

# Design guideline for magnetic integration in LLC resonant converters

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**Abstract** -- The aim of this paper is to present a comprehensive design methodology for the magnetic integration of the series and shunt inductances of the resonant tank in an LLC resonant converter within the transformer.

The design procedure applies to symmetrical core-bobbin structures with two separate slots for the primary and secondary windings. A specific leakage inductance  $A_L$  (per square turn) that depends on geometrical parameters only is derived. This is used, together with the cross section and winding area of the core, to define two other core parameters that allow the designer to identify the minimum ferrite core size suitable for the application.

**Index Terms** – First harmonic approximation (FHA), LLC resonant converter, zero-voltage switching (ZVS), magnetic integration, transformer “all primary referred” (APR) model.

## I. INTRODUCTION

The interest in resonant topologies is continuously growing in the power conversion market, due to better efficiency and lower EMI pollution, with respect to traditional PWM solutions, inherent to these kinds of converters. This, in turn, allows for higher operating frequencies and consequently for a size reduction of the magnetic components and, generally, of the overall volume of the power supply.

One of the most popular resonant topologies nowadays is the multi-resonant LLC converter in its half-bridge implementation, which directly derives from the LC series resonant converter, by adding a shunt inductor in parallel to the load (see Fig. 1).

The LC series resonant converter suffers from a few drawbacks, which limit its use in several applications: in fact it only allows for buck operation (step-down) and cannot regulate the output voltage when unloaded; furthermore, at light load (when the resonant tank current is very low), the ZVS (zero voltage switching) behavior that minimizes power consumption is lost.

The LLC converter overcomes all of these problems, as shown in [1], where its behavior is analyzed, demonstrating that the resonant tank can be designed in order to always operate the half-bridge MOSFETs in ZVS condition, while regulating the output voltage down to zero-load. This operation is achieved essentially for free, as the additional shunt inductor can be implemented simply by introducing an air gap in the transformer core

and, furthermore, the transformer leakage inductance can be used as the series resonant inductor.

The aim of this paper is to give guidelines for the magnetic integration of the series and shunt inductances of the resonant tank inside the transformer, in order to get a resonant circuit composed only of a capacitor and a resonant transformer.

The article starts illustrating the LLC resonant converter principle of operation, recalling the FHA (first harmonic approximation) approach used in [1] and the design procedures proposed in [1] and [2] to calculate the resonant tank. Then, we discuss various transformer models, including the APR (all-primary-referred) one, which better fits with the basic LLC resonant tank circuit. Afterwards, considering the case of a symmetrical two-slot coil-former, the specific leakage inductance of the core-bobbin structure is introduced, a parameter that only depends on the shape and size of the ferrite core.

Finally, we outline the proposed design process for the magnetic integration; and we define two core specific parameters,  $K_{GM}$  and  $K_{GW}$ , which allow the designer to choose the minimum ferrite core size that meets the electrical requirements and, at the same time, guarantee a maximum specified temperature rise of the transformer.

## II. CIRCUIT DESCRIPTION

A resonant converter basically applies a square voltage or current generated by a power switch network to a resonant circuit; the energy circulates in the resonant tank and part of it is delivered to the load. Fig. 1 shows a typical LLC resonant half-bridge converter: the half-bridge MOSFETs, driven on and off symmetrically with 50% duty cycle, generate a square waveform with peak-to-peak amplitude  $V_{dc}$ , and an average value of  $V_{dc}/2$ . This voltage is applied to the resonant tank, composed of the series capacitor  $C_r$ , the series inductor  $L_r$  and the shunt inductor  $L_m$ , so that energy can be transferred to the load, which is coupled to the resonant tank by the ideal transformer  $T$ . The capacitor  $C_r$  acts both as a resonant and a dc blocking capacitor: so, at the end, the alternate square voltage across the resonant tank has an amplitude of  $\pm V_{dc}/2$ . The transformer also provides insulation from the mains.

The design procedure of an LLC converter is not as straightforward as for a PWM converter; in [1] a design guideline is presented that is based on the FHA approach; this tremendously simplifies the circuit model, leading to

a linear circuit, which can be dealt with through the classical complex ac-circuit analysis. The FHA approach is based on the assumption, that the power transfer from the source to the load through the resonant tank is almost completely associated to the fundamental harmonic of the Fourier expansion of the currents and voltages involved, in accordance to the selective nature of a resonant circuit.

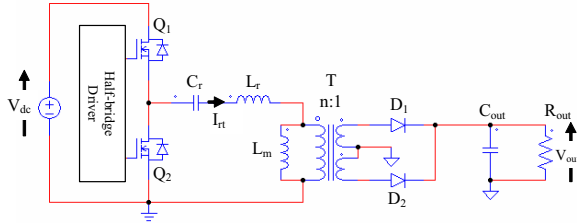


Figure 1. Half-bridge LLC resonant converter schematic

Fig. 2 shows the linear equivalent circuit, derived from the FHA assumption, where  $V_{i,FHA}$  and  $V_{o,FHA}$  are the first harmonic of the Fourier series expansion of the input and output square voltages of the resonant converter, respectively, while  $R_{o,ac}$  is the ac equivalent resistance of the output load:

$$V_{i,FHA} = \frac{\sqrt{2}}{\pi} V_{dc} \quad V_{o,FHA} = \frac{2\sqrt{2}}{\pi} V_{out} \quad R_{o,ac} = \frac{8}{\pi^2} \frac{V_{out}^2}{P_o} = \frac{8}{\pi^2} R_{out}$$

The ten-step procedure proposed in [1], allows the designer to calculate the resonant-tank circuit parameters,  $C_r$ ,  $L_r$ ,  $L_m$  and  $n$ , which fulfill a set of input/output specifications, including the ZVS condition and the no-load operation, and assuming that the circuit operates at resonance frequency when the input voltage has the nominal value. The resonance frequency and the maximum operating frequency are input parameters, chosen by the designer.

