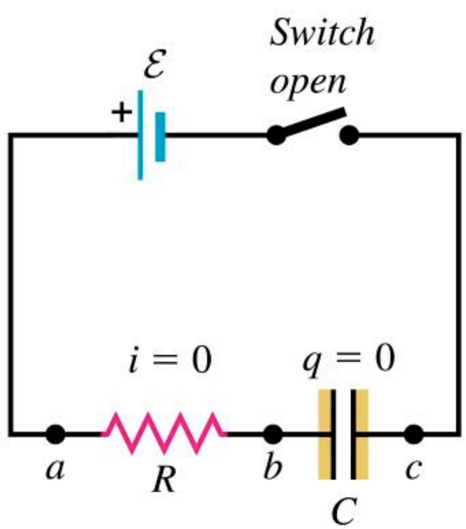
Time dependent circuits - The RC circuit

Example 1 – Charging a Capacitor-

Up until now we have assumed that the emfs and resistances are ***constant*** *in time, so that all potentials, currents and powers are constant in time. However, whenever we have a capacitor that is being charged, or discharged, this is not the case.*

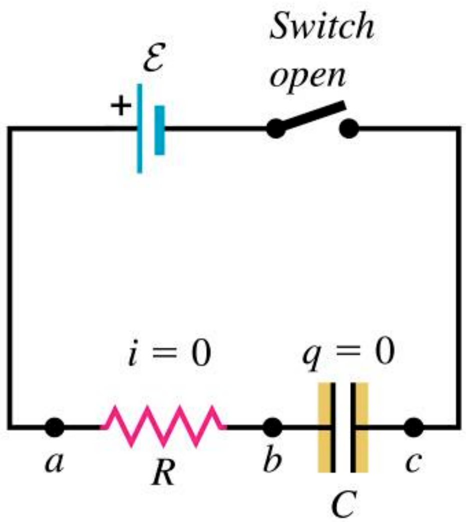
Now, consider a circuit that consists of an emf, a resistor and a capacitor, but with an open switch

With the switch open the current in the circuit is zero and zero charge accumulates on the capacitor

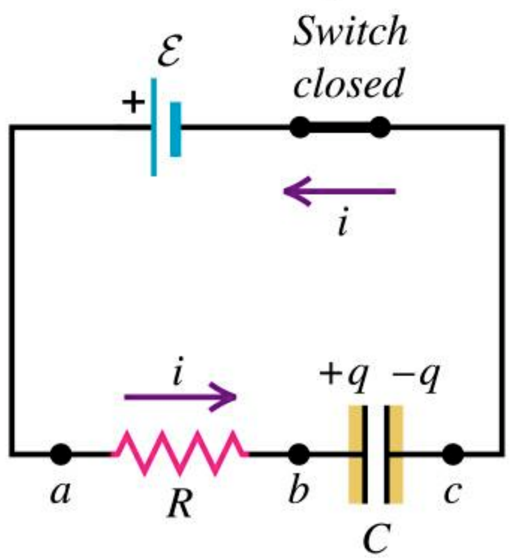


Time dependent circuits - The R-C circuit

Example 1 – Charging a Capacitor cont.

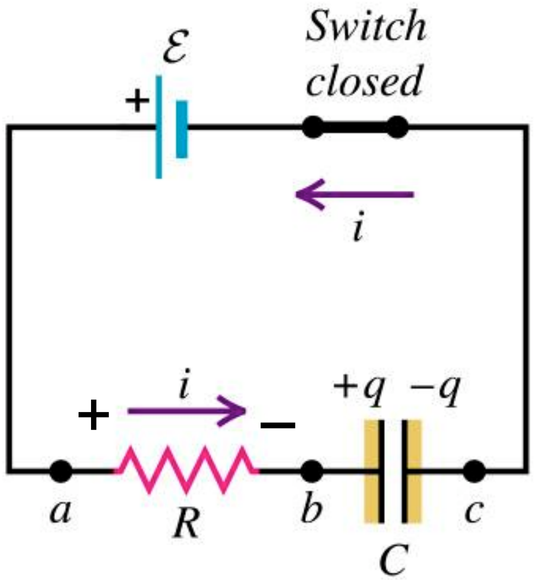


Capacitor has no net charge at t=0



Voltage, current and charge are functions of time (we will use lower case letters to denote time-varying quantities).

