

# **Antenna Engineering**

## **EC 544**

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## Types of Uniform Linear Array According to the Direction of Main Maximum

1. Broadside

$$(\theta_m = 90^\circ)$$

2. Ordinary End-Fire

$$(\theta_m = 0^\circ, 180^\circ)$$

3. Phased (Scanning)

$$(0^\circ \leq \theta_m \leq 180^\circ)$$

4. Hansen-Woodyard End-fire

$$(\theta_m = 0^\circ, 180^\circ)$$

## THE ARRAY FACTOR OF BROADSIDE N – ELEMENT LINEAR UNIFORM ARRAY

- It is desirable in many applications to have the maximum radiation of an array directed normal to the axis of the array (broadside,  $\theta_m = 90^\circ$ ).
- To optimize the design, the maxima of the single element and of the array factor should both be directed toward  $\theta_m = 90^\circ$ . The requirements of the single elements can be accomplished by the judicious choice of the radiators and those of the array factor by the proper separation and excitation of the individual radiators.

Referring to the array factor, the first maximum of the array factor occurs when:  $m = 0$ , So

$$\psi \Big|_{\theta=90^\circ} = (kd \cos \theta + \beta) \Big|_{\theta=90^\circ} = 0 + \beta = 0$$

$$\boxed{\beta = 0}$$

## THE ARRAY FACTOR of BROADSIDE N – ELEMENT LINEAR UNIFORM ARRAY

To ensure that there are no principle maxima in other directions, which are referred to as grating lobes, the separation between the elements should not be equal to multiples of a wavelength ( $d \neq n\lambda$ ,  $n = 1, 2, 3, \dots$ ) when  $\beta = 0$ .

Avoid  $d = n\lambda$  because

$$\psi \Big|_{\substack{\beta=0 \\ d=n\lambda}} = (kd \cos \theta + \beta)_{\substack{\beta=0 \\ d=n\lambda}} = 2\pi n \cos \theta$$

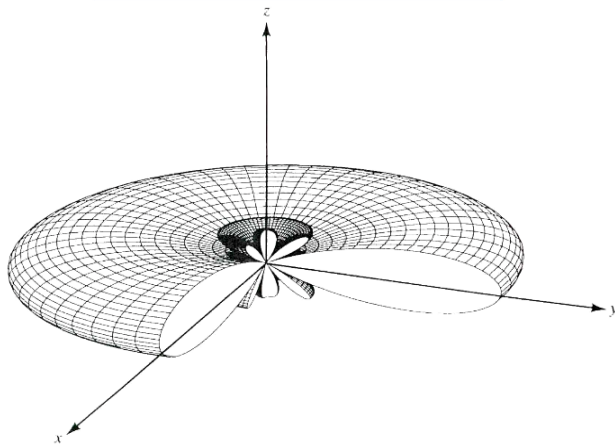
$$\psi = 2\pi n \cos \theta \Big|_{\theta=0^\circ, 180^\circ} = \pm 2\pi n$$

This value of  $\psi$  when substituted in the array factor makes the array factor attain its maximum value. Thus for a uniform array with  $\beta = 0$  and  $d = n\lambda$ , in addition to having the maxima of AF directed broadside ( $\theta_m = 90^\circ$ ) to the axis of the array, there are additional maxima.

# THE ARRAY FACTOR of BROADSIDE N – ELEMENT LINEAR UNIFORM ARRAY

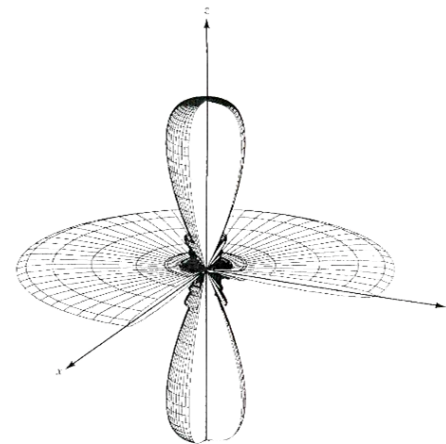
For  $d = \lambda$ , the additional maxima are directed along the axis ( $\theta_g = 0^\circ, 180^\circ$ ) of the array, since  $\cos \theta_g = +1$  and  $-1$ .

