

Integral Control of PWM DC-DC Buck-Derived Converters

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Abstract — A concept of integral control is applied for multioperating point buck-derived pulse-width modulated dc-dc converters. A new large-signal circuit model of the power stage is introduced which allows for eliminating the load resistance from the system matrix. Two closed-loop continuous-conduction mode push-pull converters are designed using pole-placement technique: a conventional converter and a converter with a small inductor; what is achievable by a bidirectional power flow. The large-signal dynamic performance of these converters is investigated by simulations in the time domain. The bidirectional power-flow converter with integral control was found to be suitable in switched load applications, e.g., radar systems, dot-matrix printers, and power supplies for random-access computer memories.

I. INTRODUCTION

Pulse-width modulated (PWM) dc-dc converters are nonlinear circuits due to the switching action of power semiconductor switches. Among many methods of modeling and analyzing dc-dc converters, the state-space averaging method developed by Middlebrook and Čuk [1] has been the most popular one. Sacrificing some accuracy, this method leads to linear models of power stages which makes it possible the application of linear control theory techniques in designing of switching dc-dc regulators. The voltage-feedforward control and the current-mode control [2] allows for obtaining good dynamic performance of single operating point converters. Attempts to deal with large-signal disturbances in the load current were made in [3] using a bilinear systems theory. A general algorithm of designing the control loop for single operating point converters with RHP poles in control-to-output transfer function was proposed in [4] based on a LQR problem approach. Some power dc-dc converters must be able, however, to maintain a good dynamic regulation under switched load conditions, e.g., computer random-access memories, radar systems, printheads of dot-matrix printers, some radio transmitters, etc. Thus, there is a need in applying the linear control theory (which is well known to most engineers and has a good software support in many CAD packages) for designing switching regulators with dynamic loads.

The objective of this paper is to describe the concept of the integral control of multioperating point buck-derived dc-dc converters, along with simulation results. As a starting point, a circuit linear model [5] of the power stage of the push-pull converter is obtained in Section II. The inverse absorption theorem is applied to this model. The current

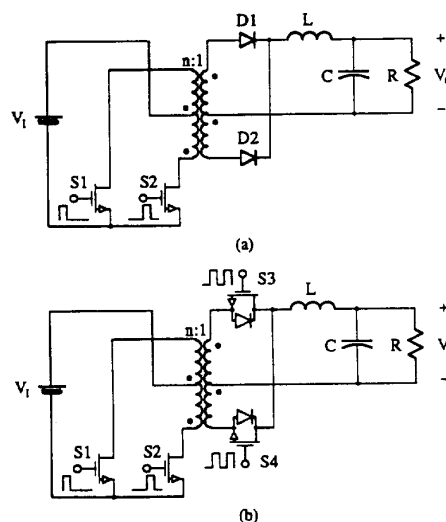


Figure 1: PWM push-pull converter. (a) conventional circuit. (b) bidirectional power-flow circuit.

of the resulting output current sink is an unknown function of time and is considered to be a disturbance to the system. Making the system matrix independent of the load resistance is an important contribution of this paper. In Section III, the state-space model is augmented by adding a reference input. Then, an integral control scheme is applied [6], which is robust, has a zero steady-state error and allows for tracking the reference input. Feedback loop gains are calculated by means of the pole-placement technique with an extensive use of CAD packages. In Section IV, the performance of two closed-loop systems with a switched load is examined. The first circuit is a conventional continuous conduction mode PWM push-pull converter and the other is a bidirectional power-flow PWM push-pull converter. It is shown that the second converter exhibits much better dynamic performance. Conclusions and suggestions for further investigations are given in Section V.

II. LINEAR MODEL OF THE POWER STAGE

A circuit of the push-pull converter is shown in Fig. 1(a). It consists of a dc input voltage source V_i , two MOSFETs

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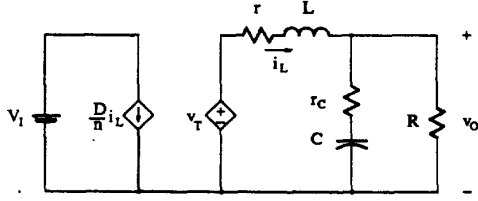


Figure 2: Large-signal circuit model of the push-pull converter power stage.