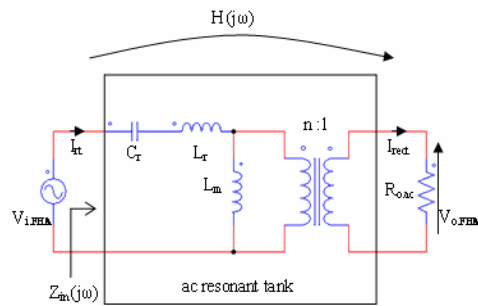


Figure 2. LLC resonant converter FHA equivalent model

The circuit in Fig. 2 is completely characterized by its input impedance  $Z_{in}(j2\pi f_{sw})$  and forward transfer function  $H(j2\pi f_{sw})$ . These are used in [1] to identify all the relationships and conditions to solve for the unknowns of the circuit ( $C_r$ ,  $L_r$ ,  $L_m$  and  $n$ ). Fig. 3 shows the resonant tank voltage gain  $M$  ( $M = n|H|$ ) versus normalized frequency  $f_n$ , at different values of the circuit quality factor  $Q$  (which means, at different power levels). The red curve in this graph represents the no-load voltage gain. The load independent point, where all the curves touch, is the operating point at the resonance frequency

( $f_n = 1$ ), where the required gain is unity: therefore, in the frame of the ten-step procedure, this is the operating point at nominal input voltage.

The regulation of the output voltage of the resonant converter is achieved by changing the switching frequency of the square waveform: as the working region is in the inductive part of the voltage gain characteristics of the resonant tank, the frequency control acts by increasing the frequency to lower the output power and by decreasing the frequency at decreasing input voltage (because the required voltage gain increases with decreasing input voltage).

Because the FHA model of an LLC resonant converter is not so accurate in the below-resonance region (where the current waveforms exhibit both the resonance frequency of  $C_r$  with  $L_r$  and the resonance frequency of  $C_r$  with the total inductance  $L_r + L_m$ ), in [2] a further model is presented that is based on the inspection of the tank current in a particular operating mode. In this operating mode, the tank current, during the time interval when the output diodes are conducting, has a sinusoidal shape at resonance frequency (while the current in  $L_m$  is linear, because the voltage across it is clamped to the output voltage reflected at primary). In the time interval when diodes are not conducting, the tank current (and therefore the magnetizing current, too) is imposed to stay flat, equal to the value at the end of the previous time interval: in this way, the induction level  $B$ , which only depends on the current flowing in the magnetizing inductance, also does not increase, and the transformer can be designed as if it was operated at resonance frequency instead of the real operating frequency, below resonance. Also the nine-step procedure proposed in [2] allows the designer to calculate the values of the circuit parameters of Fig. 2, according to the same specification requirements as in [1].

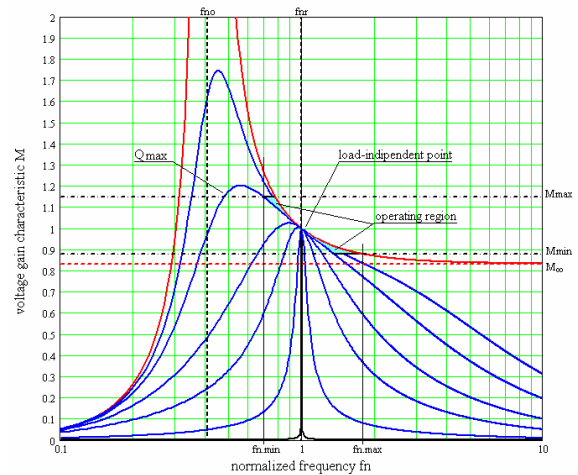


Figure 3. Voltage gain of an LLC resonant converter

Looking at Fig. 2, it is evident that the LLC resonant converter is well suited for the magnetic integration of the series and shunt inductances ( $L_r$ ,  $L_m$ ) as well as the transformer (T) into one magnetic component. In fact, the tank configuration recalls one of the most popular models

of a transformer: the APR one, where the series inductance  $L_r$  can be seen as the total leakage inductance, that is, the one measured on the primary side with the secondary windings short circuited.

### III. TRANSFORMER MODEL

Fig. 4 shows a system of two coupled inductors (as a two winding transformer can be generally termed), with  $L_1$  and  $L_2$ , respectively, primary and secondary self-inductance and  $M$  mutual inductance; we can derive an electrical-equivalent model of this magnetically coupled system, formed by the inductors  $L_a$ ,  $L_b$ ,  $L_\mu$  and an ideal transformer with turn ratio  $N$ . By comparing the branch-constitutive equations of the two circuits:

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = s \cdot \begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \Leftrightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = s \cdot \begin{bmatrix} L_\mu + L_a & L_\mu/N \\ L_\mu/N & L_\mu/N^2 + L_b \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

we can find the relationships to pass from one model to the other:

$$\begin{cases} L_1 = L_\mu + L_a \\ M = \frac{L_\mu}{N} \\ L_2 = \frac{L_\mu}{N^2} + L_b \end{cases} \Leftrightarrow \begin{cases} L_a = L_1 - L_\mu = L_1 - N M \\ L_\mu = N M \\ L_b = L_2 - \frac{L_\mu}{N^2} = L_2 - \frac{M}{N} \end{cases}$$

The equivalent model presents one degree of freedom, as it has four unknowns ( $L_a$ ,  $L_b$ ,  $L_\mu$  and  $N$ ), while the coupled inductor has only three unknowns ( $L_1$ ,  $L_2$  and  $M$ ); that means we can fix one of these arbitrarily and determine all the others: the logic choice is to fix  $N$ .