Broadside ( $d = \lambda/4$ )  $N = 10$



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Broadside/End-Fire ( $d = \lambda$ ,  $N = 10$ )



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No Grating Lobes

$$d < \lambda$$

Grating Lobes

$$d \geq \lambda$$

## NULLS OF THE ARRAY FACTOR of BROADSIDE N – ELEMENT LINEAR UNIFORM ARRAY

$$\sin\left(\frac{N\psi}{2}\right) = 0$$

$$\frac{N\psi}{2} = \sin^{-1}(0) = \pm n\pi, \quad n = 1, 2, \dots$$

$$n \neq 0, N, 2N, \dots$$

$$\frac{N}{2}(\text{kd}\cos\theta_n + \beta) = \pm n\pi$$

$$\theta_n = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2n}{N}\pi\right)\right]$$

$$\boxed{\beta = 0}$$

$$\theta_n = \cos^{-1}\left[\pm \frac{n}{N} \frac{\lambda}{d}\right]$$

$$n = 1, 2, \dots$$

$$n \neq 0, N, 2N, \dots$$

## MAXIMA (PRINCIPLE) OF THE ARRAY FACTOR of BROADSIDE N – ELEMENT LINEAR UNIFORM ARRAY

$$\sin\left(\frac{\psi}{2}\right) = 0 \Rightarrow \frac{\psi}{2} = \sin^{-1}(0) = \pm m\pi$$
$$m = 0, 1, 2, \dots$$

$$\psi = \pm 2m\pi = kd \cos \theta_m + \beta$$

$$\theta_m = \cos^{-1} \left[ \frac{\lambda}{2\pi d} (-\beta \pm 2m\pi) \right], \quad m = 0, 1, 2$$

$$\boxed{\beta = 0}$$

$$\theta_m = \cos^{-1} \left( \pm \frac{m\lambda}{d} \right)$$
$$m = 0, 1, 2$$

## HALF POWER (3 – dB) POINT OF THE ARRAY FACTOR of BROADSIDE N – ELEMENT LINEAR UNIFORM ARRAY

$$AF \approx \frac{\sin\left(\frac{N}{2}\psi\right)}{\frac{N}{2}\psi} = 0.707 \Rightarrow \frac{N\psi}{2} = \pm 1.391$$

$$\frac{N}{2}\psi = \frac{N}{2}(kd \cos \theta_h + \beta) = \pm 1.391$$

$$\theta_h \approx \cos^{-1} \left[ \frac{\lambda}{2\pi d} \left( -\beta \pm \frac{2.782}{N} \right) \right]$$

$$\boxed{\beta = 0}$$

$$\theta_h \approx \cos^{-1} \left( \pm \frac{1.391\lambda}{\pi N d} \right)$$

$$\pi d/\lambda \ll 1$$



## SECONDARY MAXIMA OF THE ARRAY FACTOR of BROADSIDE N – ELEMENT LINEAR UNIFORM ARRAY

$$\frac{N}{2}\psi \simeq \pm \left( \frac{2s+1}{2} \right) \pi, \quad s = 1, 2, 3, \dots$$

$$\frac{N}{2}(kd \cos \theta_s + \beta) = \pm \left( \frac{2s+1}{2} \right) \pi$$

$$\theta_s \simeq \cos^{-1} \left\{ \frac{\lambda}{2\pi d} \left[ -\beta \pm \left( \frac{2s+1}{N} \right) \pi \right] \right\}$$

$$\boxed{\beta = 0}$$

$$\theta_s \simeq \cos^{-1} \left\{ \pm \frac{\lambda}{2d} \left( \frac{2s+1}{N} \right) \right\} \quad s = 1, 2, 3, \dots$$

