NUMERICAL MODELING OF MICROSTRIP RADIAL STUB

T. Günel and S. Kent

*A simple numerical algorithm used to model microstrip radial stub is presented. The cascaded interconnections of uniform transmission lines with incremental length **dl** are considered for the calculation of input impedance of a radial stub. Calculated results are consistent with the theoretical and experimental results found in the literature.*

Key Words:
Microstrip lines, Radial stubs, Impedance matching

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Radial stubs (RS) are used in impedance matching portions of low and high power microwave circuits. The radial stub is very suitable to solve problems of location and parasitics of low impedance shunt stubs by using fan-shaped open stubs with the narrow end connected to the main transmission line. The RS is also a space-saving element and presents a smaller outer dimension with respect to a quarter-wavelength open-ended stub at the same frequency. While the behavior of the conventional stubs degrades due to the excitation of higher order modes, the RS gives a low impedance level at a specified point in a broad frequency band.

Sorrentino, [1985] proposed the resonant mode expansion technique and a more accurate formula was given by Giannini et al., [1984]. The fringe field effects were considered by Wolff and Knoppik, [1974]. Sorrentino and Roselli [1992] proposed a new, simple and accurate formulation under the assumption that the effective permittivities of all resonant modes ($\varepsilon_{eff,n}$) the same and equals $\varepsilon_{eff,1}$ for any $n \geq 1$. In order to calculate input impedance of a lossy radial-line stub, the Bessel and Neumann functions must be evaluated for complex arguments due to a complex wavenumber. This requires the computation of a series with infinite terms [March, 1985].Therefore, the computational efficiency decreases.

Coimbra, [1984] proposed a microstrip element with input at the vertex of an isosceles triangle. In his work, a cascaded chain of microstrip transmission line segments of increasing width was used to model the element for the computation of its input impedance. A new numerical method that is not based on the isosceles triangle assumption is presented. The RS as the cascaded interconnections of uniform transmission lines with incremental length ***dl*** *is* considered here. To account for losses, a lossy transmission line can be used in this model. Thus, the new model has the advantage of simplicity. The input impedance of this model has been computed and the results have been compared with the results given by Sorrentino and Roselli, [1992] and Giannini et al., [1984].

Formulation of RS Input Impedance

A microstrip RS is shown in Figure 1. The input impedance of the RS assuming $\varepsilon_{eff,n}=\varepsilon_{eff,1}$ for any $n\geq 1$, is given as in Sorrentino and Roselli, [1992]

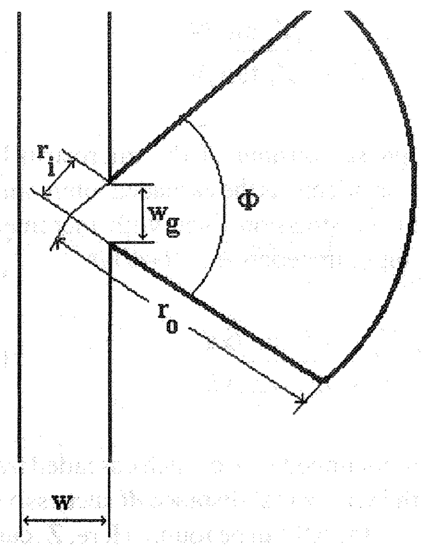

FIGURE 1: Geometry of the microstrip radial stub.

$$Z_{in}=-jZ_o(r_{ie})\cot(kr_{ie},kr_{oe})-j\omega\mu h\frac{2}{\phi}\frac{1/(r_{oe}^2-r_{ie}^2)}{k_{oo}^2-k^2\varepsilon_{eff,1}}+\frac{h}{j\omega\varepsilon_{eff,o}}\frac{2/\phi}{r_{oe}^2-r_{ie}^2} \quad (1)$$

where the suffix e denotes effective dimension. $Z_o(r_{ie})$ and cot (kr_{ie}, kr_{oe}) are computed from

$$Z_o(r_{ie})=\frac{120\pi h}{r_{ie}\phi\sqrt{\varepsilon_r}} \quad (2)$$

$$\cot(kr_{ie},kr_{oe})=\frac{N_o(kr_{ie})J_1(kr_{oe})-J_o(kr_{ie})N_1(kr_{oe})}{J_1(kr_{ie})N_1(kr_{oe})-N_1(kr_{ie})J_1(kr_{oe})} \quad (3)$$

J_i and N_i are the Bessel functions of ith order, k is the wavenumber, h is the substrate thickness, ε_r is the relative dielectric permittivity of the microstrip line.