However – Kirchhoff's Rules are still valid at any instant in Time



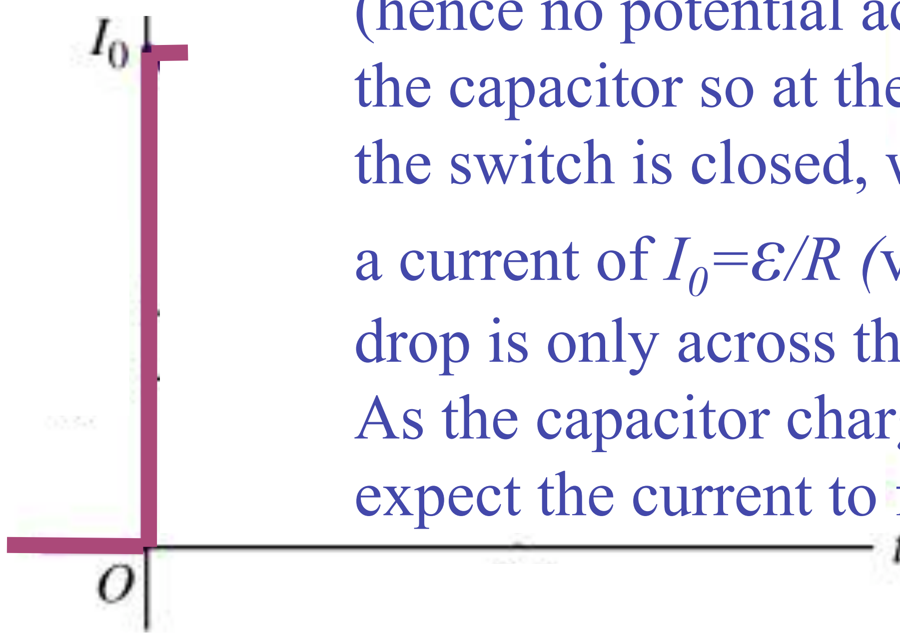
$$v_{ab} = iR$$ Ohms Law

$$v_{bc} = \frac{q}{C}$$ Definition of Capacitance

$$\varepsilon - iR - \frac{q}{C} = 0$$ Kirchhoff's Loop Rule

$$i = \frac{\varepsilon}{R} - \frac{q}{RC}$$

Close Switch at t=0



At t=0, there is no charge on (hence no potential across) the capacitor so at the "instant" the switch is closed, we expect a current of $I_0 = \mathcal{E}/R$ (voltage drop is only across the resistor). As the capacitor charges, we expect the current to fall.

Charging a capacitor cont.

$$i = \frac{\varepsilon}{R} - \frac{q}{RC}$$

- Charge will flow on the capacitor and the current will decrease until we reach $i = 0$, then:

$$\frac{\varepsilon}{R} = \frac{Q_f}{RC}$$

$$Q_f = C\varepsilon$$

- Note, the final value charge, Q_f does not depend on R

- So, we start with current $I_o = \varepsilon/R$ and the current reduced to $I_f = 0$, at the same time the charge on the capacitor increased from $Q_0 = 0$ to $Q_f = C\varepsilon$. But, what is the current or the charge on the capacitor at any instant in time?

Charging a capacitor

$$i = \frac{dq}{dt}$$

$$i = \frac{\varepsilon}{R} - \frac{q}{RC} \longrightarrow \boxed{\frac{dq}{dt} = \frac{\varepsilon}{R} - \frac{q}{RC}}$$

This is a differential equation. We can rearrange and integrate to find what we need.

$$\frac{dq}{dt} = -\frac{1}{RC}(q - C\varepsilon)$$

$$\int_0^q \frac{dq'}{q' - C\varepsilon} = \int_0^t \frac{dt'}{RC}$$

This yields:

$$\ln\left(\frac{q - C\varepsilon}{-C\varepsilon}\right) = -\frac{t}{RC}$$

Charging a capacitor cont.

$$\ln\left(\frac{q - C\varepsilon}{-C\varepsilon}\right) = -\frac{t}{RC}$$

Taking the exponents (inverse log) both sides:

$$\frac{q - C\varepsilon}{-C\varepsilon} = e^{-\frac{t}{RC}}$$

Solving for q (now we're getting to the bit you need)

$$q = C\mathcal{E}\left(1 - e^{-t/RC}\right) = Q_{\mathrm{f}}\left(1 - e^{-t/RC}\right) \qquad (26.12)$$

(*R*-*C* circuit, charging capacitor)