$\pi d/\lambda \ll 1$

# Beam widths for Uniform Amplitude Broadside Arrays

First Null Beam width (FNBW)

$$\Theta_n = 2 \left[ \frac{\pi}{2} - \cos^{-1} \left( \frac{\lambda}{Nd} \right) \right]$$

Half Power Beam width (HPBW)

$$\Theta_h \approx 2 \left[ \frac{\pi}{2} - \cos^{-1} \left( \frac{1.391\lambda}{\pi Nd} \right) \right]$$

$\pi d/\lambda \ll 1$

First Side Lobe Beam width (FSLBW)

$$\Theta_s \approx 2 \left[ \frac{\pi}{2} - \cos^{-1} \left( \frac{3\lambda}{2dN} \right) \right]$$

$\pi d/\lambda \ll 1$

## THE ARRAY FACTOR OF ORDINARY END - FIRE N – ELEMENT LINEAR UNIFORM ARRAY

Instead of having the maximum radiation broadside to the axis of the array, it may be desirable to direct it along the axis of the array (end-fire). As a matter of fact, it may be necessary that it radiates toward only one direction (either  $\theta_m = 0^\circ$  or  $\theta_m = 180^\circ$ )

### A. Maximum Toward $\theta_m = 0^\circ$

$$\psi \Big|_{\theta=0^\circ} = (kd \cos \theta + \beta) \Big|_{\theta=0^\circ} = kd + \beta = 0$$

$$\boxed{\beta = -kd}$$

### B. Maximum Toward $\theta_m = 180^\circ$

$$\psi = (kd \cos \theta + \beta) \Big|_{\theta=180^\circ} = -kd + \beta = 0$$

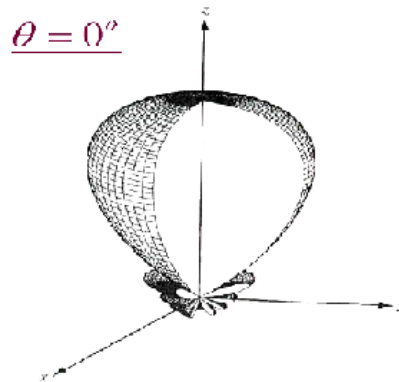
$$\boxed{\beta = +kd}$$

## THE ARRAY FACTOR OF ORDINARY END - FIRE N – ELEMENT LINEAR UNIFORM ARRAY

$$\underline{N = 10, d = \lambda / 4}$$

$$1. \beta = -kd = -\frac{2\pi}{\lambda} \left( \frac{\lambda}{4} \right) = -\frac{\pi}{2}$$

Max Toward  $\theta = 0^\circ$



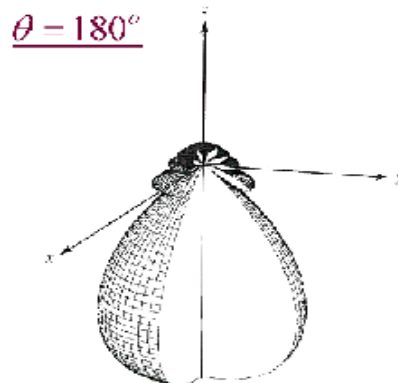
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## THE ARRAY FACTOR OF ORDINARY END - FIRE N – ELEMENT LINEAR UNIFORM ARRAY

$$\underline{N = 10, d = \lambda / 4}$$

$$2. \beta = +kd = +\frac{2\pi}{\lambda} \left( \frac{\lambda}{4} \right) = +\frac{\pi}{2}$$