The expressions r_{ie} and r_{oe} are the effective dimensions of the RS and given as in Giannini et al, [1984],

$$r_{ie}=\frac{w_{ge}}{2\sin(\phi/2)} \quad (4)$$

$$r_{oe}=r_o\left\{1+\frac{2h}{\pi r_o}\left[\ln\left(\frac{\pi r_o}{2h}\right)+1.7726\right]\right\}^{1/2}+\frac{w_{ge}-w_g}{2}\begin{Bmatrix}\{1/\sin(\phi/2)\}, & \phi\leq\pi\\ 1, & \pi\leq\phi\leq 3\pi/2\end{Bmatrix} \quad (5)$$

The effective permittivities can be computed as in Vrba, [1979]

$$\varepsilon_{eff,m}=\frac{C_{eff,m}(\varepsilon)+C_{eff,f}(\varepsilon)}{\dfrac{C_{eff,m}(\varepsilon)}{\varepsilon_r}+C_{eff,f}(\varepsilon_o)},m=0,1;\varepsilon=\varepsilon_0\varepsilon_r \quad (6)$$

$C_{eff,f}(\varepsilon)$ is given by

$$C_{eff,f}(\varepsilon)=\frac{\pi(r_o+r_i)}{\delta_m}\left[\frac{1}{Z_L v_{ph}}-\frac{(r_o-r_i)\varepsilon}{h}\right]$$

$$\delta m=\begin{cases}1, & m=0\\ 2, & m\neq 0\end{cases} \quad (7)$$

v_{ph} is the phase velocity in the microstrip line, and Z_L is the wave impedance. $C_{eff,m}(\varepsilon)$ can be calculated as follows [Vrba, 1979]:

$$C_{eff,m}(\varepsilon)=\frac{\varepsilon\phi}{2h\delta_m}\frac{1}{Z_m^2(kr_o)}\left\{r_o^2F_m(kr_o)-r_i^2F_m(kr_i)\right\} \quad (8)$$

$$F_m(kr)=Z_m^2(kr)-\left(\frac{2m}{kr}Z_m(kr)-Z_{m+1}(kr)\right)Z_{m+1}(kr),r=r_o,r_i \quad (9)$$

$$Z_{mr}(kr)=N_m(kr)-\left\{\frac{\frac{m}{kr}N_m(kr)-N_{m+1}(kr)}{\frac{m}{kr}J_m(kr)-J_{m+1}(kr)}\right\}J_m(kr),r=r_o,r_i \quad (10)$$

Numerical Model of RS

Figure 2 shows numerical model of the RS. It is considered as the cascaded interconnections of uniform transmission lines with incremental length ***dl***. As shown in Figure 2, widths of each transmission lines in Section I are computed by using Equation 11