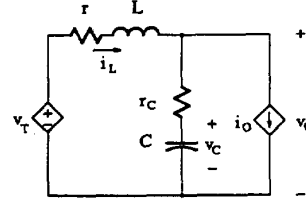


Figure 3: Simplified circuit model of the push-pull converter power stage with a variable load.

as controllable switches $S1$ and $S2$ (BJTs, IGBTs, or MCTs can also be used), an isolation transformer, two diodes $D1$ and $D2$, an inductor L , a filter capacitor C , and a load resistance R . The switches are turned alternately on and off at the switching frequency $f_s = 1/T$ with the ON duty ratio $D = t_{on}/T \leq 0.5$, where t_{on} is the interval when the switch is ON. The modeling of the converter of Fig. 1(a) is carried out under the following assumptions:

- 1) The transistor output capacitances and diode capacitances are neglected; as a result, switching losses are also neglected.
- 2) The transistor ON resistance r_{DS} is linear and the transistor OFF resistance is infinite.
- 3) The diodes in the ON state are modeled by a linear battery V_F and a linear forward resistance R_F and in the OFF state by an infinite resistance.
- 4) The transformer leakage inductances, the magnetizing inductance, the stray capacitances, and the magnetic core parallel resistance are neglected.
- 5) Passive components are linear, time-invariant, and frequency-independent.
- 6) The output impedance of the input voltage source is zero.

Fig. 2 depicts a circuit model of the push-pull converter power stage, where v_T is the voltage of the dependent voltage source, v_O is the converter output voltage, r is the equivalent averaged resistance (EAR) in series with the inductor, and r_C is the ESR of the capacitor. If the ON resistances of the transistors and the winding resistance of the primary of the transformer r_{T1} are reflected to the secondary of the transformer, the EAR in series with the inductor is

$$r = 2D \frac{r_{DS} + r_{T1}}{n^2} + \left(\frac{1}{2} + D\right)(R_F + r_{T2}) + r_L, \quad (1)$$

where r_{T2} is the winding resistance of the secondary of the transformer and r_L is the ESR of the inductor L . The voltage v_T consists of an output voltage of the idealized transformer v_T , and the averaged ON threshold voltage of the diodes and is given by

$$v_T = v_{T_s} - V_F = \frac{2D}{n} v_I - V_F. \quad (2)$$

The structure of the model of Fig. 2 is identical for all buck-derived PWM converters. Small differences are in the expressions for r and v_T only.

In many practical applications, the load resistance is not fixed. It can be a known (e.g., radar systems) or unknown

(e.g., computer memories) function of time. Using the inverse absorption theorem, the load resistance can be replaced by a current sink $i_O = v_O/R$. Henceforth, the output current i_O is treated as a disturbance to the system which results in a system matrix independent of the load resistance. Elimination of the load resistance from the system matrix is an important contribution of this paper. Taking into account the fact that the value of the input current does not affect the voltage v_T , a simplified model of the converter power stage suitable for a controller design is depicted in Fig. 3, where i_L is the current through the inductance L , v_C is the voltage across the capacitance C , and i_O is a time-varying current sink.

III. CONTROL PROBLEM

The dc voltage V_O is the output quantity of the converter of Fig. 1. The steady-state value of the output is given by an external constant or varying voltage source V_{ref} . The control circuit should stabilize the output voltage under variations in the input voltage and the load resistance. The first part of the control task can be accomplished by a feed-forward of the input voltage and the other by a state feedback control.