Max Toward  $\theta = 180^\circ$



## NULLS OF THE ARRAY FACTOR OF ORDINARY END - FIRE N – ELEMENT LINEAR UNIFORM ARRAY

$$\sin \left( \frac{N \psi}{2} \right) = 0$$

$$\frac{N \psi}{2} = \sin^{-1}(0) = \pm n\pi, \quad n = 1, 2, \dots$$

$$n \neq 0, N, 2N, \dots$$

$$\frac{N}{2} (kd \cos \theta_n + \beta) = \pm n\pi$$

$$\theta_n = \cos^{-1} \left[ \frac{\lambda}{2\pi d} \left( -\beta \pm \frac{2n}{N} \pi \right) \right]$$

Maximum Toward  $\theta=0^\circ$

$$\beta = -kd$$

$$\theta_n = \cos^{-1} \left[ 1 - \frac{n\lambda}{Nd} \right] \quad n = 1, 2, \dots$$

$$n \neq 0, N, 2N, \dots$$

Maximum Toward  $\theta=180^\circ$

$$\beta = +kd$$

$$\theta_n = \cos^{-1} \left[ -1 + \frac{n\lambda}{Nd} \right] \quad n = 1, 2, \dots$$

$$n \neq 0, N, 2N, \dots$$

## MAXIMA (PRINCIPLE) OF THE ARRAY FACTOR OF ORDINARY END - FIRE N – ELEMENT LINEAR UNIFORM ARRAY

$$\sin\left(\frac{\psi}{2}\right) = 0 \Rightarrow \frac{\psi}{2} = \sin^{-1}(0) = \pm m\pi$$

$m = 0, 1, 2, \dots$

$$\psi = \pm 2m\pi = kd \cos \theta_m + \beta$$

$$\theta_m = \cos^{-1} \left[ \frac{\lambda}{2\pi d} (-\beta \pm 2m\pi) \right], \quad m = 0, 1, 2$$

Maximum Toward  $\theta=0^\circ$

$$\boxed{\beta = -kd}$$

$$\theta_m = \cos^{-1} \left( 1 - \frac{m\lambda}{d} \right) \quad m = 0, 1, 2$$

Maximum Toward  $\theta=180^\circ$

$$\boxed{\beta = +kd}$$

$$\theta_m = \cos^{-1} \left( -1 + \frac{m\lambda}{d} \right) \quad m = 0, 1, 2$$

## THE ARRAY FACTOR OF ORDINARY END - FIRE N – ELEMENT LINEAR UNIFORM ARRAY

1.  $d < \lambda/2$ : End-fire only in one direction  
( $\theta = 0^\circ$  or  $180^\circ$ )
2.  $d = \lambda/2$ : End-fire in both directions  
( $\theta = 0^\circ$  &  $180^\circ$ )
3.  $\lambda/2 < d < \lambda$ : End-fire in one direction  
( $\theta = 0^\circ$  or  $180^\circ$ ) and maximum  
toward two other directions
4.  $d = \lambda = n\lambda$ : End-fire in both directions  
( $\theta = 0^\circ$  &  $180^\circ$ ) and broadside



## HALF POWER (3 -dB) POINT OF THE ARRAY FACTOR OF ORDINARY END - FIRE N – ELEMENT LINEAR UNIFORM ARRAY

$$AF \approx \frac{\sin\left(\frac{N}{2}\psi\right)}{\frac{N}{2}\psi} = 0.707 \Rightarrow \frac{N\psi}{2} = \pm 1.391$$

$$\frac{N}{2}\psi = \frac{N}{2}(kd \cos \theta_h + \beta) = \pm 1.391$$

$$\theta_h \approx \cos \left[ \frac{\lambda}{2\pi d} \left( -\beta \pm \frac{2.782}{N} \right) \right]$$

Maximum Toward  $\theta=0^\circ$

$$\boxed{\beta = -kd}$$

$$\theta_h \approx \cos^{-1} \left( 1 - \frac{1.391\lambda}{\pi N d} \right) \quad \pi d/\lambda \ll 1$$

Maximum Toward  $\theta=180^\circ$

$$\boxed{\beta = +kd}$$

$$\theta_h \approx \cos^{-1} \left( -1 + \frac{1.391\lambda}{\pi N d} \right) \quad \pi d/\lambda \ll 1$$