Taking the time derivative, to get the instantaneous current, i

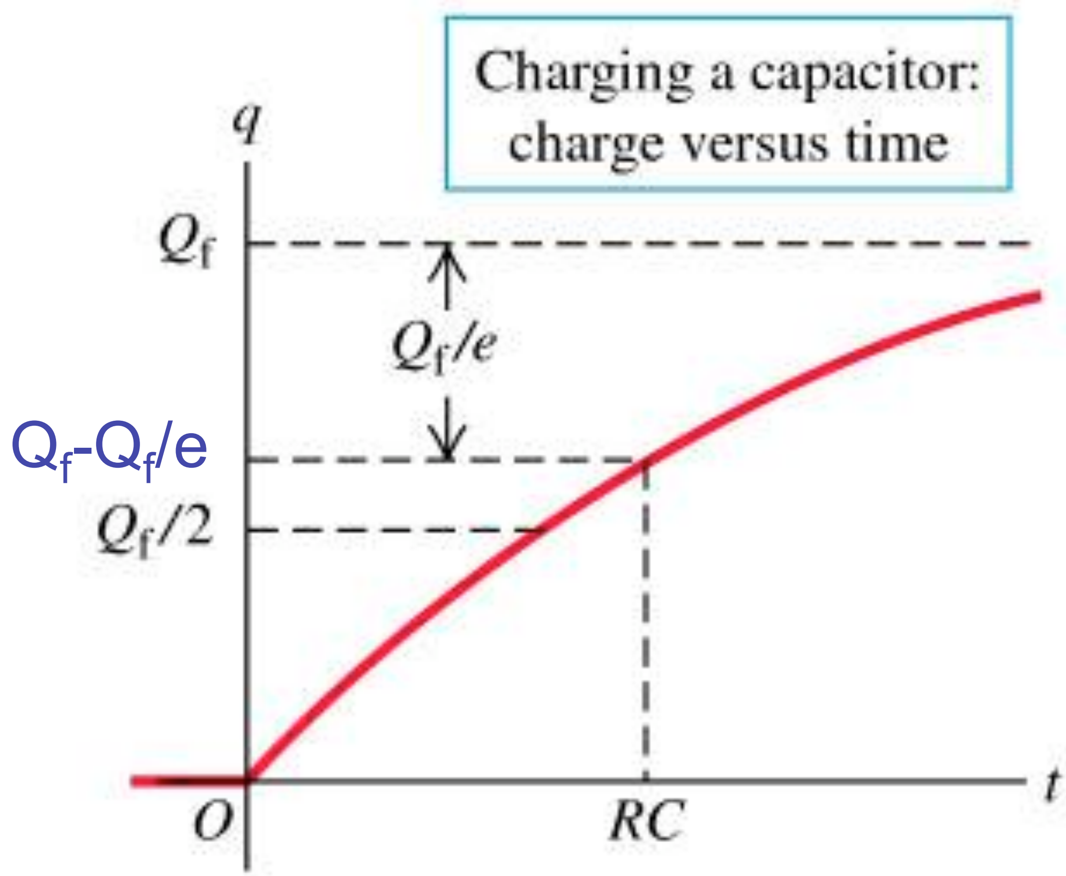
$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R}e^{-t/RC} = I_0 e^{-t/RC} \qquad (26.13)$$

(*R*-*C* circuit, charging capacitor)

Charging a capacitor instantaneous charge

$$q = C\mathcal{E}(1 - e^{-t/RC}) = Q_f(1 - e^{-t/RC}) \qquad (26.12)$$

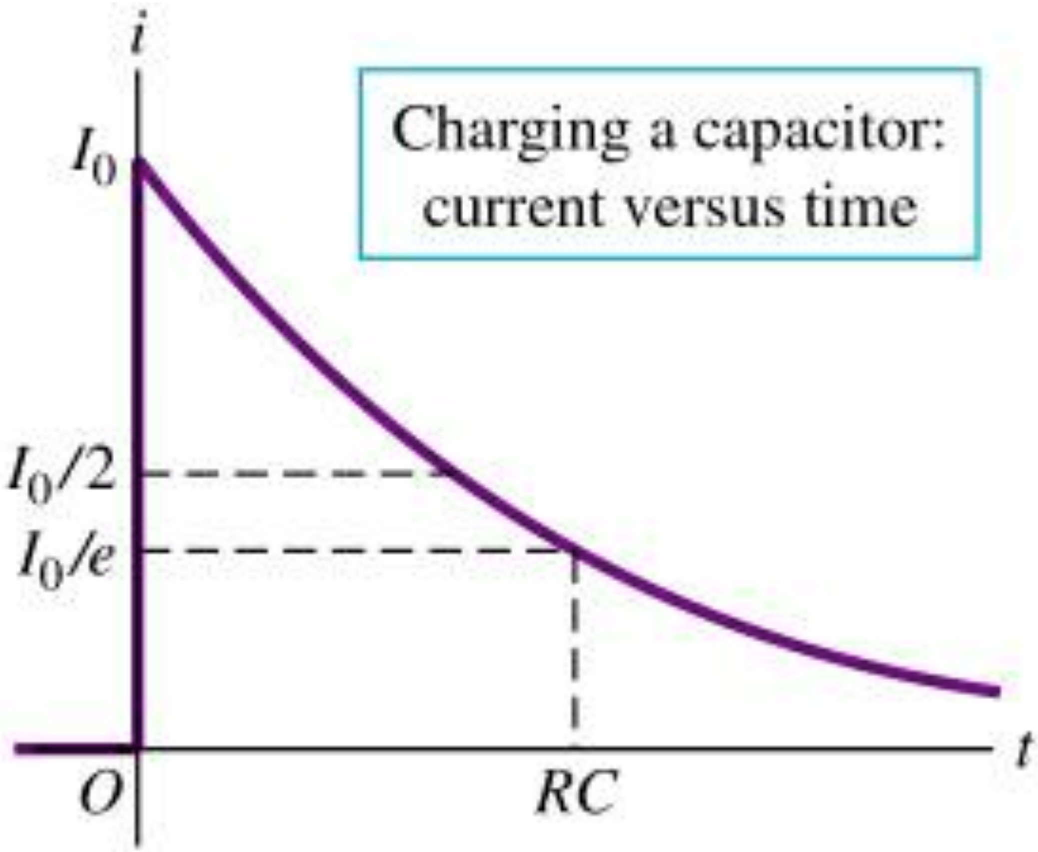
(*R*-*C* circuit, charging capacitor)



Remember:
e=2.718....