The model of Fig. 3 is a linear time-invariant system and can be described by a continuous-time state equation

$$\frac{dx(t)}{dt} = Ax(t) + Bu(t) + Ew(t) \quad (3)$$

where x is the $n \times 1$ state vector, u is the $p \times 1$ input vector, w is the $q \times 1$ disturbance vector, A is the $n \times n$ constant state matrix, B is the $n \times p$ constant input matrix, and E is the $n \times q$ constant disturbance matrix. The output equation corresponding to (3) takes the form

$$y(t) = Cx(t) + Du(t) + Fw(t), \quad (4)$$

where y is the $m \times 1$ output vector, and C , D , and F are constant matrices. For the model of Fig. 3, $x = [i_L \ v_C]^T$, $u = v_T$, $w = i_O$, $y = v_O$,

$$A = \begin{bmatrix} -\frac{r+r_C}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \quad E = \begin{bmatrix} \frac{r_C}{L} \\ -\frac{1}{C} \end{bmatrix} \quad (5)$$

and

$$C = [r_C \ 1] \quad D = 0 \quad F = -r_C. \quad (6)$$

For steady-state, $i_L = i_O$ and $v_O = v_C$.

Let us assume that an $m \times 1$ constant vector y_{ref} is the desired output of the system described by (3) and (4). For a constant w , the control problem is to design u such that $\frac{dz}{dt} \rightarrow 0$ as $t \rightarrow \infty$ for system stability, $y \rightarrow y_{ref}$ as $t \rightarrow \infty$ for zero steady-state errors. To solve this problem, an augmented new state vector [6] is defined as

$$z = \begin{bmatrix} x - x_s \\ q - q_s \end{bmatrix}, \quad (7)$$

where

$$q = \int_0^t (y - y_{ref}) dt \quad (8)$$

and x_s, q_s are the steady-state values of x and q , respectively. Using (3) and (4), one obtains the matrix form of a system with reference inputs and disturbances

$$\frac{dz}{dt} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \begin{bmatrix} x \\ q \end{bmatrix} + \begin{bmatrix} B \\ D \end{bmatrix} u + \begin{bmatrix} E & 0 \\ F & -I \end{bmatrix} \begin{bmatrix} w \\ y_{ref} \end{bmatrix}, \quad (9)$$

where I is $m \times m$ unity matrix. The steady-state equation corresponding to (9) is

$$0 = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix} \begin{bmatrix} x_s \\ q_s \end{bmatrix} + \begin{bmatrix} B \\ D \end{bmatrix} u_s + \begin{bmatrix} E & 0 \\ F & -I \end{bmatrix} \begin{bmatrix} w \\ y_{ref} \end{bmatrix}, \quad (10)$$

where u_s is the steady-state value of the control vector u . Substituting (10) into (9) and defining $v = u - u_s$, yields a standard form of state equation

$$\frac{dz}{dt} = Gz + Hv, \quad (11)$$

where

$$G = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix}, \quad H = \begin{bmatrix} B \\ D \end{bmatrix}. \quad (12)$$

Thus, the control problem is reduced to the well-known form of stabilization of the system described by (11). For this purpose, the constant-gain state feedback matrix K can be used, which leads to the control input given by

$$v = -Kz. \quad (13)$$

Partitioning appropriately $K = [K_1 \ K_2]$ yields

$$u - u_s = -K_1(x - x_s) - K_2(q - q_s). \quad (14)$$

Using (8) and taking into account that steady-state terms balance, one obtains from above equation

$$u = -K_1 x - K_2 \int_0^t (y - y_{ref}) dt, \quad (15)$$

which is the control law for the system described by (3) and (4) with nonzero reference input. The block diagram of the closed-loop system with $D = 0$ is depicted in Fig. 4. The control of this system is robust, i.e., $y - y_{ref} = 0$ in the steady-state as long as the system remains stable under its parameter violations.