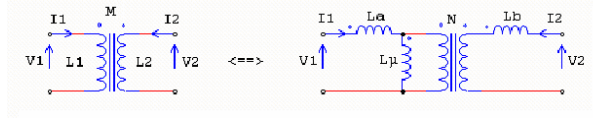


Figure 4. Coupled inductor equivalent model

In case  $N$  equals the real winding turns ratio  $n_t = N_1/N_2$  (where  $N_1$  and  $N_2$  are the number of turns of the two windings), the model represents the real physical model of the coupled inductor (see Fig. 5), where  $L_M$  is the magnetizing inductance (associated to the flux that mutually links the two windings), while  $L_{\sigma 1}$  and  $L_{\sigma 2}$  are the primary and secondary leakage inductances (which are associated to the primary and secondary leakage fluxes). This physical model is exactly equal to that coming from the reluctance model approach. From a physical standpoint it must be verified that:

$$M \leq \sqrt{L_1 L_2}$$

where the equality sign means perfect coupling between the windings (which cannot be reached in reality, as leakage fluxes are always present, though they can be very small). A coupling coefficient  $k$  ( $k < 1$ ) is therefore introduced, so that it is possible to write:

$$M = k \sqrt{L_1 L_2}$$

and it can be demonstrated that  $k$  is the geometric mean

of the primary and secondary coupling coefficients  $k_1$  and  $k_2$  (which represent the portions of flux generated by the primary and secondary windings, respectively, that link to the other winding):

$$k = \sqrt{k_1 k_2}$$

In fact, by the definitions of the coupling coefficients, we have:

$$\frac{M}{L_1} = \frac{N_2}{N_1} k_1 \quad \text{and} \quad \frac{M}{L_2} = \frac{N_1}{N_2} k_2 \quad (*)$$

By multiplying the two expressions above and solving for  $M$ , we get:

$$M = \sqrt{k_1 k_2} \sqrt{L_1 L_2} = k \sqrt{L_1 L_2}$$

Therefore, for the physical model we find the following relationships:

$$n_t = \frac{N_1}{N_2} \quad \begin{cases} L_M = k_1 L_1 \\ L_{\sigma 1} = (1 - k_1) L_1 \\ L_{\sigma 2} = (1 - k_2) L_2 \end{cases} \quad \text{where} \quad \begin{cases} k_1 = \frac{M}{L_1} n_t \\ k_2 = \frac{M}{L_2} \frac{1}{n_t} \end{cases}$$

Another interesting transformer model comes out when  $N$  is chosen equal to the square root of primary to secondary inductance ratio (see Fig. 5) which leads to the following set of relationships:

$$n_e = \sqrt{\frac{L_1}{L_2}} \quad \begin{cases} L_\mu = k L_1 \\ L_{S1} = (1 - k) L_1 \\ L_{S2} = (1 - k) L_2 = L_{S1} / n_e^2 \end{cases} \quad k = \frac{M}{\sqrt{L_1 L_2}}$$

where  $n_e$  is the effective turns ratio. In this case,  $L_\mu$ ,  $L_{S1}$  and  $L_{S2}$  do not have the physical meaning of magnetizing and leakage inductances, but they are only model parameters. Furthermore, it is worth noting that the secondary side series inductance reflected to the primary side equals the primary side series inductance:

$$L_{S1} = L_{S2,p} = n_e^2 L_{S2}$$



Figure 5. Coupled inductor equivalent models for  $N = n_t$  and  $N = n_e$

The two models with  $N = n_t$  and  $N = n_e$  are perfectly identical in case that  $k_1 = k_2$ , which might happen when the system of coupled inductors is symmetrical, such as in a two-slot symmetrical bobbin, with each winding of equal number of turns wound in each slot. In fact, by dividing the two expressions above in (\*) and root squaring, we get:

$$n_e = \sqrt{\frac{L_1}{L_2}} = \frac{N_1}{N_2} \sqrt{\frac{k_2}{k_1}} = n_t \sqrt{\frac{k_2}{k_1}}$$

Another very useful equivalent model of the coupled inductor is the APR one illustrated in Fig. 6, derived by imposing the secondary side series inductance  $L_b$  (in Fig. 4) is zero, which translates to:

$$N = \frac{M}{L_2} = \frac{M}{\sqrt{L_1 L_2}} \sqrt{\frac{L_1}{L_2}} = k n_e = n$$

The resulting relationships for this model (both in direct and inverse form) are:

$$\begin{cases} L_m = n M = k^2 L_1 \\ L_r = L_1 - n M = (1 - k^2) L_1 \end{cases} \quad \begin{cases} L_1 = \frac{L_m}{k^2} \\ L_2 = \frac{L_m}{n^2} \end{cases} \quad M = \frac{L_m}{n}$$

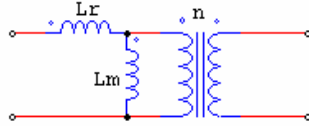


Figure 6. Transformer APR model

The transformer equivalent circuit (for  $N = n_e$  and for  $N = n = k n_e$ ) can be completely identified through three measurements: the inductances  $L_1$  and  $L_2$  of the primary and secondary windings and the inductance  $L_{tot}$  of the series connection of the two windings (such that the current has the same flowing direction through both the dotted winding terminals), that is:

$$L_{tot} = L_1 + L_2 + 2M \quad \Rightarrow \quad M = \frac{L_{tot} - (L_1 + L_2)}{2}$$

Once  $L_1$ ,  $L_2$  and  $M$  are known, you can calculate the model parameters ( $n$ ,  $L_m$ ,  $L_r$ ) and ( $n_e$ ,  $k$ ,  $L_{S1}$ ,  $L_{S2}$ ) through the above sets of equations. If the primary and secondary number of turns (or turns ratio) are also known, you can also completely define the physical model (for  $N = n_e$ ) by calculating ( $k_1$ ,  $k_2$ ,  $L_M$ ,  $L_{\sigma 1}$ ,  $L_{\sigma 2}$ ).