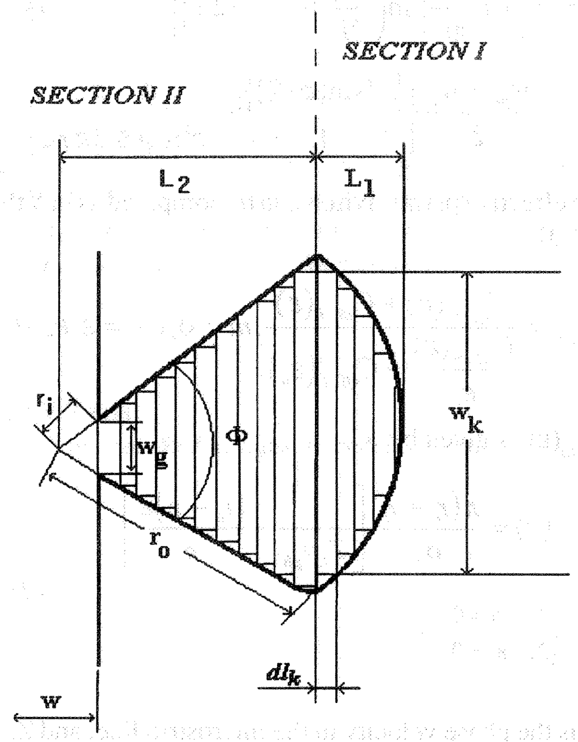

FIGURE 2: *Numerical model of the radial stub.*

$$w_k = \sqrt{8r_o(L_1 - dl * k) - 4(L_1 - dl * k)^2} \tag{11}$$
$$k = N_1, N_1 - 1, \ldots, 0$$

where N_1 is the total number of transmission lines in Section I and

$$L_1 = 2r_o \sin^2 \frac{\phi}{4} \tag{12}$$

Widths of each transmission lines in Section II (see Figure 2) are computed as follows

$$w_k = 2(L_2 - k * dl)\tan\left(\phi/2\right) \quad k = 0, 1, \ldots N_2 \tag{13}$$

where $L_2 = r_o - L_1$, and N_2 is the total number of lines in Section II. For a load impedance Z_L input impedance of a lossless transmission line with length ***l*** and a characteristic impedance of Z_o can be computed as in Ahmed, [1981]

$$Z_{in} = Z_o \frac{Z_L + jZ_o \tan \beta l}{Z_o + jZ_L \tan \beta l} \tag{14}$$

where β is the phase constant of the microstrip line. Since the line is considered as the cascaded interconnections of uniform transmission lines with incremental distance ***dl***, the input impedance at ***(l+dl)*** *is*

$$Z_{in} + dZ_{in} = Z_o \frac{Z_L + jZ_o \tan \beta dl}{Z_o + jZ_L \tan \beta dl} \tag{15}$$

Computing the input impedance of each cascaded transmission line with incremental distance ***dl*** successively, input impedance of the RS can be found. Here, Z_o can be calculated using the formulae given by Fooks and Zakarevicius, [1990] for each transmission line in terms of widths of the lines (W_k, k=0,1,...N), relative dielectric constant and height of the substrate material.

Numerical Results

The input impedance of the RS between 1 GHz and 13 GHz is computed using Equation 1 for the stub parameters given as in Sorrentino and Roselli [1992]: ϕ=60°, r_o=4.4 mm, w_g =0.360mm, ε_r =9.7, substrate thickness h=0.635 mm, metal thickness t=15 µm. In order to use numerical model, initial values of N_1 and N_2 were selected and the number of subdivisions were increased until the first resonance frequency (at which the input impedance passes through a zero) remained constant. Since RS is generally used in the resonance frequency region, N_1=10 and N_2=30 were sufficient to obtain very good agreement around the first resonance frequency within the frequency band, ±1 GHz approximately. Figure 3 shows the input reactance of the radial stub computed by the numerical model for N_1=10 and N_2=30 (dashed lines) and by the Equation 1 (solid line). If the number of segments is halved, the first resonance

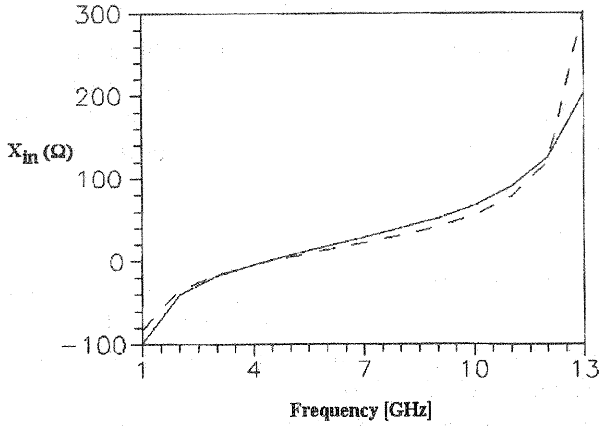

FIGURE 3: Input reactance of a radial stub computed by the numerical model for $N_1=10$ and $N_2=30$ (dashed lines) and by the Equation 1 (solid line) for $\phi=60°$, $r_o=4.4$ mm.