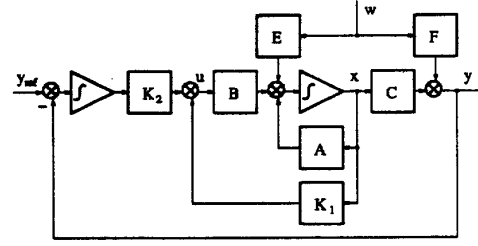


Figure 4: Block diagram of the linearized PWM dc-to-dc converter with integral control.

One of the methods for calculating K utilizes the pole-placement technique. After applying the constant-gain state feedback to system described by (11), the state equation becomes

$$\frac{dz}{dt} = (G - HK)z. \quad (16)$$

The eigenvalues of the closed-loop system matrix $(G - HK)$ can be placed in any specified locations if and only if the pair (G, H) is controllable. The pair (G, H) is controllable if and only if the pair (A, B) is controllable and the matrix

$$P = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad (17)$$

has rank $n+m$. The response of the closed-loop system can be made arbitrarily fast by placing the eigenvalues far away to the left in the s plane. But it demands large and fast control signals. The trade-off between control efforts and the system performance can be achieved by the trial and errors method using computer-aided design tools. Many programs are available for calculating the gain matrix K for given matrices G and H and desired locations of closed-loop system eigenvalues. In Section IV, an example of such a design for a PWM push-pull converter is presented.

Neglecting V_F in (2), one can see that the value of the model control voltage v_T is the averaged value of a rectangular waveform with an amplitude of v_I/n and a duty ratio of $2D$. The duty ratio is determined by comparing an error signal from the feedback with a sawtooth waveform (of the frequency $2f_s$ in the case of the push-pull converter). Since the amplitude of the aforementioned rectangular waveform is directly proportional to the line voltage v_I , the slope of the sawtooth waveform is also made directly proportional to v_I to make the control voltage v_T independent of v_I . This technique is an example of a feedforward control and allows for reducing the influence of the converter input voltage on the output voltage.

IV. COMPUTER-AIDED DESIGN

A PWM push-pull converter of Fig. 1(a) to be used in radar systems was designed to meet the following specifications: constant operating frequency, continuous conduction mode, $V_I = 180-220$ V, $V_O = 6$ V, and variable load resistance R with the output current is in a range from $I_{Omin} = 0.15$ A to $I_{Omax} = 3$ A. The parameters of the converter are: $f_s = 200$ kHz, $n = 16$, $L = 100$ μ H, $C = 220$ μ F, $r_L = 0.3$ Ω , $r_{DS} = 0.5$ Ω , $R_F = 25$ m Ω , $r_{T1} = 50$ m Ω , $r_{T2} = 10$ m Ω , $r_C = 40$ m Ω , and $V_F = 0.7$ V. The EAR in series