It is easy to recognize that the LLC resonant circuit is well suited for magnetic integration, i.e. to combine the inductors as well as the transformer in Fig. 2 into a single magnetic device; in fact, the magnetic part of the basic LLC resonant circuit is topologically identical to the APR model of a transformer.

The design flow based on the procedures proposed in [1] and [2] provides the parameters of the APR model  $L_r$ ,  $L_m$  and  $n$  (together with the resonant capacitance  $C_r$ ); hence, the further step is to determine the parameters of the physical model.

The problem is mathematically undetermined: there are four unknowns ( $n_t$ ,  $L_\mu$ ,  $L_{\sigma 1}$  and  $L_{\sigma 2}$ ) in the physical model and only three parameters in the APR one. One simplifying assumption that overcomes this issue is the magnetic circuit symmetry: flux linkage is assumed to be exactly the same for both primary and secondary windings (that is  $k_1 = k_2$ ). In this case, the physical model ( $N = n_t$ ) and the model with  $N = n_e$  are the same, which provides the missing condition for solving the system ( $L_{S1} = n_e^2 L_{S2}$ ). The complete solution is the following:

$$\begin{cases} k = \sqrt{L_m / (L_r + L_m)} \\ n_e = n_t = n / k \end{cases} \quad \begin{cases} L_1 = L_r + L_m \\ L_M = L_\mu = k L_1 \end{cases} \quad \begin{cases} L_{S1} = (1 - k) L_1 = L_{\sigma 1} \\ L_{S2} = L_{S1} / n_e^2 = L_{\sigma 2} \end{cases}$$

As previously mentioned, it is not difficult to find real-

world structures where the condition of magnetic symmetry is quite close to reality: consider for example the ferrite E-core plus slotted bobbin assembly, using side-by-side winding arrangement, as shown in Fig. 7.

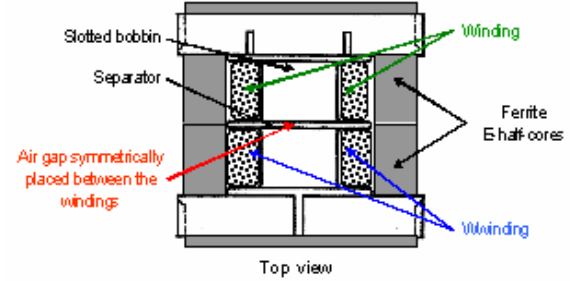


Figure 7. Transformer construction: E-cores and slotted bobbin

This type of transformer construction is well suited for this application, where the leakage inductance is used as the resonant-tank series inductance, and therefore needs to be well controlled and reliable for the circuit operation. Furthermore, the side-by-side winding technique allows for a wide range of values of leakage inductance, and hence coupling coefficient  $k$ , and is simpler and more effective than the layer winding arrangement.

Once the transformer model is completely defined, the next step is to find out the relationships to pass from the model parameters to the transformer construction parameters:  $N_1$ ,  $N_2$  and  $l_G$ , respectively, primary and secondary number of turns and thickness of the air gap. These parameters are the only ones that can be changed within a certain design to get the desired transformer, once the ferrite core and bobbin are chosen.

The inductance factor  $A_L$  (i.e. the inductance per square turn) can be alternatively used, instead of the air gap thickness  $l_G$ . Magnetic material suppliers usually specify  $A_L$  and  $l_G$  data in the form of a table in their ferrite core datasheets, or give the  $A_L$  values as a parameterised function of the air gap size.

#### IV. SPECIFIC LEAKAGE INDUCTANCE

In this section we derive a semi-empirical formula based on the energy calculation approach. It allows the designer to estimate the leakage inductance of a two-winding transformer employing a two-section coil-former.

Fig. 8 shows a symmetrical two-slot bobbin with  $N_1$  turns primary winding (in red) and  $N_2$  turns secondary (in yellow). In the next discussion we will neglect the coil former thickness and dimensions, except the safety distance  $d_s$  between primary and secondary windings, and we will assume that the two windings are equally and evenly distributed inside their respective slots.

It is possible to derive the expression of the energy associated to the leakage flux, which is stored in the winding volume. If the secondary winding is short circuited, the current  $I_1$  flowing into the primary winding causes a current  $I_2 = I_1 N_1 / N_2$  flowing into the secondary, such that the flux inside the magnetic core is almost zero.