Charging a capacitor - instantaneous current

$$i = I_0 e^{-t/RC}$$

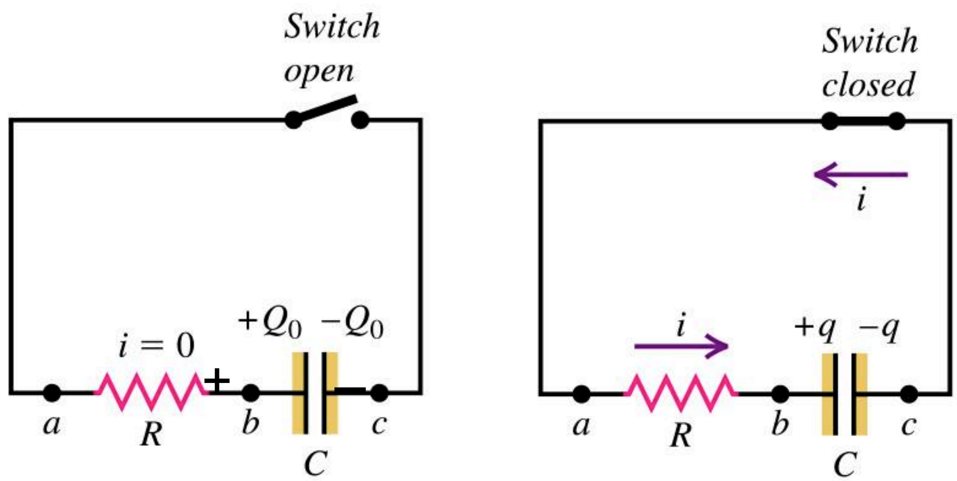


R-C circuits - time constant

- After a time *RC*, the current in the R-C circuit has decreased to 1/e = 0.368 of its initial value.
- At this time, the capacitor charge has reached (1-1/e) = 0.632 of its final value $Q_f = C\varepsilon$.
- The product *RC* is therefore a measure of how quickly the capacitor charges.
- We call *RC* the **time constant** (τ) of the circuit.

$$\tau = RC$$

Discharging a capacitor



Now we start with a charged capacitor in series with an open switch (we have removed the battery from the earlier charging circuit).

Now we close the switch and current will flow. We define time t=0 as the time we close the switch and $q=Q_0$ at that time. We discharge the capacitor through the resistor fully until $q=0$

