IFFT with extending $\mathbf{X}$ to a JN-length vector by inserting $(J-1)N$ zeros in its middle, i.e.

$$\mathbf{X}_e = \left[X_0, \ldots, X_{N/2-1}, \underbrace{0, \ldots, 0}_{(J-1)N}, X_{\frac{N}{2}}, \ldots, X_{N-1} \right]^T. \tag{3}$$

It is clear that $\mathbf{x} = \mathrm{IFFT}_{JN}\{\mathbf{X}_e\}$. For a large N (e.g. $N \ge 64$), the real and imaginary parts of x_n may be approximated as Gaussian random variables with zero mean and a variance σ^2. Based on this assumption, the signal amplitude $|x_n|$ follows a Rayleigh distribution with the probability density function (PDF) as

$$f_{|x_n|}(x) = \frac{2x}{\sigma^2} e^{-\frac{x^2}{\sigma^2}}, \quad x \ge 0. \tag{4}$$

The cumulative density function (CDF) of $|x_n|$ is therefore

$$F_{|x_n|}(x) = \mathrm{Prob}\{|x_n| \le x\} = \int_0^x \frac{2y}{\sigma^2} e^{-\frac{y^2}{\sigma^2}} dy = 1 - e^{-\frac{x^2}{\sigma^2}}, \ x \ge 0. \tag{5}$$

The PAPR of OFDM signal in a given frame is defined as

$$\mathrm{PAPR}_{\mathbf{x}} = \frac{\max_{n\in[0,JN-1]}\left\{|x_n|^2\right\}}{E\left\{|x_n|^2\right\}}. \tag{6}$$

It is more helpful to consider the PAPR as a random variable and utilize a statistical description given by the complementary cumulative density function (CCDF), defined as the probability that the PAPR of $\mathbf{x}$ exceeds an assigned level $\gamma_0 > 0$, i.e.

$$\mathrm{CCDF}_{\mathbf{x}}(\gamma_0) = \mathrm{Prob}\{\mathrm{PAPR}_{\mathbf{x}} > \gamma_0\} = 1 - \left(1 - e^{-\gamma_0}\right)^N. \tag{7}$$

The principle of NCT is described as follows. The original signal x_n is companded before converted into analog waveform and amplified by the HPA. The companded signal is denoted as $y_n = h(x_n)$, where $h(\cdot)$ is the companding function that only changes the amplitude of x_n. In the case of additive Gaussian white noise (AWGN) channel, the received signal $r_n = y_n + v_n$ can be recovered by the de-companding function $h^{-1}(\cdot)$, namely, $x'_n = h^{-1}(y_n + v_n) = x_n + h^{-1}(v_n)$, where v_n is channel noise.

III. New Algorithm Description

The basic idea of the proposed algorithm is to transform the statistics of the amplitude $|x_n|$ into the desirable PDF defined by a piecewise function, which consists of two linear functions with an inflexion point. Assume the inflexion point and cutoff point of the PDF of $|y_n|$ are $cA (0 < c < 1)$ and $A (A > 0)$, respectively. Thus, the desirable target PDF can be expressed as

$$f_{|y_n|}(x) = \begin{cases} k_1 x, & 0 \le x \le cA \\ k_2 x + (k_1 - k_2)cA, & cA < x \le A \end{cases}, \tag{8}$$

where two slopes $k_1 > 0$ and $k_2 < 0$ are variable parameters that determine the desired companding form i.e. the ultimate PAPR, while controlling the average output power in this transform. Based on the definition of PDF $\int_{-\infty}^{+\infty} f_{|y_n|}(x)dx = 1$, we have

$$k_1 = \frac{2 - A^2 k_2 (c-1)^2}{A^2 c(2-c)}. \tag{9}$$

From (8), the CDF of $|y_n|$ can be represented as

$$F_{|y_n|}(x) = \begin{cases} \frac{k_1}{2}x^2, & 0 \le x \le cA \\ \frac{k_2}{2}x^2 + (k_1-k_2)cAx - \frac{k_1-k_2}{2}(cA)^2, & cA < x \le A \\ 1, & x > A \end{cases}. \tag{10}$$