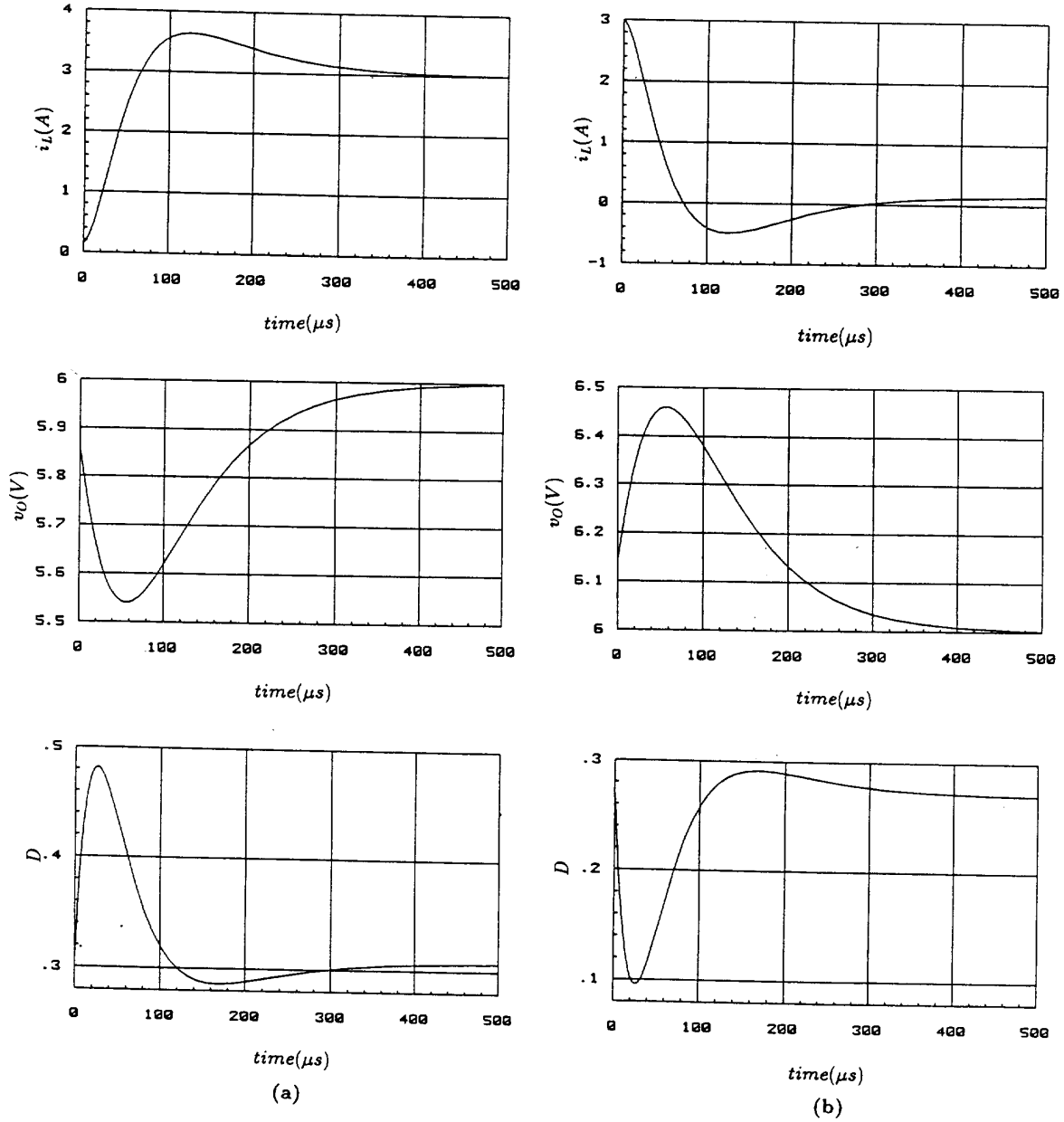


Figure 5: Dynamic response of the inductor current, the output voltage, and the duty ratio of the conventional push-pull converter of Fig. 1(a). (a) to the step in the load current from 0.15 A to 3 A. (b) to the step in the load current from 3 A to 0.15 A.