Mr. Kirchhoff says (loop rule): $$\varepsilon - iR - \frac{q}{C} = 0$$

but, there's no EMF, rearranging we get: $$i = \frac{dq}{dt} = -\frac{q}{RC}$$

Discharging a capacitor - graphs

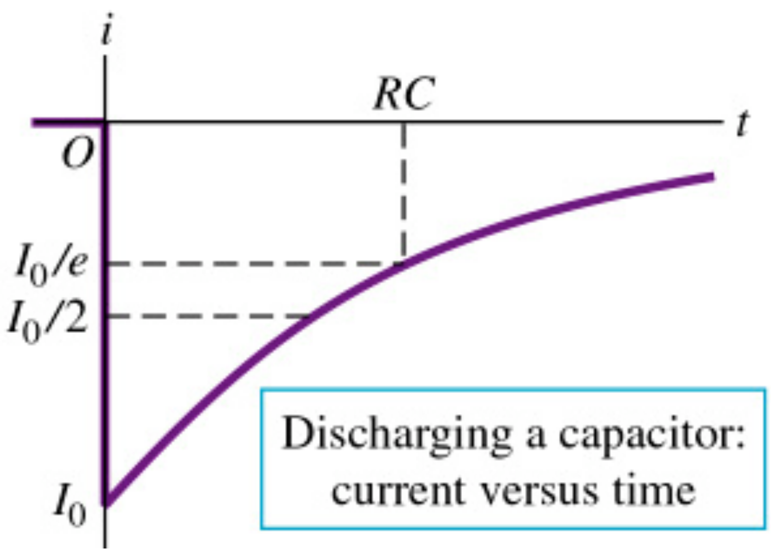
$$i = \frac{dq}{dt} = -\frac{q}{RC}$$

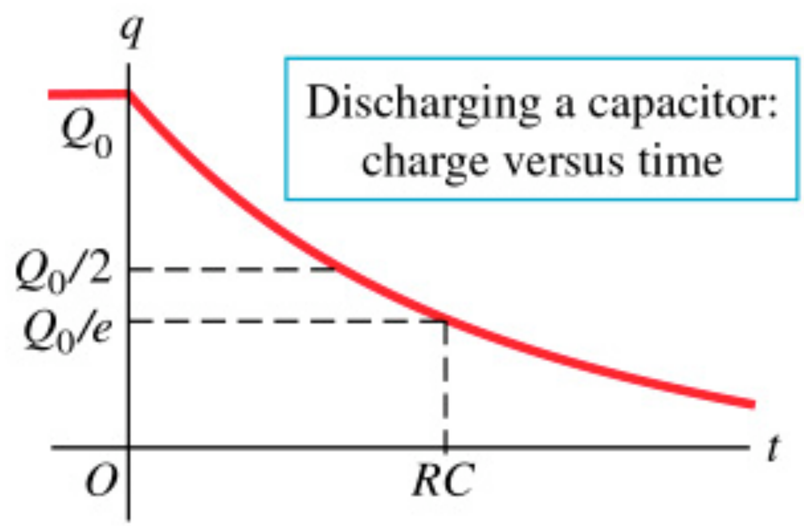
$$I_0 = -\frac{Q_0}{RC}$$

Solving the same way as for charging (see Y&F p1000) we obtain

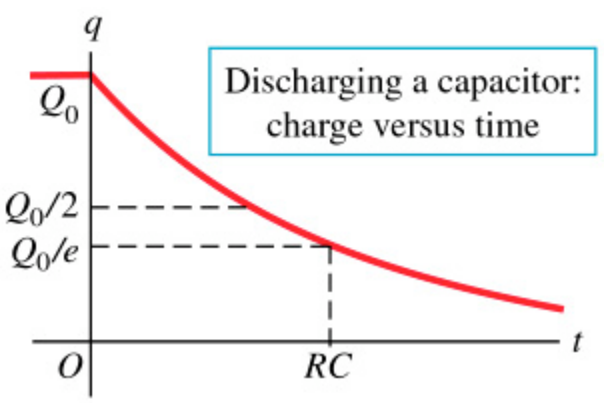
$$\boxed{q = Q_0 e^{-t/RC}}$$

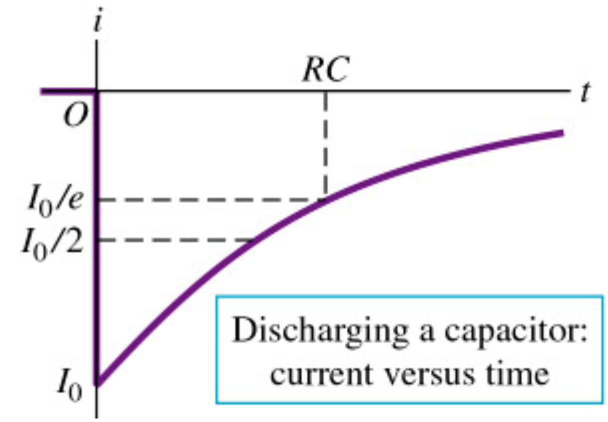
$$\boxed{I = I_0 e^{-t/RC}}$$

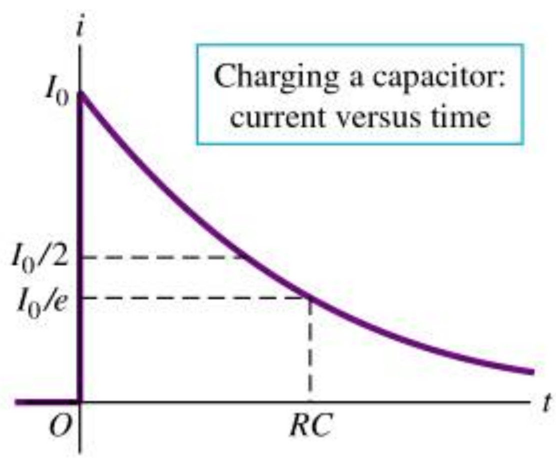


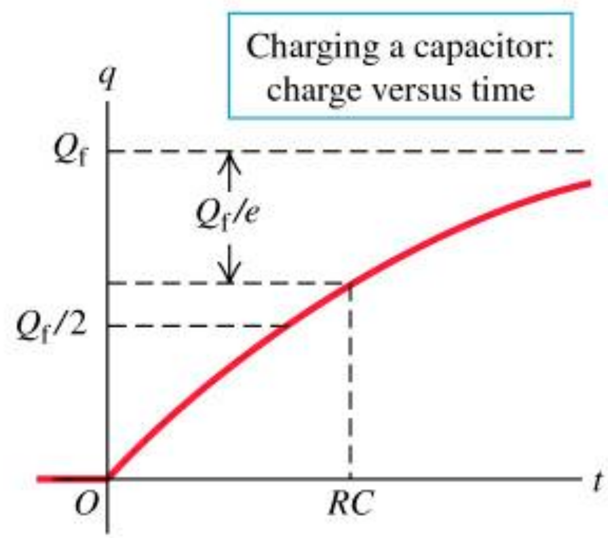


Charging/discharging capacitors
Which curve?









Charging/discharging capacitors
Which curve?