Clearly, CDF is a strictly monotonic increasing function and has the corresponding inverse function as follows.

$$F^{-1}_{|y_n|}(x) = \begin{cases} \sqrt{\frac{2x}{k_1}}, & x \le \frac{k_1}{2}(cA)^2 \\ \frac{1}{k_2}\Big((k_2-k_1)cA \\ \quad + \sqrt{(k_1-k_2)k_1c^2A^2 + 2k_2x}\Big), & x > \frac{k_1}{2}(cA)^2 \end{cases}. \tag{11}$$

Given that $h(x)$ is also a strictly monotonic increasing function, we can obtain the following relationship.

$$\begin{aligned} F_{|x_n|}(x) &= \mathrm{Prob}\left\{|x_n| \le x\right\} = \mathrm{Prob}\left\{h\left(|x_n|\right) \le h(x)\right\} \\ &= F_{|y_n|}\left(h(x)\right). \end{aligned} \tag{12}$$

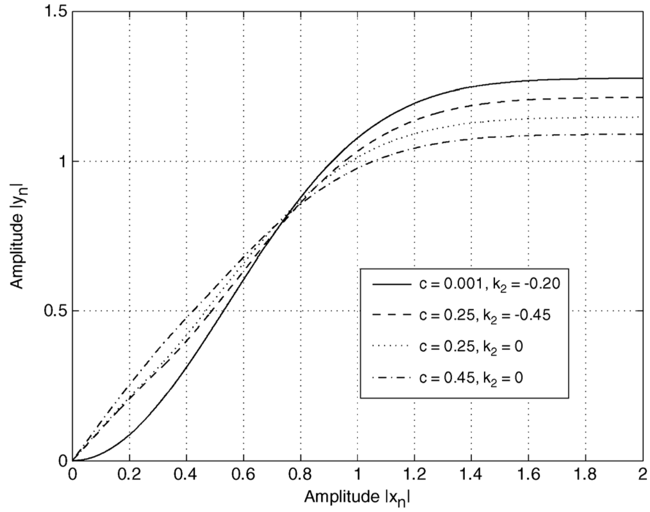

Fig. 1. Transfer curves of the proposed companding function.

Thus, the proposed companding function is given by (13), shown at the bottom of the page. where $\chi_0 = \sigma(-\ln(1-(k_1/2)c^2A^2))^{1/2}$. Additionally, in order to keep the input and output signal with a constant average power level, namely $E\{|y_n|^2\} = E\{|x_n|^2\} = \sigma^2$, we can obtain

$$A = \left(\frac{1}{2\zeta_2}\left(\left(\zeta_1^2 - 4\zeta_0\zeta_2\right)^{\frac{1}{2}} - \zeta_1\right)\right)^{\frac{1}{2}}, \tag{14}$$

where $\zeta_0 = 12\sigma^2(c-2), \zeta_1 = -2(c^3-4)$ and $\zeta_2 = k_2(c^3-3c+2)$. The transfer curves of this algorithm with various parameters are plotted in Fig. 1, from that we can see that the transform can achieve more reduction in the PAPR with k_2 or c increasing. Especially, it is noteworthy that the EC [9] and TC [13] are two special cases of the proposed algorithm. More specifics about the selection criteria of parameters are shown in Section IV.

At the receiver side, the companded signal can be recovered by the corresponding de-companding function as seen in (15), shown at the bottom of the page. In practice, since actual signal processed at the transmitter and receiver are the quantized signal with finite set of values, the functions in (13) and (15) can be numerically pre-computed and performed via the look-up tables [14]. Thus, its implementation complexity can be, therefore, significantly reduced.