frequency shifts 4% in the frequency region between 1-20 GHz as shown in Figure 4.

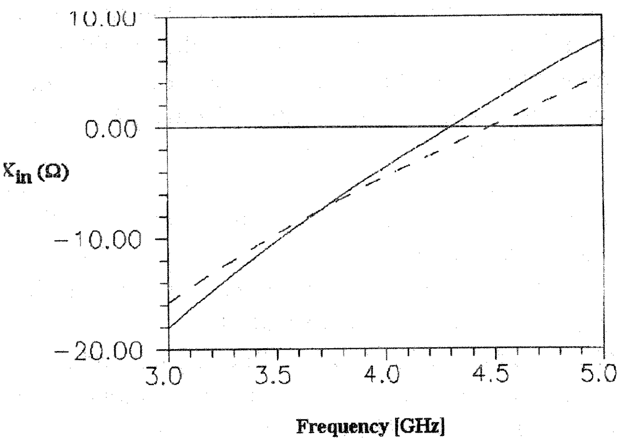

FIGURE 4: The first resonance frequency shift in the frequency region between 1-20 GHz. Dashed line (by numerical model), Solid line (by Equation 1).

In order to reduce discrepancies in other regions of the frequency band (1 - 13 GHz), the values of N_1 and N_2 were increased to 20 and 60, respectively. The results obtained by the analytical and numerical approaches are presented in Figure 5.

When the number of subdivisions is increased, the resonance frequency is not strongly sensitive to the number of subdivisions. When the frequency dependence of the characteristic impedances of the lines is not to be taken into account, the first resonance frequency above 20 GHz, shifts 3% with respect to the result obtained by the closed form expression Equation 1. Figure 6 shows the resonance frequency shift above 20 GHz.

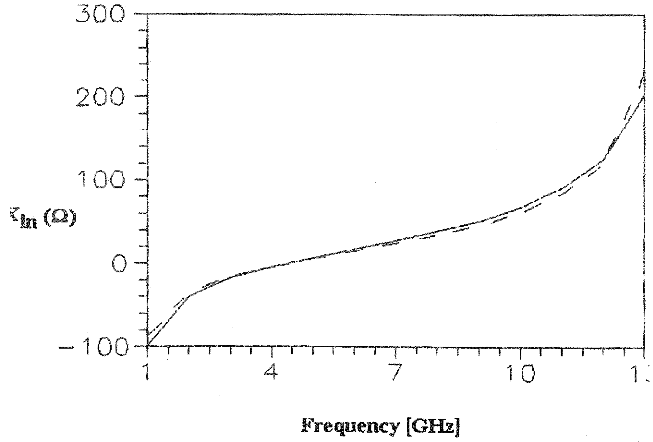

FIGURE 5: Input reactance of a radial stub computed by the numerical model for $N_1=20$ and $N_2=60$ (dashed lines) and by the Equation 1 (solid line) for $\phi=60°$, $r_o=4.4$ mm.