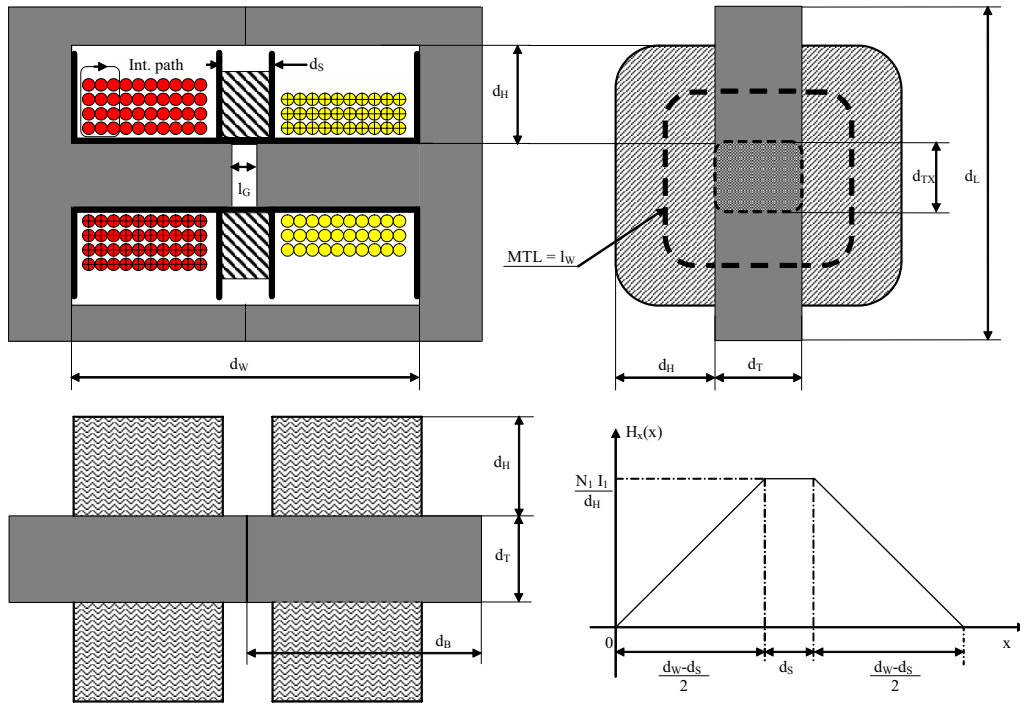


Figure 8. Two section slotted transformer and magnetic field distribution along winding section width direction

Due to the high permeability of ferrite materials, the field intensity,  $H$ , inside the magnetic core is negligible too; therefore, we can neglect the energy stored into the core and assert that the energy of the magnetic field, generated by the primary current  $I_1$ , is only dislocated within the winding volume. Referring to Fig. 6, we can also assert that this energy equals the one associated to the leakage inductances  $L_{\sigma 1}$  and  $L_{\sigma 2}$  of the transformer model, when the secondary is short circuited, that is (due to symmetry assumption) it is equal to twice the energy associated to  $L_{\sigma 1}$ :

$$E = \frac{1}{2} L_{\sigma 1} I_1^2 + \frac{1}{2} L_{\sigma 2} \left( \frac{N_1 I_1}{N_2} \right)^2 = L_{\sigma 1} I_1^2 = \frac{1}{2} \int_{Vol} B H dv = \frac{\mu_o}{2} \int_{Vol} H^2 dv$$

The field intensity  $H$  (see Fig. 8), considered approximately uniform along the window height ( $d_H$ ) direction, is derived through Ampere's law:

$$\oint H d\ell = N_1 I_1 \Leftrightarrow H_x(x) = \frac{N_1 I_1}{d_H} \frac{2x}{d_W - d_S} \quad 0 \leq x \leq \frac{d_W - d_S}{2}$$

In fact, considering an integration path as indicated in Fig. 8, which encircles more and more current along the  $x$  direction, the  $H_x$  field increases linearly from zero to the maximum value ( $N_1 I_1 / d_H$ ) when all the turns are encircled, then it stays constant through the safety distance spacer and at last decreases linearly again down to zero, when all of the secondary turns are encircled.

The differential volume element  $dv$  is equal to the product of  $dx$  times the area  $A_x$  of the cross section of the winding area orthogonal to the  $x$  axis (see Fig. 8), which is a sort of ring surface, whose area is approximately the product of the mean turn length  $l_W$  times the window area

height  $d_H$ ; therefore the energy stored in the magnetic field is:

$$\begin{aligned} E &= L_{\sigma 1} I_1^2 = \frac{\mu_o}{2} \int_0^{d_W} H_x^2(x) A_x dx = \frac{\mu_o}{2} \int_0^{d_W} H_x^2(x) l_W d_H dx = \\ &= \frac{\mu_o}{2} (N_1 I_1)^2 \frac{l_W}{d_H} \left[ 2 \int_0^{\frac{d_W - d_S}{2}} \left( \frac{2x}{d_W - d_S} \right)^2 dx + \int_{\frac{d_W - d_S}{2}}^{\frac{d_S}{2}} dx \right] = \\ &= \frac{\mu_o}{6} l_W \frac{d_W + 2d_S}{d_H} (N_1 I_1)^2 \end{aligned}$$

The average length of turn ( $l_W$ ), in case of an E core type (square center leg), can be evaluated as follows:

$$l_W = \frac{A_{ext.T} - A_{CS}}{d_H} = \frac{2}{d_H} (d_H^2 + d_H(d_T + d_{TX})) = d_H + d_T + d_{TX}$$

while in the case of an ETD core type (round center leg diameter  $D_{CL}$ ) it can be approximated by the expression:

$$l_W = \frac{A_{ext.T} - A_{CS}}{d_H} = \frac{1}{d_H} \frac{\pi}{4} ((d_H + D_{CL})^2 - D_{CL}^2)$$

where  $A_{CS}$  is the area of the center leg cross section of the core, while  $A_{ext.T}$  is the area of the most external winding turn. From the above calculated energy expression, we can obtain the series inductance  $L_r$  in the transformer APR model:

$$L_r = A_\sigma (1+k) N_1^2 \quad \text{where} \quad A_\sigma = \frac{\mu_o}{6} l_W \frac{d_W + 2d_S}{d_H}$$

The term  $A_\sigma$  is the "specific leakage inductance per squared turn" associated with the core and winding construction: it depends only on the geometrical parameters of the chosen core and coil-former and, in

spite of all simplifying assumptions, its expression provides quite accurate results, within 10-15% of experimental measurements.