## SECONDARY MAXIMA OF THE ARRAY FACTOR OF ORDINARY END - FIRE N – ELEMENT LINEAR UNIFORM ARRAY

$$\frac{N}{2}\psi \simeq \pm \left( \frac{2s+1}{2} \right) \pi, \quad s = 1, 2, 3, \dots$$

$$\frac{N}{2}(kd \cos \theta_s + \beta) = \pm \left( \frac{2s+1}{2} \right) \pi$$

$$\theta_s \simeq \cos^{-1} \left\{ \frac{\lambda}{2\pi d} \left[ -\beta \pm \left( \frac{2s+1}{N} \right) \pi \right] \right\}$$

Maximum Toward  $\theta=0^\circ$

$$\beta = -kd$$

$$\theta_s \simeq \cos^{-1} \left\{ 1 - \frac{\lambda}{2d} \left( \frac{2s+1}{N} \right) \right\} \quad s = 1, 2, 3, \dots$$

$\pi d/\lambda \ll 1$

Maximum Toward  $\theta=180^\circ$

$$\beta = +kd$$

$$\theta_s \simeq \cos^{-1} \left\{ -1 + \frac{\lambda}{2d} \left( \frac{2s+1}{N} \right) \right\} \quad s = 1, 2, 3, \dots$$

$\pi d/\lambda \ll 1$

## Beam-widths for Uniform Amplitude End-Fire Arrays

First Null Beam-width (FNBW)

$$\Theta_n = 2 \cos^{-1} \left( 1 - \frac{\lambda}{Nd} \right)$$

Half Power Beam-width (HPBW)

$$\Theta_h \simeq 2 \cos^{-1} \left( 1 - \frac{1.391\lambda}{\pi dN} \right)$$
$$\pi d/\lambda \ll 1$$

First Side Lobe Beam-width (FSLBW)

$$\Theta_s \simeq 2 \cos^{-1} \left( 1 - \frac{3\lambda}{2Nd} \right)$$
$$\pi d/\lambda \ll 1$$

## THE ARRAY FACTOR OF SCANNING PHASE N – ELEMENT LINEAR UNIFORM ARRAY

If the maximum radiation of the array is required to be oriented at an angle  $\theta_o$  ( $0^\circ \leq \theta_o \leq 180^\circ$ ). To accomplish this, the phase excitation  $\beta$  between the elements must be adjusted so that

$$\left. \psi \right|_{\theta=\theta_o} = \left. (kd \cos \theta + \beta) \right|_{\theta=\theta_o} = kd \cos \theta_o + \beta = 0$$

$$\beta = -kd \cos \theta_o$$

Thus by controlling the progressive phase difference between the elements, the maximum radiation can be squinted in any desired direction to form a scanning array. Since in phased array technology the scanning must be continuous, the system should be capable of continuously varying the progressive phase between elements.

## THE HALF POWER BEAM-WIDTH OF THE ARRAY FACTOR OF SCANNING PHASE N – ELEMENT LINEAR UNIFORM ARRAY

$$AF = \frac{\sin\left(\frac{N}{2}\psi\right)}{\frac{N}{2}\psi} = 0.707 \Rightarrow \frac{N\psi}{2} = \pm 1.391$$

$$\frac{N}{2}\psi = \frac{N}{2}(kd \cos \theta_o + \beta) = \pm 1.391$$

$$\theta_o = \cos^{-1} \left[ \frac{\lambda}{2\pi d} \left( kd \cos \theta_o \pm \frac{2.782}{N} \right) \right]$$