IV. Performance Study

In this section, the theoretical performance of the proposed algorithm is characterized with two main evaluation criteria: the achievable reduction in PAPR and the impact of companding distortion on the BER performance at the receiver.

A. Achievable Reduction in PAPR

By making appropriate substitution in (6), the ultimate PAPR of the companded signal with the new algorithm is given by

$$\mathrm{PAPR}_{\mathbf{y}} = \frac{\max_{n\in[0,JN-1]}\left\{|y_n|^2\right\}}{E\left\{|y_n|^2\right\}} = \frac{A^2}{\sigma^2} = \frac{\left(\zeta_1^2 - 4\zeta_2\zeta_0\right)^{\frac{1}{2}} - \zeta_1}{2\zeta_2\sigma^2}. \tag{16}$$

Furthermore, a transform gain G is defined as the ratio of the PAPR of original signal to that of the companded signal [8], i.e.

$$\mathrm{G(dB)} = 10\log_{10}\frac{\mathrm{PAPR}_{\mathbf{x}}}{\mathrm{PAPR}_{\mathbf{y}}} = 10\log_{10}\frac{2\zeta_2 A_{i\max}^2}{(\zeta_1^2 - 4\zeta_2\zeta_0)^{\frac{1}{2}} - \zeta_1}, \tag{17}$$

where $A_{i\max} = \max_{0\le n\le JN-1}\{|x_n|\}$. The theoretical results of $\mathrm{PAPR}_{\mathbf{y}}$ and G are depicted in Fig. 2(a) and (b), respectively. As can be seen, this algorithm offers an adequate flexibility in the PAPR reduction by adjusting the values of k_2 and c. Consequently, the ultimate PAPR of the companded signal can be effectively confined in the interval [4.1 dB, 5.7 dB], or in other words, the achievable transform gain G in the PAPR is from 6 dB to 7.7 dB.

Moreover, substituting (16) into (7), the CCDF of the PAPR with the proposed algorithm can be written as follows.

$$\begin{aligned}\mathrm{CCDF}_{\mathbf{y}}(\gamma_0) &= \mathrm{Prob}\{\mathrm{PAPR}_{\mathbf{y}} > \gamma_0\} \\ &= \mathrm{CCDF}_{\mathbf{x}}\left(\frac{2\zeta_2 A_{i\max}^2}{(\zeta_1^2 - 4\zeta_2\zeta_0)^{\frac{1}{2}} - \zeta_1}\gamma_0\right).\end{aligned} \tag{18}$$

$$h(x) = \mathrm{sgn}(x)\cdot F_{|y_n|}^{-1}\left(F_{|x_n|}(x)\right)\begin{cases}\mathrm{sgn}(x)\sqrt{\frac{2}{k_1}\left(1-e^{-\frac{|x|^2}{\sigma^2}}\right)}, & |x|\le\chi_0\\ \mathrm{sgn}(x)\frac{1}{k_2}\left((k_2-k_1)cA+\sqrt{(k_1-k_2)k_1c^2A^2+2k_2\left(1-e^{-\frac{|x|^2}{\sigma^2}}\right)}\right), & |x|>\chi_0\end{cases} \tag{13}$$

$$h^{-1}(x) = \begin{cases}\mathrm{sgn}(x)\sigma\sqrt{-\ln\left(1-\frac{k_1|x|^2}{2}\right)}, & |x|\le cA\\ \mathrm{sgn}(x)\sigma\sqrt{-\ln\left(-\frac{k_2}{2}|x|^2+(k_2-k_1)cA|x|+1-\frac{c^2A^2}{2}(k_2-k_1)\right)}, & |x|>cA\end{cases} \tag{15}$$