The above relationships have been found in the hypothesis in which the windings completely fill the available area, but they are still valid when the winding area is not completely used; they allow us to pass from the model parameters to the construction ones (once a magnetic core and bobbin have been chosen) according to the following expressions:

$$N_1 = \sqrt{\frac{L_r}{A_\sigma} \frac{1}{1+k}} \quad N_2 = N_1 \frac{k}{n} = \frac{N_1}{n_t} \quad A_L = \frac{L_{tot}}{(N_1 + N_2)^2} \quad (1)$$

where  $L_{tot}$  is the total inductance of the primary and secondary windings series connected (with the flowing current path entering the dotted terminals):

$$L_{tot} = L_1 + L_2 + 2M = L_1 + L_2 + 2k \sqrt{L_1 L_2}$$

In literature there are several models and expressions for the  $A_L$  value of the core versus the air gap thickness  $l_G$ ; the one proposed by McLyman [3] results in very good agreement with empirical data, and is suitable when the thickness of the air gap is larger than 0.1 mm. The expression below takes the fringing flux around the air gap into account, whose effect (increasing with  $l_G$ ) is to increase the effective cross section of the magnetic path:

$$A_L = \mu_o \frac{A_{CS}}{l_G} \left[ 1 + \frac{l_G}{\sqrt{A_{CS}}} \ln \left( \frac{2d_w}{l_G} \right) \right]$$

where  $A_{CS}$  and  $d_w$  are the ferrite core cross section area and the winding area width, respectively: the above expression shows that  $A_L$  only depends on geometrical parameters (like the specific leakage inductance  $A_\sigma$ ).

## V. DERIVATION OF THE $K_{GM}$ AND $K_{GW}$ CONSTANTS

In this section, a set of basic constraining equations is presented that, if combined together, allows selecting the minimum core size of the transformer that fulfills the electrical specifications and guarantees a maximum temperature rise. The first constraint is derived by Faraday's law applied on the transformer secondary side:

$$N_2 \Delta\Phi = N_2 \Delta B A_e = \int_0^{T/2} v_o(t) dt = \frac{V_{out} + V_F}{2 f_{op}} = \frac{V_o}{2 f_{op}}$$

where  $\Delta\Phi$  is the flux generated by the magnetizing current that is linked to all primary and secondary turns; the corresponding peak value of the flux density  $B_{pk} = \Delta B/2$  is responsible for the core losses. The worst condition occurs at the resonance frequency, when the resonant tank circuit design is made according to the procedure presented in [2], otherwise the minimum operating frequency has to be chosen, in which case the above equation overestimates the flux density: here we follow the  $f_{op} = f_{res}$  option. Taking the primary to secondary turn ratio into account, the above relationship becomes:

$$N_1 = \frac{n V_o}{4 k f_{res} B_{pk} A_e} \quad (2)$$

where  $A_e$  is the effective ferrite core cross section and  $V_o$  is the transformer output voltage, which includes the diode voltage drop  $V_F$  as well. Of course, the peak operating flux density  $B_{pk}$  must be lower than the saturation value  $B_{sat}$ .

The second constraint is represented by the modified Steinmetz equation that expresses the core losses as a function of the core volume  $V_e$ , the operating frequency  $f_{op}$  and the peak flux density  $B_{pk}$  (supposed sinusoidal):

$$P_{fe} = \left( \frac{8}{\pi} \right)^{\alpha-1} V_e K_m f_{op}^\alpha B_{pk}^\beta \quad (3)$$

The parameters  $K_m$ ,  $\alpha$  and  $\beta$  depend on the chosen material grade and can be derived from the data or loss charts provided by the ferrite manufacturers for the specific material. The numerical coefficient  $(8/\pi^2)^{\alpha-1}$  is a form factor taking into account that the applied voltage (and hence induction) is a square waveform with 50% duty cycle.

A further constraint is that the temperature rise caused by the total losses should not exceed the maximum allowed by specification, that is:

$$\Delta T_{max} \leq R_{th} P_{diss} = R_{th} (P_{fe} + P_{cu}) \quad (4)$$

where  $P_{fe}$  and  $P_{cu}$  are respectively the core and copper losses; it is assumed that the temperature rise will be accordingly apportioned:

$$\Delta T_{cu} = \frac{P_{cu}}{P_{diss}} \Delta T_{max} = K_{cu} \Delta T_{max} \quad (5)$$

$$\Delta T_{fe} = \frac{P_{fe}}{P_{diss}} \Delta T_{max} = (1 - K_{cu}) \Delta T_{max}$$

where  $\Delta T_{cu}$  and  $\Delta T_{fe}$  are the amount of temperature rise associated to the copper losses and to the ferrite core losses, respectively. The thermal resistance  $R_{th}$  of the magnetic structure is usually specified by the supplier: an empirical formula is also available, which estimates the thermal resistance as a function of the core area product AP (product of the cross section area  $A_e$  and the winding area  $A_w$  of the ferrite core):

$$R_{th} = 23 AP^{-0.37} \text{ } ^\circ\text{C/W} \quad (\text{with AP in cm}^4)$$

Another constraint is constituted by a further empirical equation that relates the current density  $J$  (responsible for the temperature rise  $\Delta T_{cu}$  inside the winding area) to the area product AP (expressed in cm<sup>4</sup>):

$$J = J_{30} \sqrt{\frac{\Delta T_{cu}}{30^\circ\text{C}}} AP^{-0.24} = J_{30} \sqrt{K_{cu} \frac{\Delta T_{max}}{30^\circ\text{C}}} AP^{-0.24} \quad (6)$$

where  $J_{30}=420 \text{ A/cm}^2$  is the current density that produces 30°C temperature rise in the windings; it is worth pointing out that AP in (6) is dimensionless: it just represents the number of cm<sup>4</sup> of core area product.