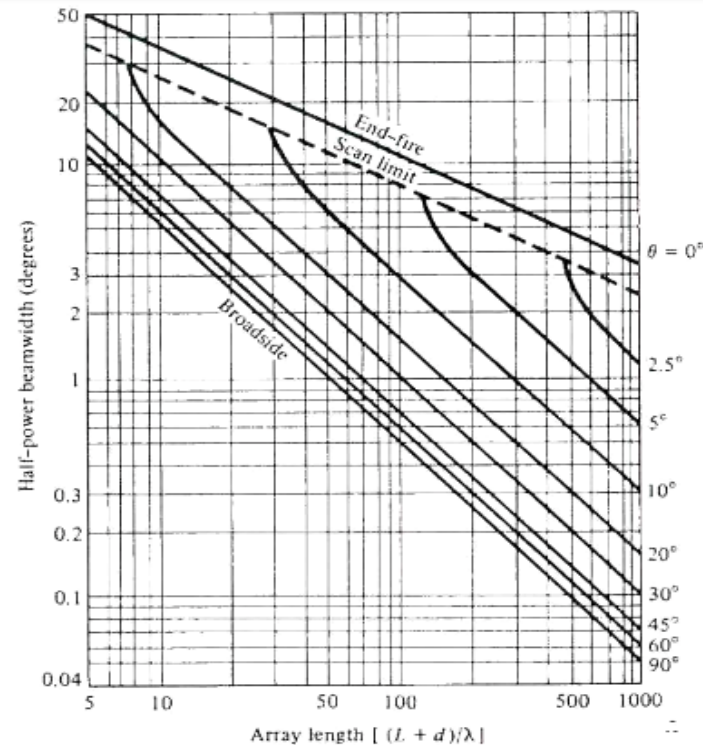
### Half-Power Beamwidth

$$\Theta_h = \cos^{-1} \left[ \cos \theta_o - 0.443 \frac{\lambda}{(L + d)} \right] \\ - \cos^{-1} \left[ \cos \theta_o + 0.443 \frac{\lambda}{(L + d)} \right]$$

Not Valid For End-Fire Arrays

$$(\theta_o \neq 0^\circ, 180^\circ)$$

# HALF-POWER BEAM-WIDTH FOR BROADSIDE, ORDINARY END-FIRE AND SCANNING N – ELEMENT LINEAR UNIFORM ARRAY



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## THE ARRAY FACTOR OF HANSEN-WOODYARD END-FIRE N – ELEMENT LINEAR UNIFORM ARRAY

To enhance the directivity of an end –fire array without destroying any of the other characteristics, Hansen and Woodyard proposed that the required phase shift between closely spaced elements of a very long array should be

### A. Maximum Toward $\theta = 0^\circ$

$$\beta = - \left( kd + \frac{2.94}{N} \right) \cong - \left( kd + \frac{\pi}{N} \right)$$

$$d = \left( \frac{N - 1}{N} \right) \frac{\lambda}{4}$$

### B. Maximum Toward $\theta = 180^\circ$

$$\beta = + \left( kd + \frac{2.94}{N} \right) \cong + \left( kd + \frac{\pi}{N} \right)$$

$$d = \left( \frac{N - 1}{N} \right) \frac{\lambda}{4}$$

## THE ARRAY FACTOR OF HANSEN-WOODYARD END-FIRE N – ELEMENT LINEAR UNIFORM ARRAY

To realize the increase in directivity as a result of the Hansen-Woodyard conditions on  $\beta$ ,  $\psi$  assumes values of

$$\text{Maximum Toward } \theta = 0^\circ \left[ \beta = -(kd + \frac{\pi}{N}) \right]$$

$$1. \quad |\psi|_{\theta=0} = \frac{\pi}{N}$$

$$2. \quad |\psi|_{\theta=180^\circ} \cong \pi$$

$$1. \quad |\psi|_{\theta=0} = |kd \cos \theta + \beta|_{\theta=0} = \left| kd - kd - \frac{\pi}{N} \right| = \frac{\pi}{N}$$

$$2. \quad |\psi|_{\theta=180^\circ} = |kd \cos \theta + \beta|_{\theta=180^\circ} = |-kd - kd - \frac{\pi}{N}| \cong \pi$$

$$2kd + \frac{\pi}{N} \cong \pi$$

$$d \cong \frac{\lambda}{4} \left( \frac{N-1}{N} \right)^{N \rightarrow \text{large}} \cong \frac{\lambda}{4}$$