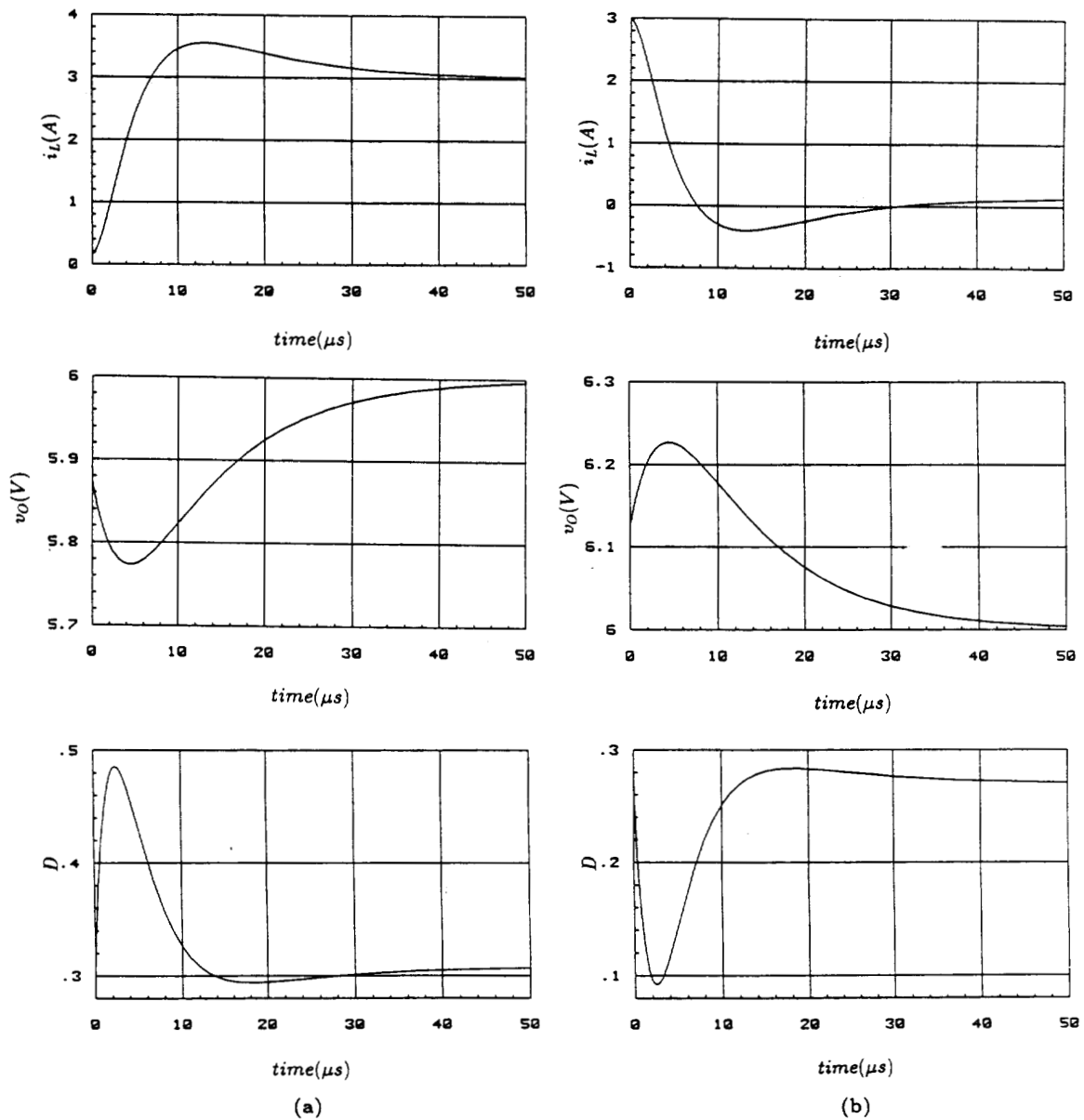


Figure 6: Dynamic response of the inductor current, the output voltage, and the duty ratio of the bidirectional push-pull converter of Fig. 1(b). (a) to the step in the load current from 0.15 A to 3 A. (b) to the step in the load current from 3 A to 0.15 A.

with inductor given by (1) is dominated by r_L . Assuming the steady-state value of $D = 0.25$, r is equal to 330 m Ω .