Considering that the primary ampere-turns equal the current density times the total primary conductor area, we can write:

$$N_1 I_{p,rms} = K_{ut} A_w J \quad (7)$$

where  $I_{p,rms}$  is the rms value of the resonant tank current



flowing in the transformer primary winding and  $K_{ut}$  is the window utilization factor (that is the ratio between the primary winding copper area to the winding area); this factor, in case Litz wire is used for windings (the real case in this application type, in order to reduce the copper losses due to skin and proximity effect), can be as low as 0.2.

The last constraint is the relationship, already found in the previous section, between the specific leakage inductance  $A_\sigma$ , the number of primary turns and the total leakage inductance, here rewritten for convenience:

$$L_r = A_\sigma (1+k) N_1^2 = \Lambda_\sigma \mu_o (1+k) N_1^2 \quad (8)$$

It is worth pointing out that the quantity  $\Lambda_\sigma = A_\sigma / \mu_o$  is dimensionally a length and only depends on the core and coil-former geometry.

Combining the equations (2) to (5) with (8), it is possible to write the following expression:

$$\frac{A_e^2}{\Lambda_\sigma} \left( \frac{1}{V_e R_{th}} \right)^{\frac{2}{\beta}} = \frac{\mu_o L_r}{1-k} \left( \frac{n V_o}{4k f_{res}} \right)^2 \left[ \frac{K_m f_{res}^\alpha}{(1-K_{cu}) \Delta T_{max}} \left( \frac{8}{\pi^2} \right)^{\alpha-1} \right]^{\frac{2}{\beta}} \quad (9)$$

Combining the equations (5) to (8), a second expression can be found:

$$A_w^2 \Lambda_\sigma A P^{-0.48} = \mu_o \left( \frac{I_{p,rms}}{K_{ut} J_{30}} \right)^2 \frac{1+k}{K_{cu}} \frac{30^\circ C}{\Delta T_{max}} \quad (10)$$

The terms on the left side of equations (8) and (9) depend on the core geometry (and  $\beta$  coefficient), while the terms on the right side depend on the circuit and application parameters and chosen material grade. Therefore the left side of above equations can be defined as the core geometrical constants  $K_{GM}$  and  $K_{GW}$ :

$$K_{GM} = \frac{A_e^2}{\Lambda_\sigma} \left( \frac{1}{V_e R_{th}} \right)^{\frac{2}{\beta}} \quad K_{GW} = A_w^2 \Lambda_\sigma A P^{-0.48} \quad (11)$$

Hence, to design a transformer which integrates the series and shunt inductances for the LLC resonant converter application, first the suitable material grade has to be chosen, depending on the operating frequency range of the application; then the second terms of equations (9) and (10) have to be calculated and finally, a core has to be selected, whose geometrical constants exceed these values:

$$K_{GM} \geq \mu_o \left( \frac{n V_o}{4k f_{res}} \right)^2 \frac{L_r}{1-k} \left[ \frac{K_m f_{res}^\alpha}{(1-K_{cu}) \Delta T_{max}} \left( \frac{8}{\pi^2} \right)^{\alpha-1} \right]^{\frac{2}{\beta}} \quad (12)$$

$$K_{GW} \geq \mu_o \left( \frac{I_{p,rms}}{K_{ut} J_{30}} \right)^2 \frac{1+k}{K_{cu}} \frac{30^\circ C}{\Delta T_{max}}$$

The first core parameter,  $K_{GM}$ , is related to the capability of the chosen ferrite core to handle the losses into the magnetic material, while assuring the required leakage: therefore, in case the inequality related to this parameter is not verified with a chosen material grade, the designer can try to use a better one, if available, with lower specific losses, and check again if the first

inequality in (12) is satisfied. The second core parameter,  $K_{GW}$ , is related to the core capability to dissipate the Joule losses generated inside the copper windings. In this procedure, the parameter  $K_{cu}$  (apportionment of copper to total losses) is arbitrarily chosen: it can be initially set to 50% and then, in case only one of the two inequalities above is verified, it can be changed in the attempt to get both of them verified for the chosen core. For example, in case the second inequality is not verified, it is possible to gradually increase  $K_{cu}$  and check if in the end both the relationships are true, that is, the ferrite core is suitable to manage the total amount of power dissipation without exceeding the maximum allowed temperature rise.

## VI. DESIGN PROCEDURE

Based on the analysis presented so far, the following procedure is proposed to design the integrated transformer in Fig. 2. The input data are the values of parameters  $n$ ,  $L_r$ ,  $L_m$ , previously calculated according to [2] in order to fulfill the converter specifications. Further input data are the maximum allowed temperature rise  $\Delta T_{max}$ , the output voltage  $V_o$  (including the output diode forward drop), the rms value of current flowing into the primary and secondary windings ( $I_{p,rms}$  and  $I_{s,rms}$ ), the resonance frequency  $f_r$  and the operating frequency range ( $f_{min} - f_{max}$ ).

1- first, select a material grade suitable for the operating frequency range of the application, identifying the parameters  $K_m$ ,  $\alpha$  and  $\beta$ .

2- then, calculate the terms on the right side of expressions (12), considering that the coupling coefficient  $k$  is found through:  $k = \sqrt{L_m / (L_r + L_m)}$

3- select a core-bobbin shape and size such that the core geometrical constants  $K_{GM}$  and  $K_{GW}$ , defined in (11), satisfy the inequalities in (12). The designer can initially set the copper to total losses apportionment  $K_{cu} = 50\%$  and then play with it ( $\pm 15\%$ ) if needed, in order to get the inequalities verified. In case the inequalities are both satisfied, the designer can try to reduce the core size by using a better material grade, with lower losses in the application frequency range.

4- calculate the number of primary turns  $N_1$ , to get the required leakage inductance, by using the first equation of (1): this number is only the first attempt and may need to be fine tuned once the first sample is built and measured. Also, define the number of secondary turns  $N_2$  and the  $A_L$  value through the other two equations of (1).