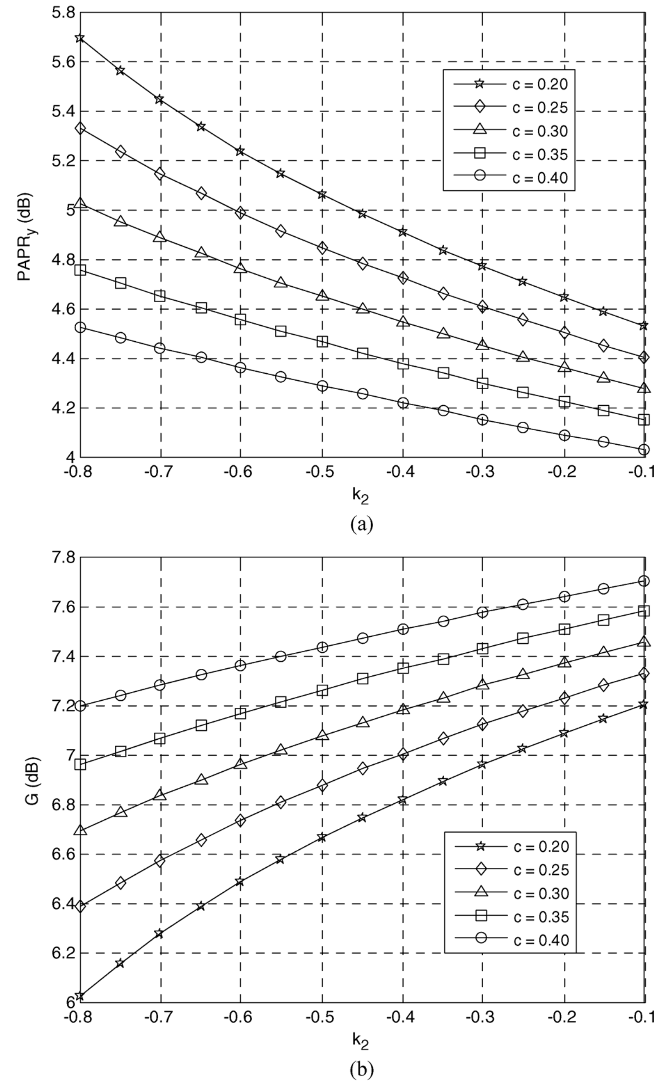

Fig. 2. Theoretical results of PAPR and G versus k_2 of the proposed algorithm. (a) The ultimate PAPR of companded signals. (b) Transform gain G.

B. Impact of Companding Distortion

In essence, NCT is an extra nonlinear operation applied to the transmitted signal. For this reason, how to minimize the impact of companding distortion on the BER performance is the key in choosing the optimal companding form and parameters. Based on the analysis results for the Gaussian signals in [15] and [16], two performance criteria: signal attenuation and companding noise b_n can be used to characterize this impact, i.e.

$$y_n = \alpha x_n + b_n, \tag{19}$$

where α is an attenuation factor, which is given by

$$\alpha = \frac{1}{\sigma^2} \int_0^{\infty} x h(x) f_{|x_n|}(x) dx. \tag{20}$$

Obviously, smaller α value corresponds to larger companding distortion and the reduced BER performance. It is shown in [11] that the noise power of b_n is also increasing as α decreases. The attenuation factor of the new algorithm can be calculated as

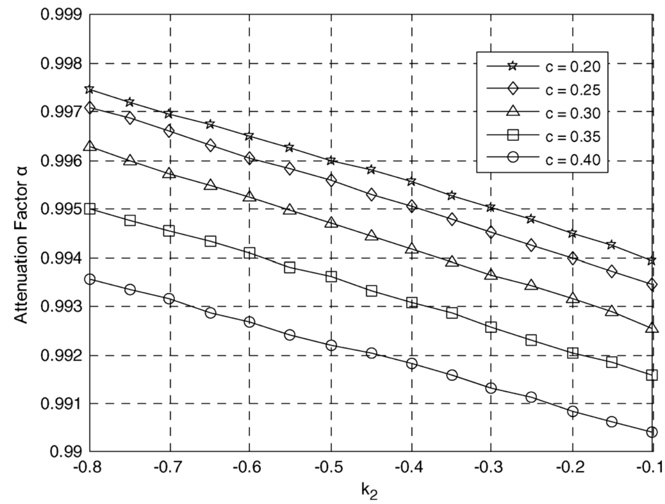

Fig. 3. Theoretical result of attenuation factor of the proposed algorithm.