Substituting circuit parameters values into (5) and (6) and using (12), one obtains the augmented system matrix G which eigenvalues are: 0, $(-1.85 + j6.48) \times 10^3$, and $(-1.85 - j6.48) \times 10^3$. Selecting the values of the closed-loop system poles, one should keep in mind that the resulting feedback gains cannot bring the control signal about exceeding its physical constraints. In the case of the push-pull converter, the duty ratio D is the control signal and it must be held in a range from 0 to 0.5. Simulations show that, for closed-loop system eigenvalues located at -1.5×10^4 , -2.5×10^4 , and -3.5×10^4 , the duty ratio D does not exceed boundaries given above for the worst case of operation, e.g., for the step change in the load current from 0.15 A to 3 A or vice versa. Such closed-loop system eigenvalues result in original system state-feedback gains $K_1 = [7.13 \ 35.51]$ and the integrator gain $K_2 = 2.89 \times 10^5$ which are easily achievable with operational amplifiers. The plots illustrating the dynamic behavior of the inductor current i_L , the output voltage v_O , and the duty ratio D under step load changes are depicted in Fig. 5. One can observe that the dynamic response of the system is not satisfactory, i.e., the settling time about 300 μ s is too long for many switched load applications and the output voltage during transitions differs by almost 0.5 V from its steady-state value. This is because a change in an inductor current takes some time according to the Faraday's law $di_L/dt = V/L$. During the transition time, the entire difference between the charge taken by the output and the charge transferred from the input must be taken from or put into the output filter capacitor resulting in the filter capacitor voltage change and, consequently, the output voltage change. The natural way to reduce the output voltage changes is to make the transition time shorter. For a constant value of V in the Faraday's law, a shorter transition time can be accomplished by reducing the inductance L . The value of the inductance of the push-pull converter given as an example at the beginning of this section, cannot be made smaller because of the condition for continuous conduction mode. Looking, however, at the simulated inductor current waveform for the load current step from 3 A to 0.15 A, one can notice that the inductor current is negative during a part of the transition time. The negative inductor current is impossible in the power stage of Fig. 1(a). To allow for a bidirectional power flow (i.e., the negative inductor current), the diodes must be replaced by bidirectional controllable switches as shown in Fig. 1(b). In the power stage configuration of Fig. 1(b), one does not need to worry anymore about the continuous conduction mode condition. The output ripple voltage due to the ESR of the filter capacitor becomes the only constraint for the minimum value of the inductance L .

Let us change in the aforementioned example of the push-pull converter the value of the inductance into $L = 10 \mu$ H and the value of the capacitance into $C = 47 \mu$ F, leaving all others parameters unchanged. The eigenvalues of a new matrix G are now 0, $(-1.85 + j4.23) \times 10^4$, and $(-1.85 - j4.23) \times 10^4$. A selection of closed loop system eigenvalues at -1×10^5 , -3×10^5 , and -4×10^5 results in original system state-feedback gains $K_1 = [7.63 \ 77.70]$ and the integrator gain $K_2 = 5.64 \times 10^6$. Fig. 6 shows the dynamic behavior of the inductor current i_L , the output voltage v_O , and the duty ratio D under step load changes. It can be seen that the settling time is now only about 23 μ s what makes the converter suitable for supplying loads switched with a frequency in the kilohertz range. Another worth noting feature of the bidirectional converter is that

the shorter settling time reduces the difference between the steady-state value of the output voltage and the maximum/minimum transition value of the output voltage despite of the decrease in the value of the filter capacitance by a factor of more than four. Practically, this difference can be made arbitrarily small without a significant change in the settling time by selecting a sufficient filter capacitance. There are, however, at least two reasons why a small filter capacitor is desirable. One is a power density factor, especially important in automotive, air, space, and robotics applications. The second is a fast dynamic response to changes in the reference input in a case of programmable power supplies.

V. CONCLUSIONS

A concept of an integral control of multioperating point buck-derived PWM dc-dc converters operating at fixed frequency was presented. A new model of a power stage leading to a system matrix independent of the load resistance is introduced. This model is suitable for designing the closed-loop system. Two PWM push-pull converters with a switched load were analyzed: a conventional converter and a bidirectional power-flow converter. It was found that the control scheme proposed for the application in buck-derived converters is very promising. The simulation results revealed that the bidirectional power-flow converter had much better dynamic performance than the conventional one because of a smaller inductance. Specifically, the settling time was reduced by a factor of 10, the variation of the output voltage during the transition were lowered, and both inductor and filter capacitor were made smaller, at the expense of higher ripple current through the inductor and a control circuit capable of driving two additional transistors. Further investigations should be focused on an experimental verification and the application of the integral control for other dc-dc converters.

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