5- define the wire section for primary and secondary windings. High frequency copper losses must be particularly addressed, because eddy currents and proximity losses are considerable and, consequently, suggest the use of Litz wires or multi-strand wires for both primary and secondary windings. If the strand diameter and the number of strands are optimized (see [4]) to minimize the total copper losses (ohmic and eddy), usually this leads to select a wire cross section such that the losses due to eddy currents are one half of the ohmic

losses: this means that the ohmic losses  $P_{cu,ohm}$  are 2/3 of the maximum allowed copper losses  $P_{cu}$ . From this consideration the designer can find out the required wire section of the primary and secondary windings: the last step is then to select the strand configuration (strand diameter and number of parallel strands with the calculated total cross section) that presents the lowest ac resistance.

Throughout the various sections presented so far, the transformer has been considered composed of one primary winding and one secondary winding (single ended), that is the case where the full-bridge rectification is used at converter output. In case of secondary center-tap windings and full-wave rectification (as in Fig. 1), the rms value of the current flowing into each secondary winding is  $1/\sqrt{2}$  times the current flowing through the single-ended winding of the previous output configuration: therefore the wire section of each center-tap can be half the copper section, necessary for the single-ended secondary winding (while the number of secondary turns  $N_2$  is obviously the same in both cases).

There is another important aspect to consider in case of a transformer with secondary center-tap: the two windings need to be tightly coupled to each other, so that the leakage inductance, measured on the primary side with either secondary winding short circuited, is almost the same. In fact, during each half cycle of operation, the resonant capacitor in Fig. 2 rings with the leakage inductance between the primary winding and the corresponding center-tap winding that is conducting current on the secondary side. Therefore, in case these two leakage inductances are substantially different, the tank current will be not symmetrical and the circuit will exhibit two different resonance frequencies during each half cycle, that can make it very difficult (or even impossible) to compensate the feedback loop of the converter. In order to get good coupling between the center-tap windings, a good practice is to wound them in parallel (in bifilar).

An integrated transformer with secondary center-tap for the LLC resonant converter specified in Table I has been designed, following the proposed procedure and a prototype has been built and tested on the bench. The resonant tank parameters, calculated through the procedure in [2], are summarized in Table II and used as input data for the transformer design. The chosen ferrite core is ETD49, grade 3F3 Ferroxcube (with  $\alpha=1.6$ ,  $\beta=2.5$   $K_m=0.25$  W/m<sup>3</sup>), for which the core parameters are:  $K_{GM}=829.3$  cm<sup>3</sup>(W/°C m<sup>3</sup>)<sup>2/β</sup>,  $K_{GW}=29.6$  cm<sup>5</sup> (assuming  $R_{th}=8$ °C/W and  $\Lambda_o=5.05$  cm). The second terms of (9) and (10), calculated for  $K_{cu}=0.5$  and  $\Delta T_{max}=40$ °C, make the inequalities (12) verified according to:  $K_{GM} \geq 737.4$  cm<sup>3</sup>(W/°C m<sup>3</sup>)<sup>2/β</sup> and  $K_{GW} \geq 10.6$  cm<sup>5</sup>. The prototype has been built with  $N_1=23$  primary turns (calculated 21.4) of litz wire 30x0.2 mm,  $N_2=4$  secondary turns of Litz wire 75x0.2 mm, for each center-tap winding, and a gap of about 0.45mm. The coil-former has a spacer between the two slots, whose thickness is  $d_s=3$  mm.

TABLE I  
DESIGN SPECIFICATION OF THE EXEMPLARY LLC CONVERTER

Parameter	Symbol	Value	Unit
Input voltage range	$V_{inmin} - V_{inmax}$	320 - 430	V
Nominal input voltage	$V_{innom}$	390	V
Regulated output voltage	$V_{out}$	36	V
Maximum (peak) output current	$I_{out}$	8.35	A
Expected efficiency (@ $V_{inmin}$ )	$\eta$	95	%
Parasitic capacitance of node HB	$C_{HB}$	200	pF
Dead time of driver circuit	$T_d$	200	ns

TABLE II  
RESONANT TANK ELECTRICAL AND OPERATING PARAMETERS

Parameter	Symbol	Value	Unit
Resonance frequency	$f_r$	120	kHz
Turn ratio of transformer in Fig. 2	$n$	5.335	-
Shunt inductance (see Fig. 2)	$L_m$	305	μH
Series inductance (see Fig. 2)	$L_r$	56	μH
Coupling coefficient	$k$	0.92	-
Tank primary current (rms value)	$I_{p,rms}$	2.1	A
Total rectified output current (rms value)	$I_{s,rms}$	9.3	A

## CONCLUSIONS

In this article a design procedure has been outlined to completely define the parameters of a transformer ( $N_1$ ,  $N_2$ ,  $I_G$ ) that integrates all the magnetic parts of an LLC resonant converter (Fig. 2) into a single component. This procedure allows the designer to choose the minimum ferrite core size to satisfy the electrical specifications and guarantee a maximum temperature rise of the transformer, once a suitable ferrite material grade has been selected for the operating frequency range. For this purpose, the two constants  $K_{GM}$  and  $K_{GW}$  have been defined, related to the core and bobbin geometry, that estimate the capability of the core-bobbin assembly to handle the total losses, while assuring the required leakage inductance, necessary for the correct operation of the resonant tank. These two constants also depend on a further parameter of the core-bobbin set: the specific leakage inductance  $\Lambda_o$  (or equivalently  $\Lambda_o$ ), that has been estimated (in section IV) using the energy calculation approach, that is, by evaluating the energy associated to the leakage flux in the winding volume.

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