## THE ARRAY FACTOR OF HANSEN-WOODYARD END-FIRE N – ELEMENT LINEAR UNIFORM ARRAY

Maximum Toward  $\theta = 180^\circ$   $\left[ \beta = +(kd + \frac{\pi}{N}) \right]$

$$1. |\psi|_{\theta=180^\circ} = \frac{\pi}{N}$$

$$2. |\psi|_{\theta=0^\circ} \cong \pi$$

$$1. |\psi|_{\theta=180^\circ} = |kd \cos \theta + \beta|_{\theta=180^\circ} = \left| -kd + kd + \frac{\pi}{N} \right| = \frac{\pi}{N}$$

$$2. |\psi|_{\theta=0^\circ} = |kd \cos \theta + \beta|_{\theta=0^\circ} = \left| kd + kd + \frac{\pi}{N} \right| \cong \pi$$

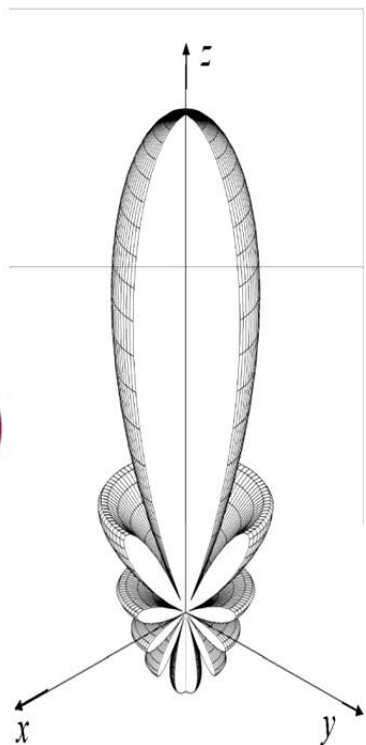
$$2kd + \frac{\pi}{N} \cong \pi$$

$$d \cong \frac{\lambda}{4} \left( \frac{N-1}{4} \right)^{N \rightarrow \text{large}} \cong \frac{\lambda}{4}$$

## Hansen-Woodyard End-Fire Array

$$N = 10$$

$$\beta = -(kd + \pi/N)$$

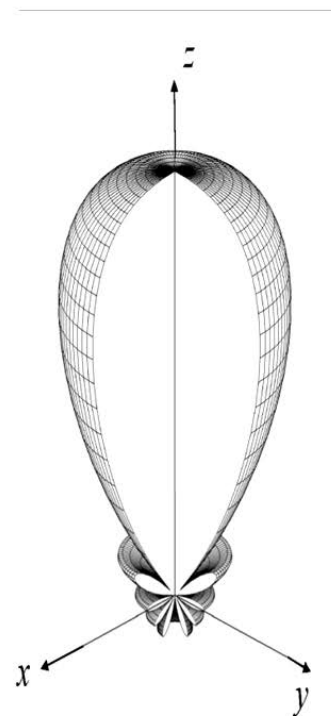


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## Ordinary End-Fire Array

$$N = 10$$

$$\beta = -kd$$



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## NULLS OF THE ARRAY FACTOR OF HANSEN-WOODYARD END-FIRE N – ELEMENT LINEAR UNIFORM ARRAY

$$\sin \left( \frac{N\psi}{2} \right) = 0$$

$$\frac{N\psi}{2} = \sin^{-1}(0) = \pm n\pi, \quad n = 1, 2, \dots$$

$$n \neq 0, N, 2N, \dots$$

$$\frac{N}{2}(kd \cos \theta_n + \beta) = \pm n\pi$$

$$\theta_n = \cos^{-1} \left[ \frac{\lambda}{2\pi d} \left( -\beta \pm \frac{2n}{N} \pi \right) \right]$$