$$\alpha = \frac{2}{\sigma^4} \int_0^{\chi_0} x^2 e^{-\frac{x^2}{\sigma^2}} \left(\frac{2}{k_1} \left(1 - e^{-\frac{x^2}{\sigma^2}} \right) \right)^{\frac{1}{2}} dx + \frac{2}{k_2 \sigma^4} \int_{\chi_0}^{\infty} x^2 e^{-\frac{x^2}{\sigma^2}} \times \left((k_2 - k_1) c A + \sqrt{(k_1 - k_2) k_1 c^2 A^2 + 2 k_2 \left(1 - e^{-\frac{x^2}{\sigma^2}} \right)} \right) dx \tag{21}$$

The theoretical result of α is depicted in Fig. 3, from that we can see that α gradually tends to 1 as k_2 and c decrease. As a result, Fig. 2 and Fig. 3 demonstrate that, to obtain an expected PAPR reduction, it may be preferable for this algorithm to make the undesired signal distortion as small as possible by choosing proper parameters. This conclusion is quite helpful to design the optimal companding form to offer an effective trade-off between the PAPR reduction and BER performance in practice.

V. Simulation Results

To evaluate the overall system performance of the proposed algorithm, computer simulations were performed based on an OFDM system with $N = 1024$ subcarriers. In the results which follow, 10^6 random OFDM frames modulated by QPSK or 16QAM were generated to obtain the CCDFs, which have been computed with an oversampling ratio $J = 4$ to offer a better PAPR estimation. In order to investigate the performance degradation and spectral regrowth, we also consider passing the companded signal through AWGN channel, the Rician multipath fading channel [17], and the solid-state power amplifier (SSPA) model [18] with a typical value $p = 2$. Five previous classic methods, i.e. the μ-law ($\mu = 13$) [6], EC ($d = 1$) [9], PC ($c = 1/\sqrt{6}$) [11], TC ($a = 0.4$, $b = 0.1$) [12], and the hyperbolic tangent companding (HTC) method ($k_2 = 0.7$) [7] were also included in the simulations.

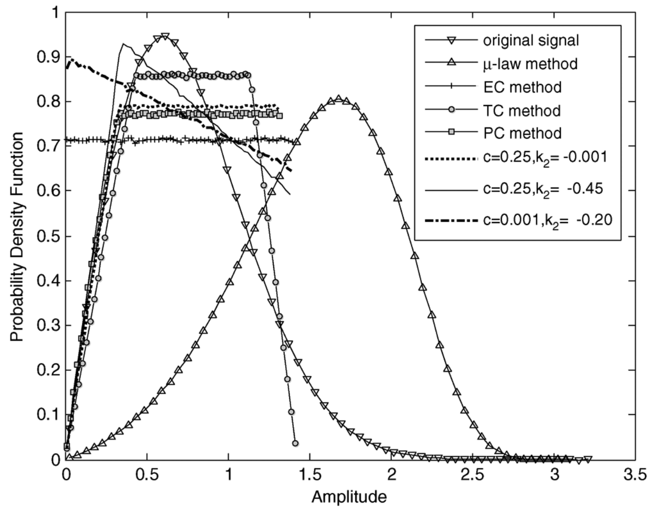

Fig. 4. The simulated PDFs of different transforms for OFDM system with $N = 1024$ and QPSK modulation.