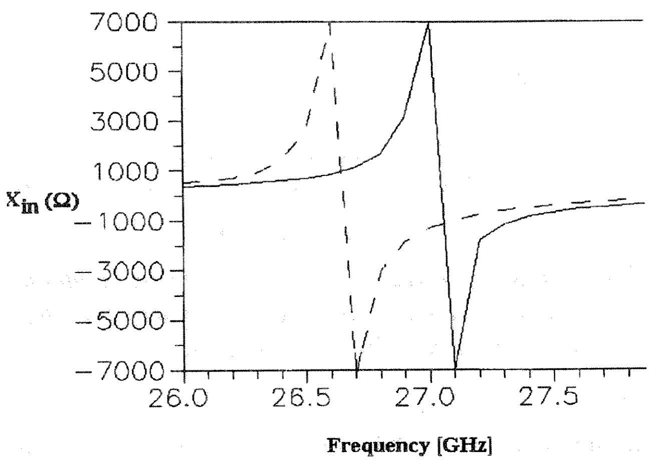

FIGURE 6: The first resonance frequency shift above 20 GHz. Dashed line (by numerical model), Solid line (by Equation 1).

In order to compare these results with the theoretical and experimental results given by Giannini et al., [1984], the input impedance of the RS on a 25-mil alumina substrate was also computed using numerical model for the stub parameters: $\phi=90°$ and 120°, $r_o=5.49$ mm, $r_o - r_i = 0.47$ cm, $N_1=20$ and $N_2=60$. The results are presented in Figure 7 for $\phi=90°$ and Figure 8 for $\phi=120°$

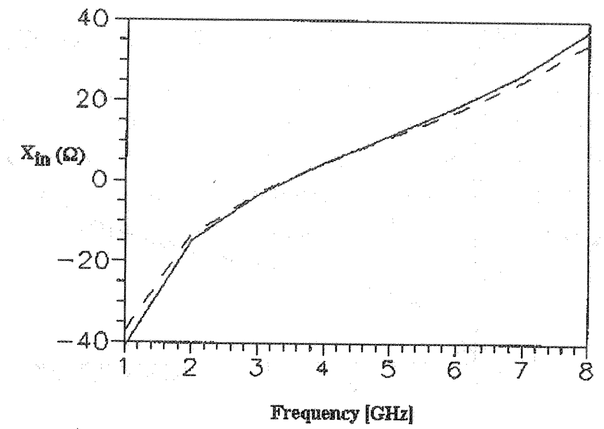

FIGURE 7: Input reactance of a radial stub computed by the numerical model (dashed line) and by Equation 1 (solid line) for ϕ=90°, r_o=5.49 mm.

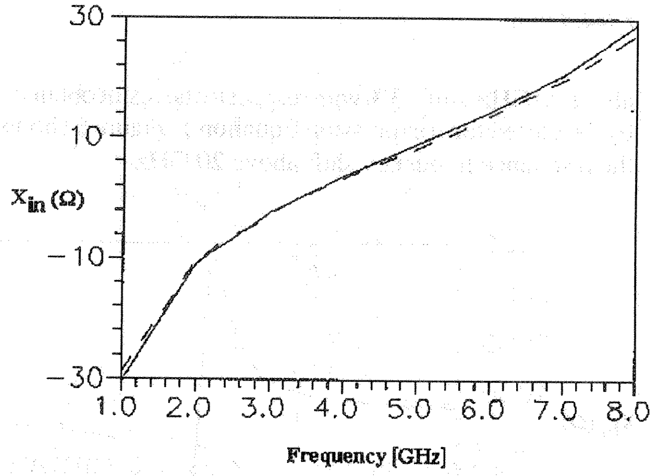

FIGURE 8: Input reactance of a radial stub computed by the numerical model (dashed line), and by Equation 1 (solid line) for ϕ =120°, r_o=5.49 mm.

The results are consistent with the results given by Giannini et al., [1984] and Sorrentino and Roselli, [1992]. For each example, very good agreement was obtained around the resonance frequencies within the frequency band, ±1 GHz approximately.

Conclusions

This new model gives a correct and simple solution for the computation of the input impedance of RS. Losses can be taken into account in this numerical model by using lossy transmission lines instead of lossless lines. Since the computer aided design of RS based on optimization algorithms requires a large number of iterations, the new approach is also very suitable because it does not need the time consuming computation of Bessel functions at each iteration. The numerical method gives the results approximately 10 times faster than the analytical one.

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