Maximum Toward  $\theta=0^\circ$

$$\beta = - \left( kd + \frac{2.94}{N} \right) \cong - \left( kd + \frac{\pi}{N} \right)$$

$$\theta_n = \cos^{-1} \left[ 1 + (1-2n) \frac{\lambda}{2Nd} \right] \quad \begin{array}{l} n = 1, 2, \dots \\ n \neq 0, N, 2N, \dots \end{array}$$

Maximum Toward  $\theta=180^\circ$

$$\beta = + \left( kd + \frac{2.94}{N} \right) \cong + \left( kd + \frac{\pi}{N} \right)$$

$$\theta_n = \cos^{-1} \left[ -1 + (-1+2n) \frac{\lambda}{2Nd} \right] \quad \begin{array}{l} n = 1, 2, \dots \\ n \neq 0, N, 2N, \dots \end{array}$$

# MAXIMA (PRINCIPLE) OF THE ARRAY FACTOR OF HANSEN-WOODYARD END-FIRE N – ELEMENT LINEAR UNIFORM ARRAY

$$\sin\left(\frac{\psi}{2}\right) = 0 \Rightarrow \frac{\psi}{2} = \sin^{-1}(0) = \pm m\pi$$

$$m = 0, 1, 2, \dots$$

$$\psi = \pm 2m\pi = kd \cos \theta_m + \beta$$

$$\theta_m = \cos^{-1} \left[ \frac{\lambda}{2\pi d} (-\beta \pm 2m\pi) \right], \quad m = 0, 1, 2$$

Maximum Toward  $\theta = 0^\circ$

$$\beta = - \left( kd + \frac{2.94}{N} \right) \cong - \left( kd + \frac{\pi}{N} \right)$$

$$\theta_m = \cos^{-1} \left[ 1 + \frac{\lambda}{2Nd} - \frac{m\lambda}{d} \right]$$

Maximum Toward  $\theta = 180^\circ$

$$\beta = - \left( kd + \frac{2.94}{N} \right) \cong - \left( kd + \frac{\pi}{N} \right)$$

$$\theta_m = \cos^{-1} \left[ -1 - \frac{\lambda}{2Nd} + \frac{m\lambda}{d} \right], \quad m = 0, 1, 2$$

$$\pi d / \lambda \ll 1$$

HALF-POWER  
POINTS

$$\theta_h = \cos^{-1} \left( 1 - 0.1398 \frac{\lambda}{Nd} \right)$$

$\pi d/\lambda \ll 1$   
 $N$  large

MINOR LOBE  
MAXIMA

$$\theta_s = \cos^{-1} \left( 1 - \frac{s\lambda}{Nd} \right)$$

$s = 1, 2, 3, \dots$   
 $\pi d/\lambda \ll 1$

# Beamwidths for Uniform Amplitude Hansen-Woodyard End-Fire Arrays

**First Null Beamwidth (FNBW)**

$$\Theta_n = 2\cos^{-1}\left(1 - \frac{\lambda}{2dN}\right)$$

**Half Power Beamwidth(HPBW)**

$$\Theta_h = 2\cos^{-1}\left(1 - 0.1398 \frac{\lambda}{Nd}\right)$$

$\pi d/\lambda \ll 1$   
 $N$  large

**First Side Lobe Beamwidth(FSLBW)**

$$\Theta_s = 2\cos^{-1}\left(1 - \frac{\lambda}{Nd}\right)$$

$\pi d/\lambda \ll 1$

For Linear Array along x - axis

$$\psi_x = kd \sin \theta \cos \phi + \beta$$

For Linear Array along y - axis

$$\psi_y = kd \sin \theta \sin \phi + \beta$$

For Linear Array along z - axis

$$\psi_z = kd \cos \theta + \beta$$