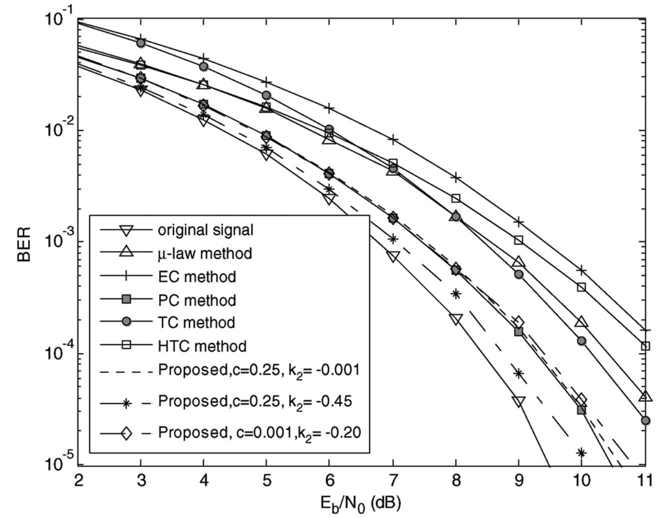

Fig. 6. BER performance of different transforms under an AWGN channel for the OFDM system with $N = 1024$ and QPSK modulation.

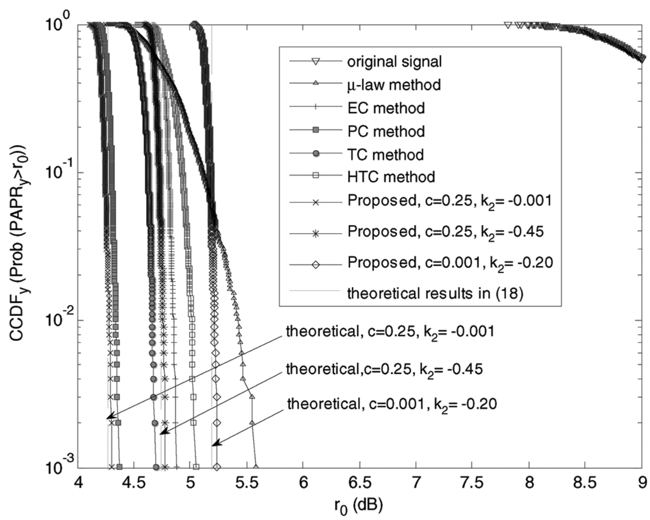

Fig. 5. PAPR reduction performance of different transforms for OFDM system with $N = 1024$, QPSK modulation, and oversampling ratio $J = 4$.

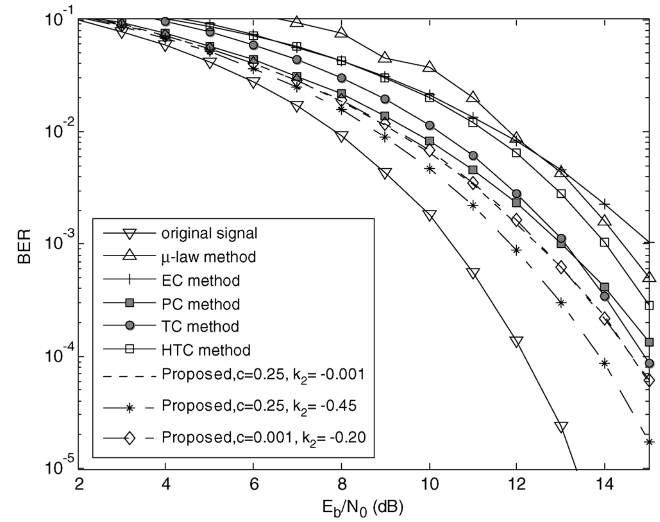

Fig. 7. BER performance of different transforms under an AWGN channel for the OFDM system with $N = 1024$ and 16QAM modulation.

A. Performance in PAPR Reduction

Fig. 4 plots the simulated PDFs of the companded signal with different transforms. As shown in this figure, by adjusting the variable parameters, this algorithm transforms the original Gaussian distribution into the desired PDF profile. In particular, unlike the EC and UC methods, this algorithm can effectively refrain from greatly increasing the probability of large signals.