Strategy for understanding which curve is correct:

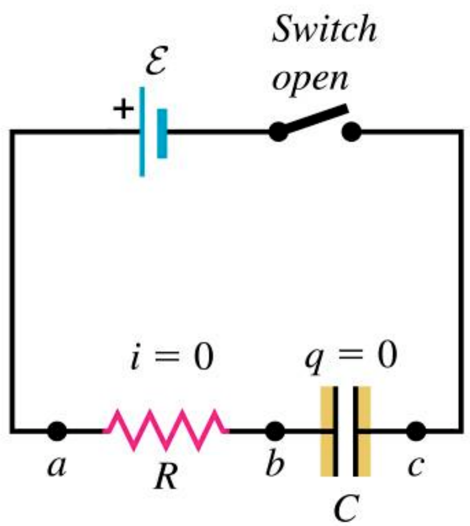
Try to figure out what the current (or charge) would be just when the switch is closed (or opened)

$$t = 0$$

Then try to figure out what the current (or charge) would be aftera very long time

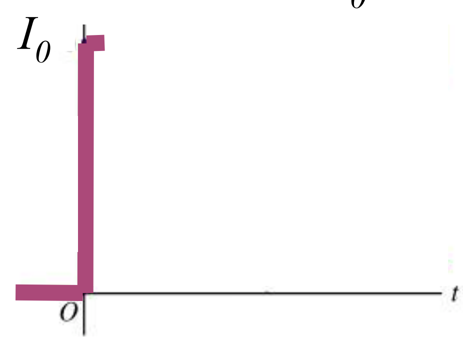
$$t = \infty$$

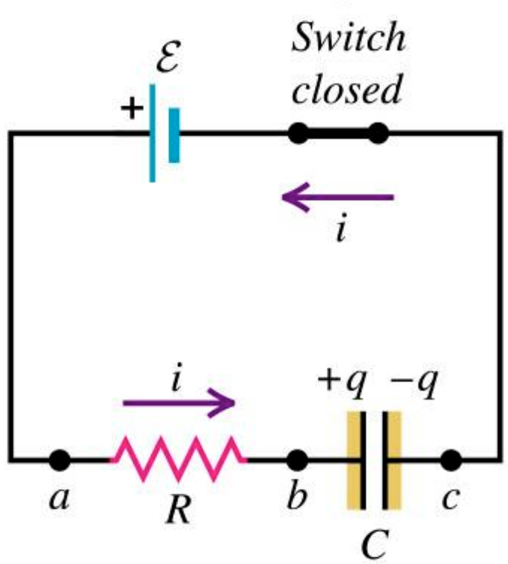
Figuring out charging a capacitor



At t=0 there is NO charge on the capacitor – and therefore no voltage across capacitor

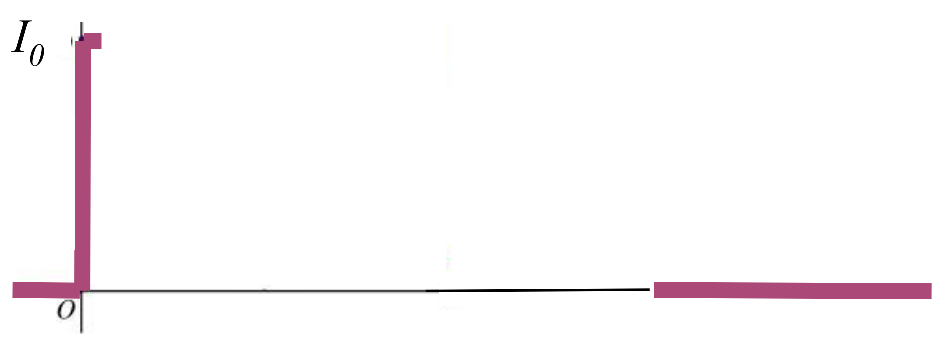
$$I_0=\varepsilon/R$$



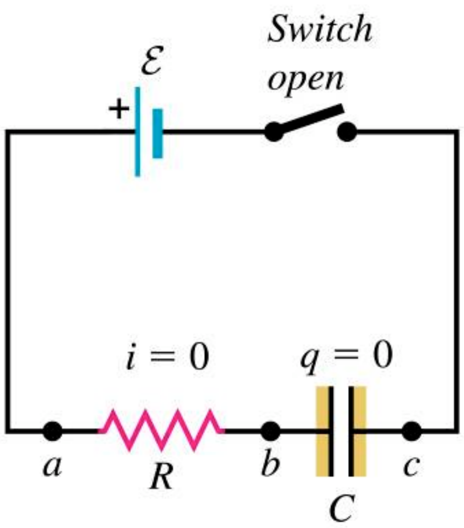


At t=∞ the capacitor is fully charged and there is no current:

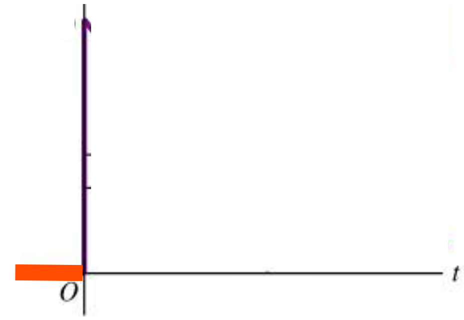
$$I_\infty=0$$

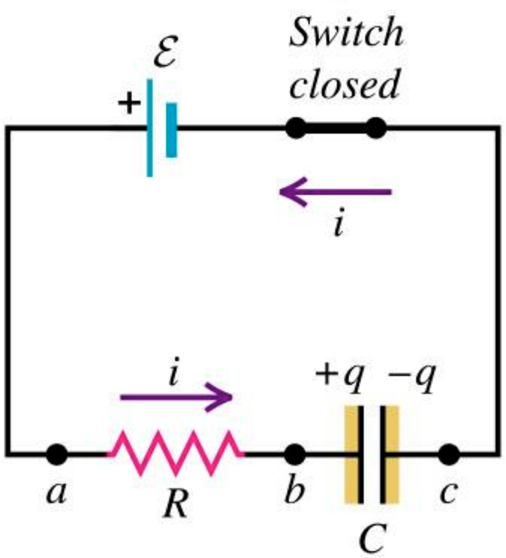


Figuring out charging a capacitor

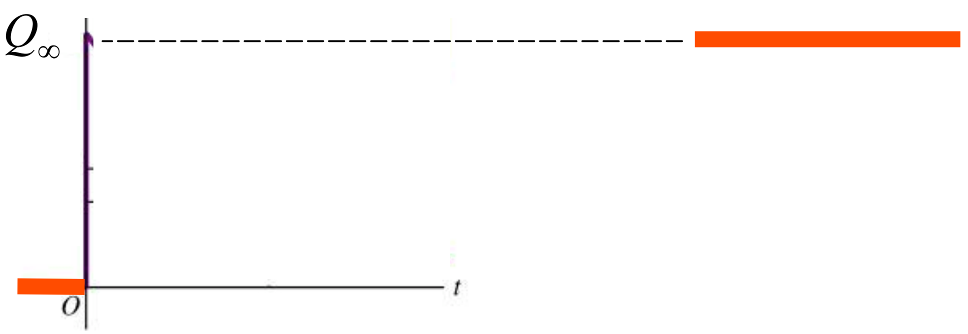


At t=0 there is NO charge on the capacitor – therefore Q_0=0





At t=∞ the capacitor is fully charged and therefore $Q_\infty=C\varepsilon$



Example: Ex 26.42 from Y&F

26.42 In the circuit shown in Fig. 26.45, $C = 5.90\ \mu F$, $\mathcal{E} = 28.0\ V$, and the emf has negligible resistance. Initially the capacitor is uncharged and the switch S is in position 1. The switch is then moved to position 2, so that the capacitor begins to charge. a) What will be the charge on the capacitor a long time after the switch is moved to position 2? b) After the switch

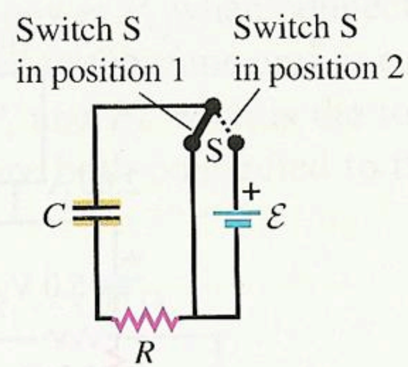
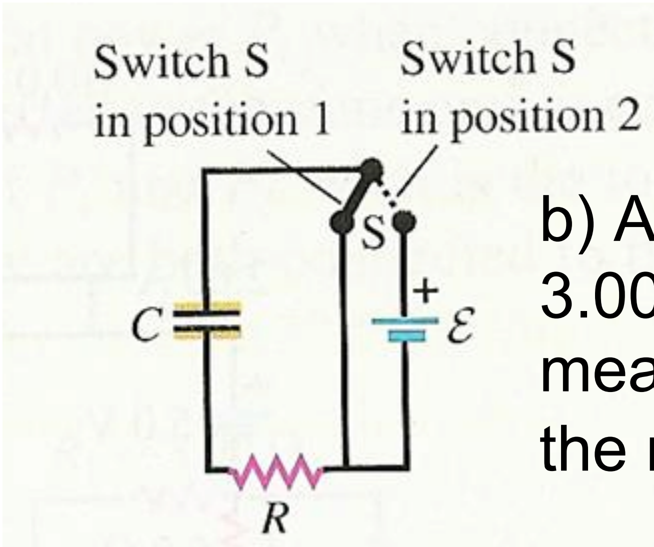


Figure 26.45 Exercises 26.42 and 26.43.

