



FIGURE 14.21. Two-pole root locus.

poles must leave the real axis somewhere in the middle (exactly in the middle, in this case) of the real-axis segment. Invoking the symmetry argument, the asymptotes are actually exactly a part of the locus.

Note that, strictly speaking, a two-pole negative feedback system is never unstable in the BIBO sense, yet its damping gets progressively smaller as  $k$  increases. For such a system, the real part stays constant once the poles become complex, implying that the exponential envelope of the impulse response has a constant shape. However, the imaginary part increases as  $k$  increases, so that more oscillations per unit time occur within the exponentially decaying envelope.

Since the exponential envelope doesn't change shape as  $k$  increases, the settling time does not change, either. Just because a system becomes less stable does not necessarily imply a degradation in settling time.

Observations and conclusions such as these simply don't emerge from gain and phase margin calculations, illustrating the importance of having several ways to assess system stability. Additional viewpoints accelerate the development of intuition.

### 14.13.3 EXAMPLE: TWO POLES AND A ZERO

To stabilize the previous system, we may consider the addition of a zero. From a phase margin viewpoint, we regard the improvement in stability as a consequence of the positive phase contributed by the zero. An alternative view, informed by root-locus constructions, is that zeros are attractors of poles, so that properly placed zeros can bend poles away from the imaginary axis and toward more highly damped configurations.

A specific (and frequently occurring) example is sketched in Figure 14.22. The locus appears to include a circle centered on the zero. It turns out to be the correct shape, not just an artifact of the author's laziness.

In constructing this locus, make use of the real-axis rule, the breakaway/entry rule (here, we have an entry point between the finite zero shown and an infinite one), and