Fig. 5 depicts the simulation results of CCDF of the PAPR with different transforms. The theoretical results in (18) are also presented. Given that $\mathrm{CCDF} = 10^{-3}$, the PAPR of original signal with $N = 1024$ is mostly greater than 11.3 dB. As can be seen from Fig. 5 that the predicted theoretical and the experimental curves almost coincide. In addition, the new algorithm with $c = 0.25$, $k_2 = -0.001$ obtains the maximal PAPR reduction, and that with $c = 0.25$, $k_2 = -0.45$ is roughly 0.3 dB and 0.1 dB inferior to the PC and TC in [12], but surpasses the EC and μ-law methods by 0.1 dB to 0.8 dB. It will be shown in below, however, that with $c = 0.25$, $k_2 = -0.45$ leads to far less BER degradation than others at the receiver.

B. BER Performance

Fig. 6 and Fig. 7 present the BER versus $\mathrm{E_b/N_0}$ curves with different transforms under an AWGN channel using QPSK and 16QAM, respectively. As a reference, the curve of 'original signal' is the ideal performance bound. As it is apparent, while NCT methods effectively reduce the PAPR, BER performance is, however, degraded at the receiver side. It can be seen that the required $\mathrm{E_b/N_0s}$ of the new algorithm are better than the referred methods for a given BER. For example, to guarantee $\mathrm{BER} = 10^{-4}$ in Fig. 6, the required $\mathrm{E_b/N_0}$ for that with $c = 0.25$, $k_2 = -0.45$ is 8.8 dB, which is about 0.5 dB to 1.6 dB superior to others. On the other hand, the BER performance degradation in comparison with the performance bound is less than 0.3 dB in this case. As aforementioned, by choosing proper

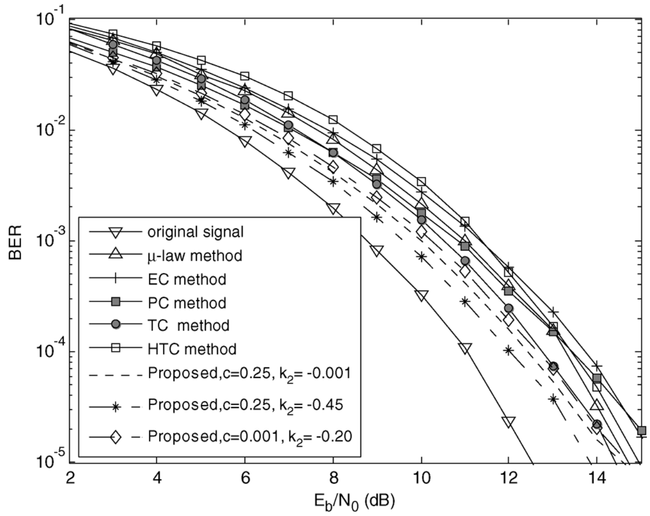

Fig. 8. BER performance of different transforms under the Rician fading channel for the OFDM system with $N = 1024$ and QPSK modulation.

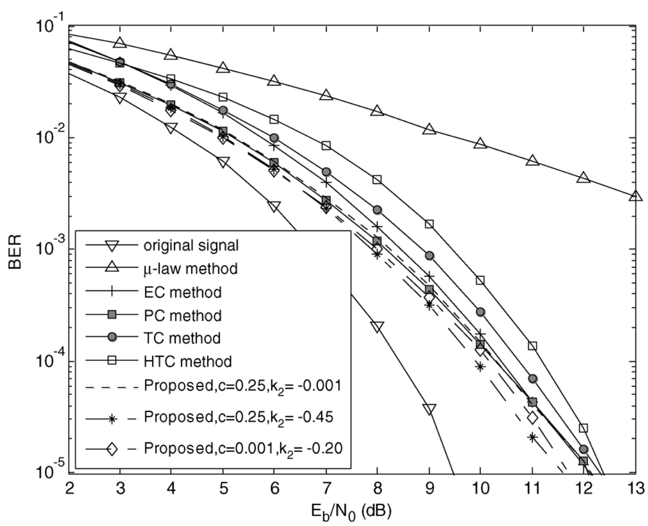

Fig. 9. BER performance of different transforms with SSPA ($p = 2$) under an AWGN channel for the OFDM system with $N = 1024$ and QPSK modulation.

transform parameters, the impact of companding distortion can be significantly reduced.