Before the switch is moved, the capacitor is uncharged: q_0=0. Moving the switch to position 2 closes the circuit and the capacitor will charge. After "a long time" i.e. $t >> RC$ the capacitor will be fully charged, no current flows and the voltage across the capacitor is equal to the emf.

$$Q_{final} = CV = (5.90 \times 10^{-6} F)(28.0V)$$

$$Q_{final} = 1.65 \times 10^{-4} C$$



b) After the switch has been in position 2 for 3.00ms, the charge on the capacitor is measured to be 110μC. What is the value of the resistance R?

$$q = Q_f(1 - e^{-t/RC}) \Rightarrow \frac{q}{Q_f} = 1 - e^{-t/RC} \Rightarrow e^{-t/RC} = 1 - \frac{q}{Q_f}$$

Need R, so take ln of both sides

$$\frac{-t}{RC} = \ln\left(1 - \frac{q}{Q_f}\right) = \ln\left(1 - \frac{110 \times 10^{-6}C}{1.65 \times 10^{-4}C}\right) = -1.1$$

$$R = \frac{t}{1.1C} = \frac{3.00 \times 10^{-3}}{1.1 \times 5.9 \times 10^{-6}} = 463\Omega$$

Chapter 26 Summary

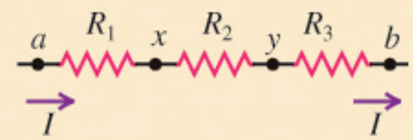
When several resistors $R_1, R_2, R_3, \ldots$, are connected in series, the equivalent resistance R_{eq} is the sum of the individual resistances. The same *current* flows through all the resistors in a series connection. When several resistors are connected in parallel, the reciprocal of the equivalent resistance R_{eq} is the sum of the reciprocals of the individual resistances. All resistors in a parallel connection have the same *potential difference* between their terminals. (See Examples 26.1 and 26.2)

$$R_{eq} = R_1 + R_2 + R_3 + \cdots \qquad (26.1)$$

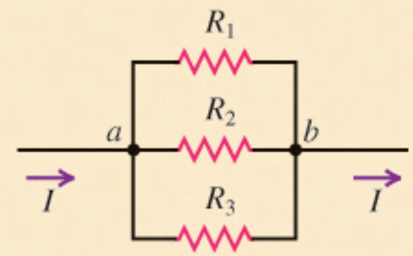
(resistors in series)

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots \qquad (26.2)$$

(resistors in parallel)



R_1, R_2, and R_3 in series



R_1, R_2, and R_3 in parallel

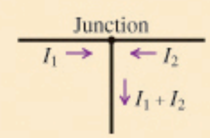
Kirchhoff's junction rule is based on conservation of charge. It states that the algebraic sum of the currents into any junction must be zero. Kirchhoff's loop rule is based on conservation of energy and the conservative nature of electrostatic fields. It states that the algebraic sum of potential differences around any loop must be zero. Careful use of consistent sign rules is essential in applying Kirchhoff's rules.
(See Examples 26.3 through Example 26.7)

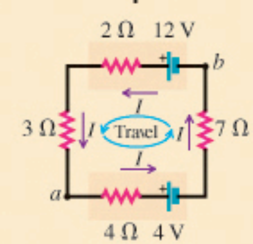
$$\sum I = 0 \qquad (26.5)$$

(junction rule)

$$\sum V = 0 \qquad (26.6)$$

(loop rule)





Chapter 26 Summary cont.

When a capacitor is charged by a battery in series with a resistor, the current and capacitor charge are not constant. The charge approaches its final value asymptotically and the current approaches zero asymptotically. The charge and current in the circuit are given by Eqs. (26.12) and (26.13). After a time $\tau = RC$, the charge has approached within $1/e$ of its final value. This time is called the time constant or relaxation time of the circuit. When the capacitor discharges, the charge and current are given as functions of time by Eqs. (26.16) and (26.17). The time constant is the same for charging and discharging. (See Examples 26.12 and 26.13)

$$q = C\mathcal{E}(1 - e^{-t/RC}) = Q_f(1 - e^{-t/RC}) \quad (26.12)$$

(capacitor charging)

$$i = \frac{dq}{dt} = \frac{\mathcal{E}}{R}e^{-t/RC} = I_0 e^{-t/RC} \quad (26.13)$$

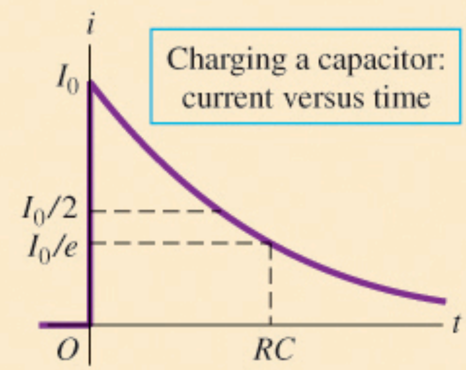
(capacitor charging)

$$q = Q_0 e^{-t/RC} \quad (26.16)$$

(capacitor discharging)

$$i = \frac{dq}{dt} = -\frac{Q_0}{RC}e^{-t/RC} = I_0 e^{-t/RC} \quad (26.17)$$

(capacitor discharging)



End of Chapter 26

You are responsible for the material covered in Y&F Sections 26.1-26.4

You are expected to:

- Understand the following terms:

  Equivalent Resistance, Junction, Kirchhoff junction rule, Kirchhoff's loop rule, R-C circuit, R-C time constant

- Be able to calculate equivalent resistances in simple geometries

- Be able to apply Kirchoff's rules to determine currents and voltages in more complex geometries

- Be able to determine the current and voltage as a function of time in R-C circuits.

Recommended Y&F Exercises chapter 26:

$\boxed{\text{19, 20, 21, 36, 39, 40, 43}}$