Fig. 8 and Fig. 9, in turn, present the BER versus $\mathrm{E_b/N_0}$ curves under the Rician multipath fading channel [17], and with the SSPA model ($p = 2$) under an AWGN channel, respectively. As can be seen from two figures, the proposed new algorithm still significantly outperforms other referred methods.

C. Spectral Characteristics

The spectral regrowth comparison is shown in Fig. 10. We can find that the new algorithm with $c = 0.25$, $k_2 = -0.45$ can produce about 2 dB to 4 dB lower out-of-band interference than other referred methods at normalized frequency 0.4.

One more observation from the simulations is, unlike the EC, UC and PC methods whose performances are almost fixed; the proposed algorithm is flexible in its companding form to satisfy various design requirements for OFDM systems.

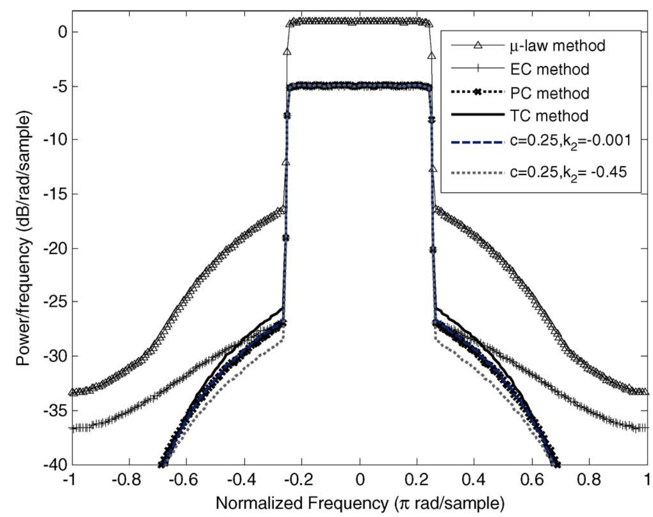

Fig. 10. Simulated PSDs of different transforms for the OFDM system with $N = 1024$, QPSK modulation, and oversampling ratio $J = 4$.

VI. Conclusions

Due to its simplicity and effectiveness, NCT is an attractive solution to reduce the PAPR of OFDM signal. In this paper, we investigate a new NCT algorithm which changes the statistics of original signal from the complex Gaussian to a desirable PDF defined as a linear piecewise function. Thus, an effective and flexible trade-off between the PAPR and BER performance can be achieved to satisfy various system requirements. Theoretical performance of this algorithm is characterized by means of the achievable reduction in PAPR and signal attenuation factor. It is shown that this algorithm can offer the transform gain in PAPR of 6.0 dB to 7.7 dB compared to the original signal. In addition, by choosing proper transform parameters, the impact caused by companding distortion can be significantly reduced. Computer simulations show the new algorithm substantially outperforms the existing NCT methods in the overall performance of OFDM system regarding the reduction in PAPR, BER and out-of-band interference under the multipath fading channel or with the HPA.

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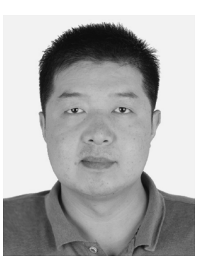

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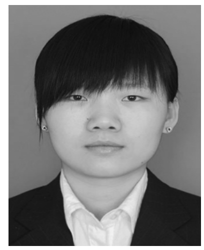

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